Discrete Event Systems

Course code: SSY165

Examination 2020-10-24

Time: 8:30-12:30, Location: Zoom

Teacher: Bengt Lennartson, phone 3722 and 0730-79 42 26

The examination includes 25 points, where grade three requires 10 points, grade four 15 points and grade five 20 points.

The result of this examination is announced and inspection of the grading is done on Tuesday *November 10* and Wednesday *November 11*, 12:30-13:00 at the division.

- All aids are allowed at the examination except collaboration with other students.
- Upload your solution in Canvas as an Assignment and only as **one document** latest 13:00.

Good luck!

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1

Prove the following predicate implication

$$\forall x[P(x)] \vee \forall x[Q(x)] \Rightarrow \forall x[P(x) \vee Q(x)]$$

and give an intuitive explanation of this implication. (4 p)

2

A PIN (Personal Identification Number) code is an n-digit number (often $n = 4$) that is meant to identify a person or a group of persons. For instance, entrances to a building may be locked and can only be unlocked by punching a given PIN code on a numeric terminal. In that case, the door unlocks if the specific sequence of numbers is entered, irrespective of the numbers that have been punched before that sequence.

Let the event label a_k represent the entering of number k, where $0 \leq k \leq 9$, and formulate an automaton that models such a PIN code reader that accepts the PIN code 7742. The automaton shall reach the desired marked state when this sequence of numbers has been entered. (4 p)

3

Consider the following predicate

$$\begin{aligned}
&(x' = x + 1 \wedge y' = y - 1 \wedge y > 0 \wedge e = a) \vee \\
&(x' = x - 1 \wedge x > 0 \wedge y' = y + 1 \wedge e = b) \vee \\
&(z' = z + 1 \wedge y' = y - 1 \wedge y > 0 \wedge e = c) \vee \\
&(z' = z - 1 \wedge z > 0 \wedge y' = y + 1 \wedge e = d)
\end{aligned}$$

where the variable $e \in \{0, a, b, c, d\}$ represents the possible events that may occur for this system, where $e = 0$ when no event occurs. The prime symbol on a variable, say x', denotes the next value of this variable, and the initial value of the variables are $x = 0, y = 3$, and $z = 0$.

a) Formulate a Petri net that represents this predicate. (2 p)

b) Generate an automaton that models the same behavior as the predicate. (2 p)

c) What type of system does this predicate represent? (1 p)

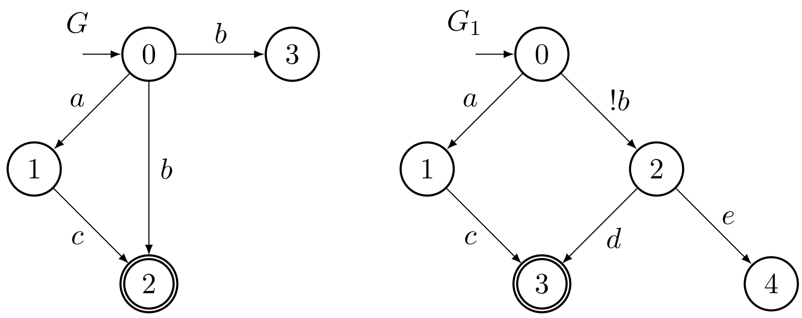
4

The automaton G that includes a nondeterministic choice can be interpreted as a controlled system that communicates with an environment. The controller then selects the next event (action), while the next (target) state is chosen by the environment. This phenomenon can alternatively be modeled as a deterministic system where uncontrollable events are involved.

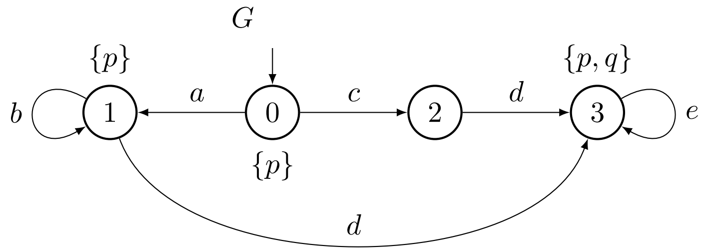
a) Explain why the proposed automaton G_1, where the event b is uncontrollable, does not model the same behavior as G in terms of the ability to reach the marked state. (2 p)

b) Suggest an alternative deterministic automaton which generates a behavior that is equivalent to G in terms of the ability to reach the marked state. As in G_1, it is allowed to add both new states and new events. (1 p)

c) Generate a supervisor that makes G nonblocking. (1 p)



5

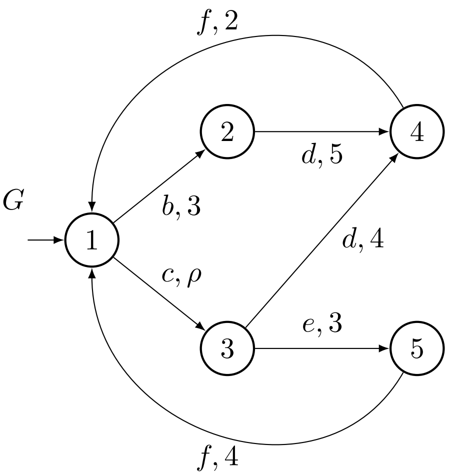


a) Determine by μ-calculus for which states the following CTL formulas are valid: $\varphi = \exists\Box p$, $\varphi = \forall\Box p$, $\varphi = \exists\Diamond q$, and $\varphi = \forall\Box\exists\Diamond q$. (3 p)

b) Which CTL formulas φ are valid in G, i.e. for which φ does $G \models \varphi$. (1 p)

6

Consider the automaton G, including immediate rewards $\rho(x, a)$ on each transition from state x with action a.



Determine based on dynamic programming and the Q-function $Q(x, a)$ for which values of the immediate reward $\rho(1, c)$ where action c is the optimal choice that maximizes the total reward

$$\sum_{k=0}^{\infty} \gamma^k \rho(x_k, a_k)$$

for the discounting factor $\gamma = 0.9$. (4 p)

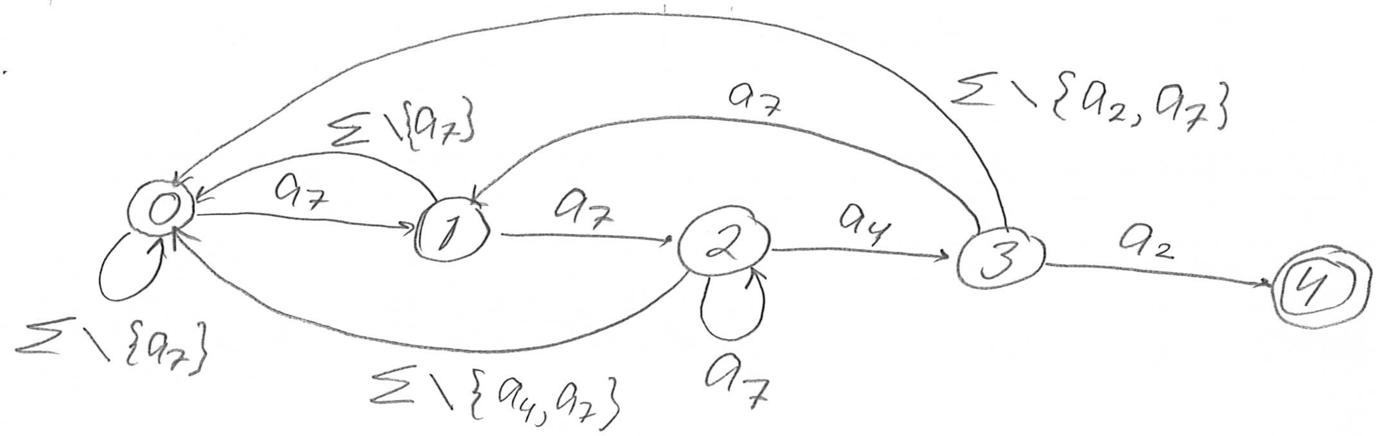
BL

Solution to DES exam 201024

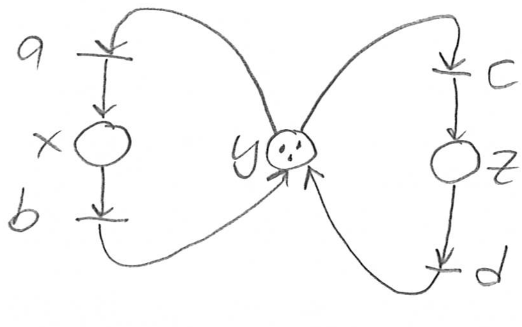
1. $$\forall x[P(x)] \vee \forall x[Q(x)] \Leftrightarrow (P(x_1)\wedge ..\wedge P(x_n)) \vee (Q(x_1)\wedge \ldots \wedge Q(x_n)) \Leftrightarrow (P(x_1)\vee Q(x_1))\wedge (P(x_1)\vee Q(x_2)) \wedge \ldots \wedge (P(x_2)\vee Q(x_2)) \wedge \ldots \wedge (P(x_n)\vee Q(x_n)) \underset{\mathcal{I}_1}{\Rightarrow} (P(x_1)\vee Q(x_1))\wedge \ldots \wedge (P(x_n)\vee Q(x_n)) \Leftrightarrow \forall x[P(x)\vee Q(x)]$$

$(\mathcal{I}_1: \; p\wedge q \Rightarrow p)$

2.

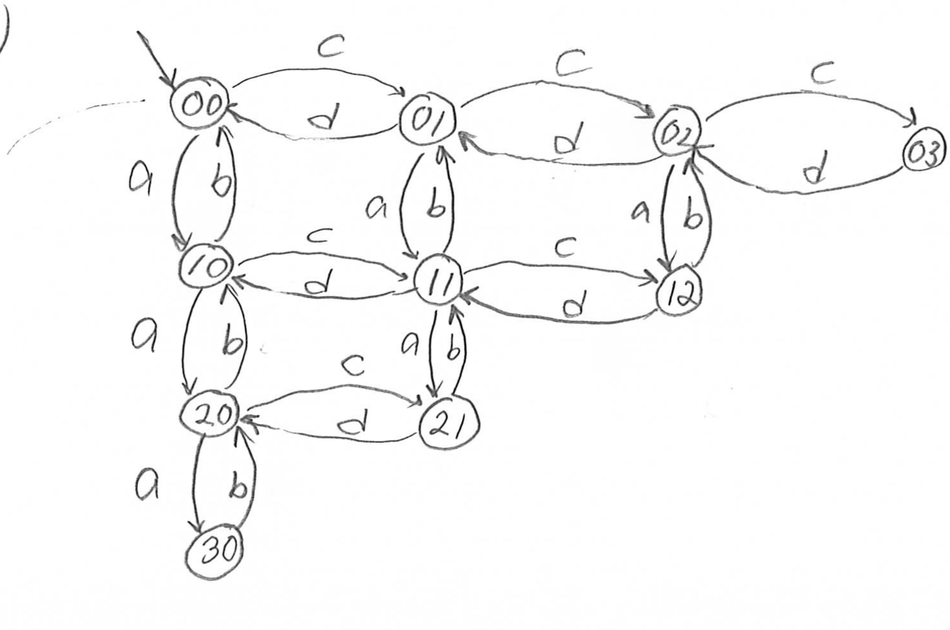


3. a)



c) Two tasks have a shared resource with capacity three.

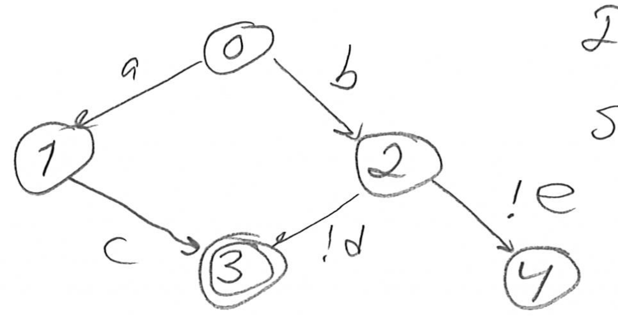
b)



state name: xz

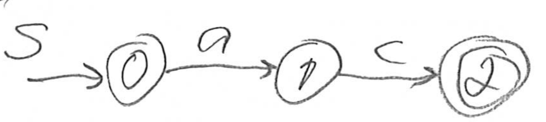
4 a) In G_1 the controller can not force the system to pass state 1, since there is an uncontrollable transition from state 0. In G the controller can decide to execute event a, which means that state 1 will be passed. If, on the other hand, event b is taken in state 0 in G, the plant can decide to move to the blocked state 3, while in G_1 the transition to the blocking state is controllable and blocking can therefore be avoided in state 2.

b)



If a is selected, state 1 is passed, while event b means that the plant can select e resulting in the blocking state 4. ∴ The same behavior as G.

c)



$G \| S = S$

5. a) $[\![\exists\Box p]\!] = [\![\nu y.\, p \wedge \exists \bigcirc y]\!] = \nu Y.\, \Psi(Y)$

$\Psi(Y) = [\![p]\!] \cap Pre^{\exists}(Y)$ $\quad Y_0 = \Sigma = \{0,1,2,3\}$

$Y_1 = \{0,1,3\} \cap Pre^{\exists}(\Sigma) = \{0,1,3\}$

$Y_2 = \{0,1,3\} \cap Pre^{\exists}(\{0,1,3\}) = \{0,1,3\} \cap \{0,1,2,3\} = Y_1 = Y^{\omega}$

$\underline{\underline{[\![\exists\Box p]\!] = \{0,1,3\}}}$

$[\![\forall\Box p]\!] = \nu Y.\, \Psi(Y)$ $\quad \Psi(Y) = [\![p]\!] \cap Pre^{\forall}(Y)$, $Y_0 = \Sigma$

$Y_1 = \{0,1,3\} \cap Pre^{\forall}(\Sigma) = \{0,1,3\}$

$Y_2 = \{0,1,3\} \cap Pre^{\forall}(\{0,1,3\}) = \{1,3\}$ since not all target states in state 0 are included in Y_1 (state 2 is missing)

$Y_3 = Y_2 = Y^{\omega} \Rightarrow \underline{\underline{[\![\forall\Box p]\!] = \{1,3\}}}$

$\underline{\underline{[\![\exists\Diamond q]\!]}} = \mu Y.\, \Psi(Y)$ $\quad \Psi(Y) = [\![q]\!] \cup Pre^{\exists}(Y)$, $Y_0 = \emptyset$

$Y_1 = \{3\} \cup Pre^{\exists}(\emptyset) = \{3\}$ $\quad Y_2 = \{3\} \cup Pre^{\exists}(\{3\}) = \{1,2,3\}$

$Y_3 = \{3\} \cup Pre^{\exists}(\{1,2,3\}) = \underline{\underline{\Sigma}}$ $\quad Y_4 = Y_3 = Y^{\omega} = [\![\exists\Diamond q]\!]$

$[\![\forall\Box\exists\Diamond q]\!] = [\![\forall\Box w]\!]$ where $[\![w]\!] = Y^{\omega} = \Sigma$

$[\![\forall\Box w]\!] = \nu Z.\, \Psi(Z)$ $\quad \Psi(Z) = [\![w]\!] \cap Pre^{\forall}(Z)$ $\quad Z_0 = \Sigma$

$Z_1 = \Sigma \cap Pre^{\forall}(\Sigma) = \Sigma$ $\quad \underline{\underline{[\![\forall\Box\exists\Diamond q]\!] = \{0,1,2,3\}}}$

b) $G \models \varphi$ when the initial state $0 \in [\![\varphi]\!]$

$G \models \exists\Box p$, $\quad G \not\models \forall\Box p$, $\quad G \models \exists\Diamond q$, $\quad G \models \forall\Box\exists\Diamond q$

6.

$$J^*(x) = \max_{a \in \Sigma(x)} \underbrace{[\rho(x,a) + \gamma J^*(\delta(x,a))]}_{Q(x,a)} \qquad \gamma = 0.9$$

$$J^*(2) = 5 + 0.9 J^*(4) = 5 + 0.9(2 + 0.9 J^*(1)) =$$

$$= 6.8 + 0.81 J^*(1)$$

$$J^*(3) = \max\{4 + 0.9 J^*(4), 3 + 0.9 J^*(5)\} =$$

$$= \max\{4 + 0.9(2 + 0.9 J^*(1)), 3 + 0.9(4 + 0.9 J^*(1))\}$$

$$= \max\{5.8 + 0.81 J^*(1), 6.6 + 0.81 J^*(1)\} =$$

$$= 6.6 + 0.81 J^*(1)$$

$$\mu(1) = \max_{a \in \{b,c\}} Q(1,a) = \arg\max\{\overbrace{3 + \underbrace{0.9(6.8 + 0.81 J^*(1))}_{\gamma J^*(2)}}^{\text{action } b},$$

$$\overbrace{\rho + \underbrace{0.9(6.6 + 0.81 J^*(1))}_{\gamma J^*(3)}}^{\text{action } c}\} = c \quad \text{when}$$

$$\rho + 0.9 \cdot 6.6 \geq 3 + 0.9 \cdot 6.8$$

$\therefore$ select c when $\rho \geq \frac{9.12}{5.94} = 3.18$