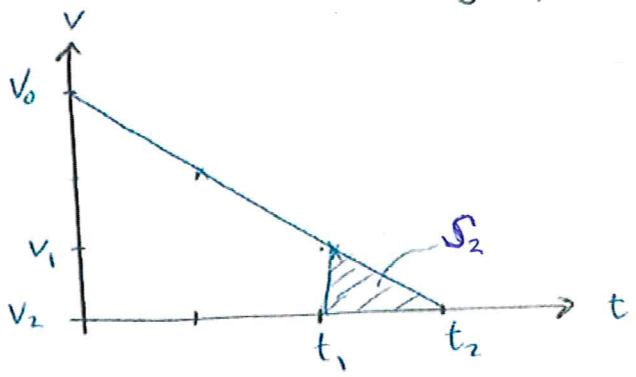
Mekanik och Termodynamik
IMS140 och IMS145
Tentamen 2024-10-26
Lösningar

1) Sökt: s_2

Givet: $v_0 = \frac{72}{3{,}6}\ \frac{m}{s}$, $t_2 = 30\,s$, $t_1 = 20\,s$

$v_2 = 0\ \frac{m}{s}$, $t_0 = 0\,s$



$$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_0}{t_2 - t_0} = -0{,}6667\ \frac{m}{s^2}$$

$$v_1 = v_0 + a t_1 = 6{,}6667\ \frac{m}{s}$$

$$s_2 = \frac{v_1 (t_2 - t_1)}{2} = 33{,}333\ m$$

Svar: $s_2 = 33\ m$

2) Sökt: $m_{låda}$

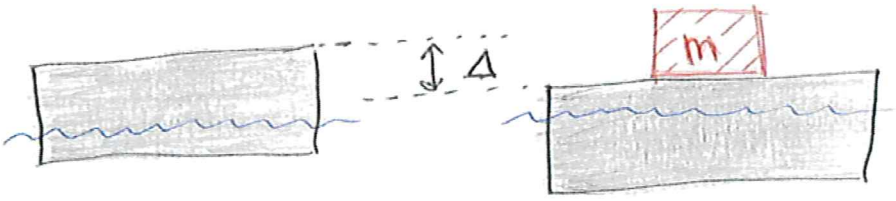
Givet: $b = 1,5\ m$, $\Delta = 0,014\ m$

$w = 2,0\ m$

$t = 0,1\ m$

$\rho_v = 998,0\ \frac{kg}{m^3}$

$\rho_{flotte} = 500,0\ \frac{kg}{m^3}$



* Arkimeds princip: $F_{flyt} = V\rho_v g$
* Tyngdkraft: $F = mg$

- Flytkraft för att bära bara lådan $F_{f,låda} = bw\Delta\rho_v g$
- Tyngdkraft från lådan på flotten $F_{mg,låda} = m_{låda} \cdot g$

$$F_{f,låda} = F_{mg,låda}$$

$$bw\Delta\rho_v g = m_{låda} g$$

$$\Rightarrow m_{låda} = bw\Delta\rho_v = 1,5 \cdot 2,0 \cdot 0,014 \cdot 998 = 41,916\ kg$$

Svar: $m_{låda} = 42\ kg$

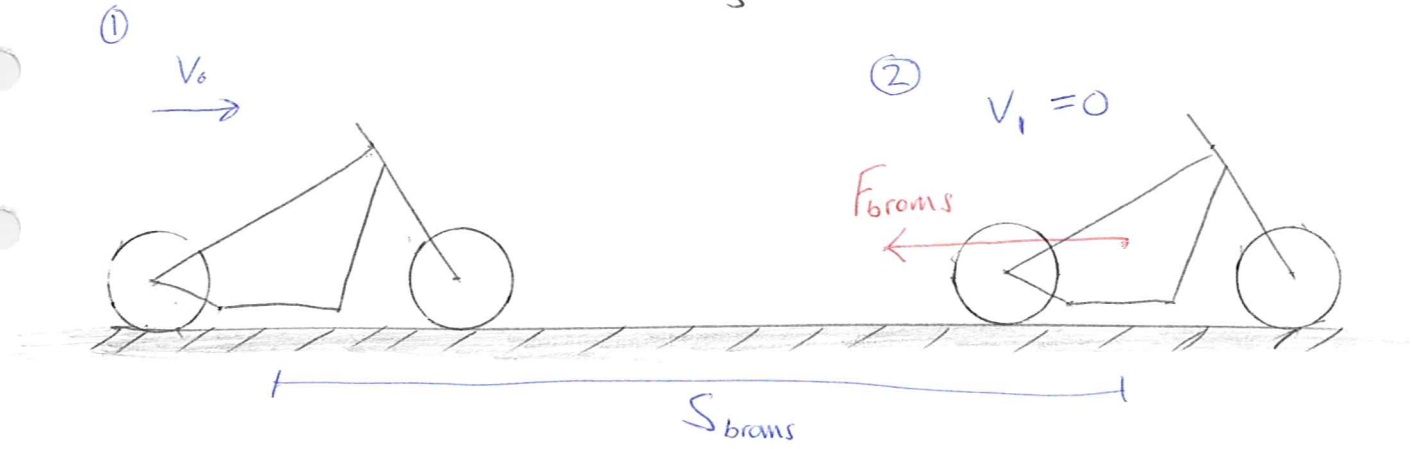
3) Sökt: a) F_{broms} b) v_{broms}

Givet: $m_{mc} = 215\ kg$, $m_{pers} = 75\ kg$

$m_{pack} = 25\ kg$, $a = 0,4 \cdot g$

$g = 9,82\ \frac{m}{s^2}$, $V_0 = \frac{100}{3,6}\ \frac{m}{s}$

$V_1 = 0\ \frac{m}{s}$



a) $F = ma$

$m_{tot} = m_{mc} + m_{pers} + m_{pack}$

$a = 0,4 \cdot g$

$\Rightarrow F_{broms} = (m_{mc} + m_{pers} + m_{pack}) \cdot 0.4 \cdot g = 1237,32\ N$

b) Energiprincip

$E_1 = E_2$

$E_{k1} = W_{broms}$

$\frac{m_t V_1^2}{2} = F_{broms}\ S_{broms}$

$\Rightarrow S_{br} = \frac{m_{tot}\ V^2}{2 \cdot F_{broms}} = 98,2186$

Svar: $F_{broms} = 1,2\ kN$, $S_{broms} = 98\ m$

4) Sökt: F

Givet: $v_0 = 30 \frac{m}{s}$, $v_1 = -8,5 \frac{m}{s}$

$m = 0,20\ kg$, $t = 0,024\ s$

Impulslagen: $I = \Delta p$

Impuls: $I = Ft$

Rörelsemängd: $p = mv$

$$Ft = mv_1 - mv_2$$

$$\Rightarrow F = \frac{m(v_1 - v_2)}{t} = \frac{0,2(30 - (-8,5))}{0,024}$$

$$= 320,833\ N$$

Svar: $F = 0,32\ kN$

5) Sökt: $P_{choklad}$

Givet: $m = 0{,}15\ kg$, $T_0 = 8^\circ C$

$T_1 = 42^\circ C$, $c_p = 5280\ \frac{J}{kg\,K}$

$t = 120\ s$

$$E = m c_p \Delta T$$

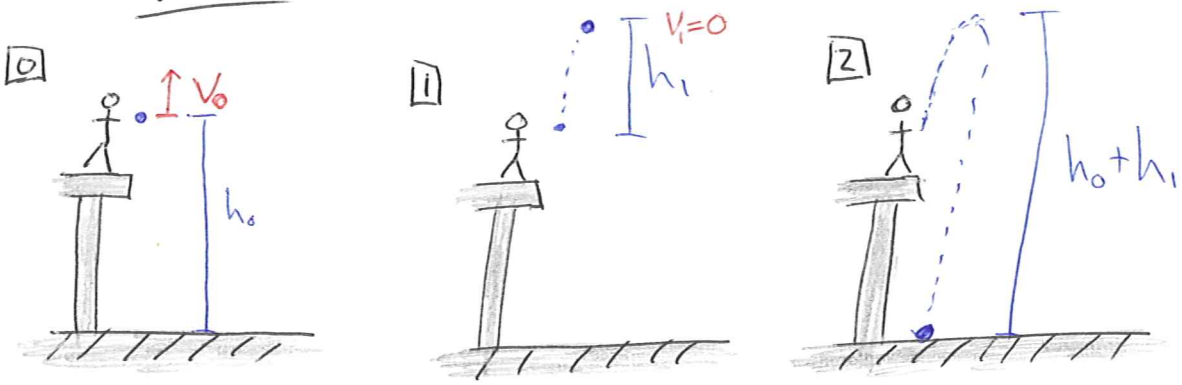
$$P = \frac{E}{t}$$

$$\Rightarrow P = \frac{m c_p (T_1 - T_0)}{t} = \frac{26\,928{,}0}{120} = 224{,}4\ W$$

Svar: P=224 W

6) Sökt: t

Givet: $h_0 = 36{,}8\ m$, $v_0 = 8{,}20\ \frac{m}{s}$, $g = 9{,}82\ \frac{m}{s^2}$



* h_1 från energiprincip

$E_0 = E_1$

$E_{p0} + E_{k0} = E_{p1} + E_{k1}$

$0 + \frac{mv_0^2}{2} = mgh_1 + 0$

$\Rightarrow h_1 = \frac{v_0^2}{2g} = 3{,}4236\ m$

* tid från 0 till 1

$s = \frac{v_0 + v_1}{2} \cdot t_1 \rightarrow h_1 = \frac{v_0}{2} t_1 \Rightarrow t_1 = \frac{2h_1}{v_0} = 0{,}835\ s$

* tid från 1 till 2

$s = v_0 t + \frac{at^2}{2} \rightarrow h_1 + h_0 = 0 + \frac{g t_2^2}{2}$

$\Rightarrow t_2 = \sqrt{2(h_0+h_1)\frac{1}{g}} = 2{,}8622\ s$

* tid från 0 till 2

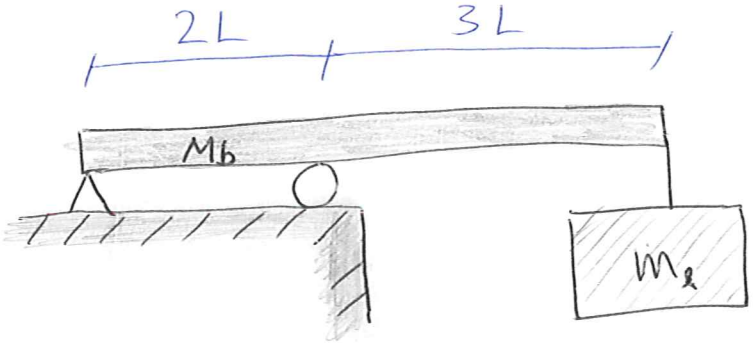
$t = t_1 + t_2 = 3{,}6972$

Svar: $t = 3{,}7\ s$

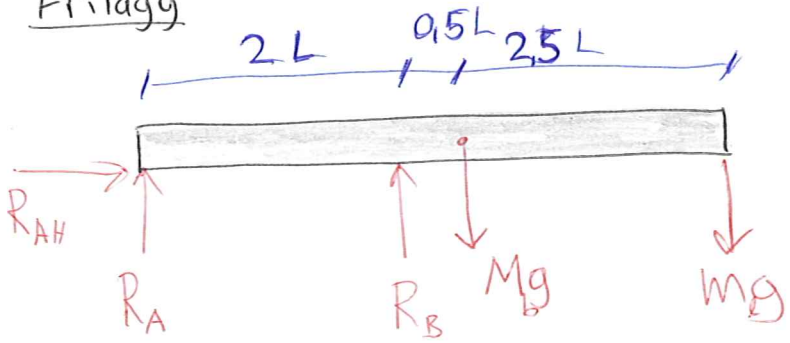
7) Sökt: $m_{låda,max}$

Givet: $M_{balk} = 52{,}0\,kg$, $L = 2{,}5\,m$

$R_{B,max} = 9600\,N$, $g = 9{,}82\,\frac{m}{s^2}$



Frilägg



Jämvikt

$\rightarrow: R_{AH} = 0$ (1)

$\uparrow: R_A + R_B - M_b g - m_\ell g = 0$ (2)

$\curvearrowright_A: -R_B \cdot 2L + M_b g\, 2{,}5L + m_\ell g\, 5L = 0$ (3)

(3) $\Rightarrow$ $m_\ell = \dfrac{R_B\, 2L - M_b g\, 2{,}5L}{g\, 5L} =$

* $R_{B,max}$ ger $m_{låda,max}$

$$m_{\ell,max} = \frac{R_{B,max} \cdot 2L - M_b g\, 2{,}5L}{g\, 5L} = 365{,}03\,kg$$

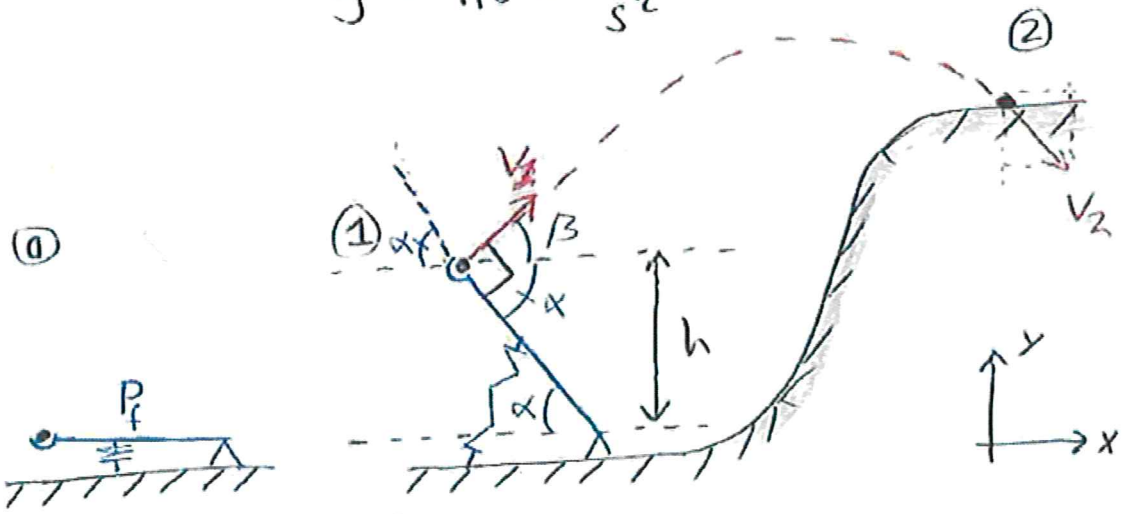
Svar: $m_{\ell,max} = 365\,kg$

8) Sökt: v_{2x}

Givet: $m = 100\,kg$, $P_f = 50000\,J$

$h = 2\,m$, $\alpha = 60°$

$g = 9{,}82\,\frac{m}{s^2}$



- Energi princip $E_0 = E_1$

$E_0 = P_f$

$E_1 = mgh + \frac{mv_1^2}{2}$

$\Big\}$ $v_1 = \sqrt{\frac{2}{m}(P_f - mgh)} = 30{,}996\,\frac{m}{s}$

- Hastighetskomposanter

$\alpha + \beta = 90° \rightarrow \beta = 90 - \alpha = 30°$

$v_{1x} = v_1 \cos(\beta) = 26{,}843\,\frac{m}{s}$

Inget luftmotstånd: $v_{1x} = v_{2x}$

SVAR: $v_{2x} = 27\,\frac{m}{s}$

9) Sökt v_1

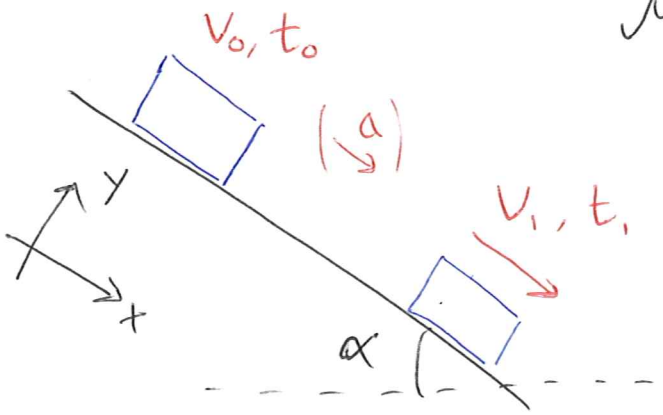
Givet $m = 15\ kg$, $\alpha = 36°$

$t_1 = 5{,}0\ s$, $t_0 = 0\ s$

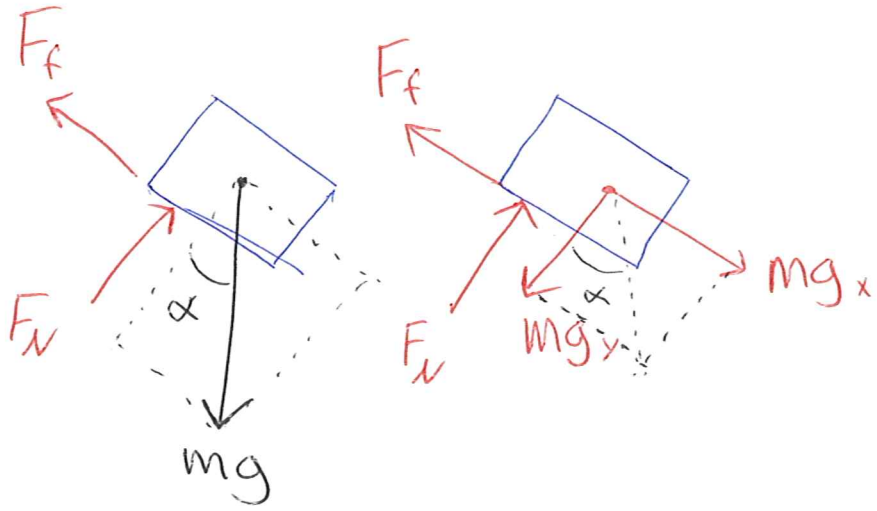
$v_0 = 0\ \frac{m}{s}$, $g = 9{,}82\ \frac{m}{s^2}$

$\mu = 0{,}5$

Rita



Frilägg



$$mg_x = mg\sin(\alpha)$$
$$mg_y = mg\cos(\alpha)$$

Jämvikt

$$\nearrow: F_N - mg\cos(\alpha) = 0 \qquad (1)$$

$$\searrow: -F_f + mg\sin(\alpha) = ma \qquad (2)$$

$$F_f = F_N \mu \qquad (3)$$

(1) $\rightarrow$ $F_N = mg\cos(\alpha)$ (4)

(4) och (3) $\rightarrow$ $F_f = mg\cos(\alpha)\mu$ (5)

(5) och (2) $\rightarrow$ $a = g(\sin(\alpha) - \mu\cos(\alpha)) = 1,7999 \frac{m}{s^2}$

Hastighet

$$v_1 = v_0 + at_1 = 0 + 1,7999 \cdot 5 = 8,999 \frac{m}{s}$$

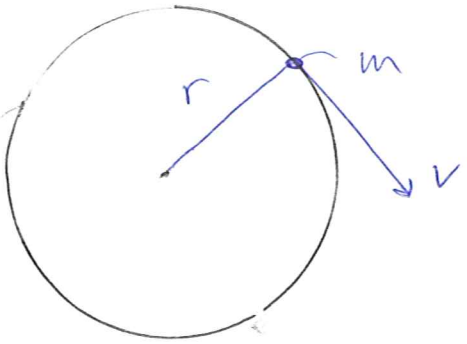
svar: $v_1 = 9,0 \frac{m}{s}$

10) Sökt : F

Givet: $m = 4\,kg$, $l_{rep} = 1{,}2\,m$

$l_{arm} = 0{,}5\,m$, $T = 2{,}2\,s$

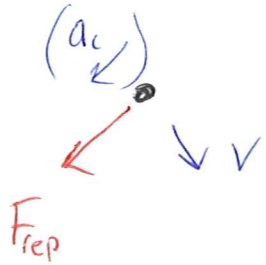
Rita



$r = l_{arm} + l_{rep}$

$v = \frac{2\pi r}{T}$

Frilägg



Jämvikt

$\swarrow$: $F = m a_c$

$a_c = \frac{v^2}{r}$

$$\Rightarrow F = \frac{m\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4 m \pi^2 r}{T^2} = 55{,}4623\,N$$

Svar $F = 55\,N$