SSY042
2020-08-28

1a) $y_s(t) = \frac{2}{3}(1 - e^{-3t})\,u(t)$

$Y_s(s) = \mathcal{L}\{y_s(t)\} = \frac{2}{3}\cdot\left(\frac{1}{s} - \frac{1}{s+3}\right)$

Insignal $x(t) = u(t) \Rightarrow X(s) = \mathcal{L}\{x(t)\} = \frac{1}{s}$

$Y_s(s) = H(s)\cdot X(s) \Rightarrow H(s) = \frac{Y_s(s)}{X(s)} = \frac{2}{3}\cdot\left(\frac{1}{s} - \frac{1}{s+3}\right)\cdot\frac{s}{1}$

$H(s) = \frac{2}{3}\left(1 - \frac{s}{s+3}\right) = \frac{2}{3}\left(\frac{s+3-s}{s+3}\right) = \frac{2}{3}\cdot\frac{3}{s+3} = \frac{2}{s+3}$

$h(t) = \mathcal{L}^{-1}\{H(s)\} = 2e^{-3t}\cdot u(t)$

produkt, 2 fkn av t

Alternativt

$h(t) = \frac{d}{dt}\{y_s(t)\} = \frac{d}{dt}\left\{\frac{2}{3}(1 - e^{-3t})\cdot u(t)\right\} =$

$= \frac{2}{3}\left[(3e^{-3t})\cdot u(t) + \underbrace{(1 - e^{-3t})\cdot\delta(t)}_{=0}\right] =$

$= 2e^{-3t}\cdot u(t)$

1b) Lägg till Im-del till $x_r(t)$

$x(t) = Be^{\beta t}\left(\cos(\omega_0 t + \phi) + j\sin(\omega_0 t + \phi)\right) =$

$= Be^{\beta t}\cdot e^{j(\omega_0 t + \phi)} = Be^{\beta t}\cdot e^{j\omega_0 t}\cdot e^{j\phi} =$

$= Be^{j\phi}\cdot e^{(\beta + j\omega_0)t} = Ae^{\alpha t}$

$A = Be^{j\phi}$; $\alpha = \beta + j\omega_0$

1c) $y(t) = \int_{-\infty}^{\infty} x(t)\,\delta(t-1)\,dt = \int_{-\infty}^{\infty} x(1)\,\delta(t-1)\,dt =$

$= x(1)\int_{-\infty}^{\infty}\delta(t-1)\,dt = 2\left(1 - \frac{1}{3}\right)\cdot 1 = \frac{4}{3}$

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2. $i_L(t) + \frac{L}{R}\frac{d}{dt}\{i_L(t)\} = i_0(t)$

$L = 1{,}0\ H$

$R_1 = 1{,}0\ \Omega$

Laplacetransformera

$I_L(s) + \frac{L}{R} s I_L(s) = I_0(s) \qquad \frac{L}{R} = 1$

$I_L(s)(1+s) = I_0(s)$

$H(s) = \frac{I_L(s)}{I_0(s)} = \frac{1}{s+1}$

$i_0(t) = e^{-2t}u(t) \overset{\mathcal{L}}{\Rightarrow} I_0(s) = \frac{1}{s+2}$

$I_L(s) = H(s) \cdot I_0(s) = \frac{1}{s+1} \cdot \frac{1}{s+2}$

P.B.U. $\frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$

$1 = A(s+2) + B(s+1) \Rightarrow s=-1:\ 1 = A(-1+2) + 0$

$A = 1$

$s=-2:\ 1 = 0 + B(-2+1)$

$B = -1$

$I_L(s) = \frac{1}{s+1} - \frac{1}{s+2}$

$i_L(t) = \mathcal{L}^{-1}\{I_L(s)\} = \left(e^{-t} - e^{-2t}\right)u(t) \quad A$

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3. $y[n] = \left(10\left(\frac{1}{3}\right)^n - 9\left(\frac{1}{2}\right)^n\right)u[n]$ z-transf.

$$Y(z) = 10\frac{z}{z-\frac{1}{3}} - 9\frac{z}{z-\frac{1}{2}} = z\cdot\frac{10(z-\frac{1}{2}) - 9(z-\frac{1}{3})}{(z-\frac{1}{3})(z-\frac{1}{2})} =$$

$$= z\cdot\frac{10z-5-9z+3}{(z-\frac{1}{3})(z-\frac{1}{2})} = z\cdot\frac{z-2}{(z-\frac{1}{3})(z-\frac{1}{2})}$$

$$x[n] = \left(\frac{1}{6}\right)^n u[n] \overset{z}{\Rightarrow} X(z) = \frac{z}{z-\frac{1}{6}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\cancel{z}(z-2)(z-\frac{1}{6})}{(z-\frac{1}{3})(z-\frac{1}{2})\cancel{z}} = \frac{z^2-\frac{13}{6}z+\frac{1}{3}}{z^2-\frac{5}{6}z+\frac{1}{6}}$$ eller

a/ $$H(z) = \frac{1-\frac{13}{6}\cdot z^{-1}+\frac{1}{3}z^{-2}}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}} = \frac{Y(z)}{X(z)}$$

$$Y(z)\left(1-\tfrac{5}{6}\cdot z^{-1}+\tfrac{1}{6}z^{-2}\right) = X(z)\left(1-\tfrac{13}{6}z^{-1}+\tfrac{1}{3}z^{-2}\right)$$ Inv. transf.

$$y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = x[n] - \frac{13}{6}x[n-1] + \frac{1}{3}x[n-2]$$

b/ $h[n] = \mathcal{Z}^{-1}\{H(z)\}$ Polynomdivision!

$$H(z) = \frac{(z^2-\frac{5}{6}z+\frac{1}{6})+\frac{5}{6}z-\frac{1}{6}-\frac{13}{6}z+\frac{1}{3}}{z^2-\frac{5}{6}z+\frac{1}{6}} = 1 - \underbrace{\frac{\frac{8}{6}z-\frac{1}{6}}{(z-\frac{1}{3})(z-\frac{1}{2})}}_{\text{P.B.U}}$$

$$\frac{\frac{8}{6}z-\frac{1}{6}}{(z-\frac{1}{3})(z-\frac{1}{2})} = \frac{A}{z-\frac{1}{3}} + \frac{B}{z-\frac{1}{2}}$$

$$\frac{8}{6}z - \frac{1}{6} = A(z-\tfrac{1}{2}) + B(z-\tfrac{1}{3})$$

$$\begin{cases} z=\frac{1}{3};\ \frac{8}{6\cdot 3}-\frac{1}{6} = A(\frac{1}{3}-\frac{1}{2}) \Rightarrow A = -\frac{5}{3} \\ z=\frac{1}{2};\ \frac{8}{6\cdot 2}-\frac{1}{6} = B(\frac{1}{2}-\frac{1}{3}) \Rightarrow B = B = 3 \end{cases}$$

$$H(z) = 1 - \left[\left(-\frac{5}{3}\right)\frac{1}{z-\frac{1}{3}} + \frac{3}{z-\frac{1}{2}}\right]$$

$$h[n] = \mathcal{Z}^{-1}\{H(z)\} = \delta[n] + \frac{5}{3}\left(\frac{1}{3}\right)^{n-1}u[n-1] - 3\left(\frac{1}{2}\right)^{n-1}u[n-1]$$

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4.

$y[n] = x[n] + x[n-1] + x[n-2]$

z-transf.

$Y(z) = X(z) + X(z) \cdot z^{-1} + X(z) z^{-2}$

$Y(z) = X(z)\left(1 + z^{-1} + z^{-2}\right) \Rightarrow H(z) = \frac{Y(z)}{X(z)} = 1 + z^{-1} + z^{-2}$

Frekvenssvar: Låt $z = e^{j\Omega}$

$H(z)\big|_{z=e^{j\Omega}} = 1 + e^{-j\Omega} + e^{-j2\Omega} = e^{-j\Omega}\left(e^{j\Omega} + 1 + e^{-j\Omega}\right)$

$H(e^{j\Omega}) = e^{-j\Omega}\left(1 + \underbrace{e^{j\Omega} + e^{-j\Omega}}_{\text{Euler}}\right) = e^{-j\Omega}\left(1 + 2\cos\Omega\right)$

Insignal $x[n] = \cos(\Omega n)$ med $\Omega = \frac{\pi}{6}$, $\forall n$

Amplitud påverkan:

$|H(e^{j\Omega})|\big|_{\Omega=\frac{\pi}{6}} = 1 \cdot \left(1 + 2\cos\frac{\pi}{6}\right) = 1 + 2 \cdot \frac{\sqrt{3}}{2} = 1 + \sqrt{3}$

Fasbidrag:

$\arg\{H(e^{j\Omega})\}\big|_{\Omega=\frac{\pi}{6}} = \arg\{e^{-j\frac{\pi}{6}}\} = -\frac{\pi}{6}$

I stationärtillstånd

$y[n] = (1+\sqrt{3})\cos\left(\frac{\pi}{6}n - \frac{\pi}{6}\right)$

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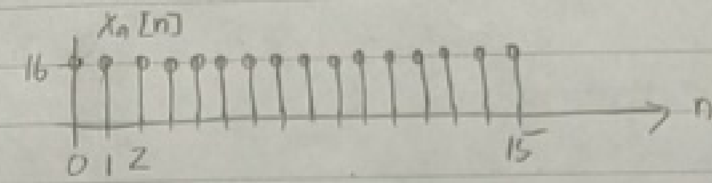
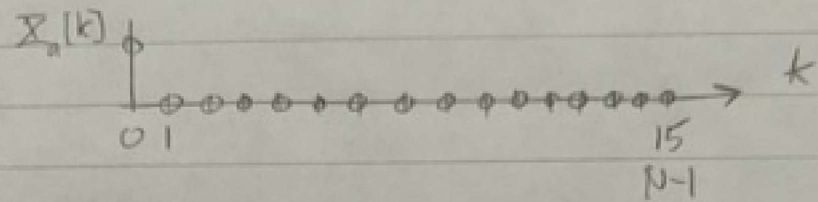
5. a/



Konstant signal $X_a[k]$ endast bidrag i $k=0$

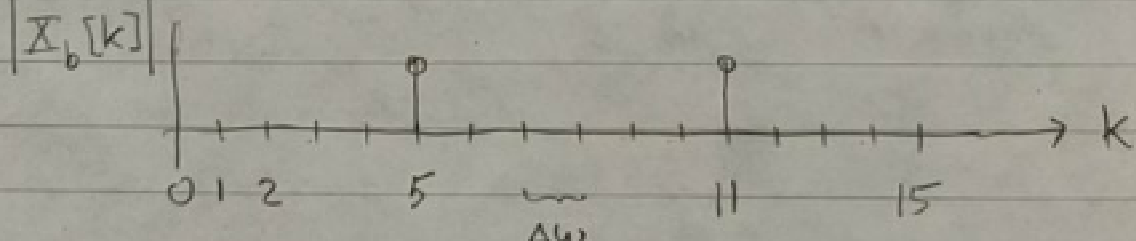


b/ $\omega_s = \frac{2\pi}{T_s} = \frac{2\pi}{3{,}125 \cdot 10^{-3}} = 2\pi \cdot 320$ rad/s

$\omega = 200\pi$ rad/s , $N = 16$

$\frac{k}{N} = \frac{\omega}{\omega_s} \Rightarrow k = \frac{\omega}{\omega_s} \cdot N = \frac{200\pi}{2\pi \cdot 320} \cdot 16 = 5$

$X_b[k]$ ger bidrag vd $k=5$ och $N-k=11$



c/ $\Delta\omega = \frac{\omega_s}{N} = \frac{2\pi \cdot 320}{16} = 2\pi \cdot 20 = 40\pi$ rad/s