problem 1

den 15 mars 2021 13:02

$$z = \frac{(2+i)\left(e^{12\pi i} - e^{\frac{\pi}{2} i}\right)}{1 - i^{4\cdot 3276+3}} = \frac{(2+i)(1-i)}{1+i}$$

$$e^{12\pi i} = \left(e^{2\pi i}\right)^6 = 1^6 = 1$$

$$e^{\pi/2\, i} = i$$

$$i^{4\cdot 3276+3} = \left(i^4\right)^{3276} i^3 = 1^{3276} i^3 = i^2 \cdot i = -i$$

$$z = \frac{(2+i)(1-i)}{1+i} = \frac{2-2i+i+i(-i)}{1+i} = \frac{2-2i+i-i^2}{1+i} = \frac{2-2i+i-(-1)}{1+i} =$$

$$= \frac{3-i}{1+i} = \frac{(3-i)(1-i)}{(1+i)(1-i)} = \frac{3-3i-i+i^2}{1^2+1^2} = \frac{3-3i-i-1}{2}$$

$$= \frac{2-4i}{2} = 1-2i$$

$$\boxed{\begin{array}{l} \mathrm{Re}(z) = 1 \\ \mathrm{Im}(z) = -2 \end{array}}$$

problem 2

den 15 mars 2021 13:29

$$f(x) = \frac{9x^3 + 2ax^2 + 7}{x^2 - 3} \approx 9x + 24 \quad x \to \infty \quad a = ?$$

$$0 = \lim_{x\to\infty}\left[f(x) - (9x+24)\right] = \lim_{x\to\infty}\left[\frac{9x^3+2ax^2+7}{x^2-3} - (9x+24)\right]$$

$$= \lim_{x\to\infty}\left[\frac{9x^3+2ax^2+7-(9x+24)(x^2-3)}{x^2-3}\right]$$

$$= \lim_{x\to\infty} \frac{\cancel{9x^3}+2ax^2+7-(\cancel{9x^3}-3\cdot 9x+24x^2-3\cdot 24)}{x^2-3}$$

$$= \lim_{x\to\infty} \frac{x^2(2a-24)+3\cdot 9\cdot x+7+3\cdot 24}{x^2-3} \quad \begin{matrix} :x^2 \\ :x^2 \end{matrix}$$

$$= \lim_{x\to\infty} \frac{2a-24+\mathcal{O}\left(\frac{1}{x}\right)}{1+\mathcal{O}\left(\frac{1}{x}\right)}$$

$$0 = 2a - 24 \implies 2a = 24 \implies \boxed{a = 12}$$

problem 3

den 15 mars 2021 13:59

$$\lim_{x\to 2} \frac{x \ln \frac{x}{2}}{x^2-4} = \lim_{x\to 2} \frac{x \ln \frac{x}{2}}{(x+2)(x-2)}$$

$$= \lim_{x\to 2}\left(\frac{x}{x+2}\cdot\frac{\ln\frac{x}{2}}{x-2}\right) = \lim_{x\to 2}\left(\frac{x}{x+2}\right)\cdot \lim_{x\to 2}\frac{\ln \frac{x}{2}}{x-2} =$$

0/0 LH

$$= \frac{2}{2+2}\cdot \lim_{x\to 2}\frac{\frac{1}{\frac{x}{2}}\cdot\frac{1}{2}}{1}.$$

$$= \frac{2}{2+2}\cdot\frac{1}{2}\cdot 2\cdot \lim_{x\to 2}\frac{1}{x} = \frac{2}{2+2}\cdot\frac{1}{2}\cdot 2\cdot\frac{1}{2}$$

$$= \frac{\cancel{2}}{2\cdot\cancel{2}}\cdot\frac{1}{\cancel{2}}\cdot\cancel{2}\cdot\frac{1}{2} = \frac{1}{4}$$

problem 4

den 15 mars 2021 14:07

$$f(x) = \begin{cases} \dfrac{x^{113} + x^{89} - 2x}{x-1} & x \neq 1 \\ c & x = 1 \end{cases}$$

$x \neq 1$, $f(x) = \dfrac{a(x)}{b(x)}$, a, b är kontinuerliga funktioner och $b \neq 0$. SATS i ADAMS säger att a/b är kontinuerlig

$x = 1$, kontinuitets villkor: $\lim\limits_{x \to 1} f(x) = f(1)$

$f(1) = c$, om $\lim\limits_{x \to 1} f(x)$ finns då kan vi välja

$$c = \lim_{x \to 1} f(x)$$

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{x^{113} + x^{89} - 2x}{x-1} = \left\{ \frac{0}{0}, \text{LH} \right\}$$

$$= \lim_{x \to 1} \frac{113 \cdot x^{112} + 89\, x^{88} - 2}{1} = \frac{113 + 89 - 2}{1}$$

$$= 113 + 89 - 2 = 111 + 89 = 110 + 1 + 89 = 110 + 90$$

$$= 200$$

$$\boxed{f = 200}$$

problem 5

den 15 mars 2021 14:13

$$f(x) = \tan\left(\frac{x\pi}{4}\right) + e^{\pi(x-1)}$$

$f\left(1+\frac{1}{100}\right) = ?$ i linjär approximation

$$f\left(1+\frac{1}{100}\right) \approx f(1) + f'(1)\,\frac{1}{100}$$

$$f(1) = \tan\frac{\pi}{4} + e^{0} = 1 + 1 = 2$$

$$f'(1) = ? \qquad f'(x) = \frac{1}{\cos^2\left(\frac{x\pi}{4}\right)} \cdot \frac{\pi}{4} + \pi e^{\pi(x-1)}$$

$$f'(1) = \frac{1}{\cos^2\left(\frac{\pi}{4}\right)} \cdot \frac{\pi}{4} + \pi e^{0} = \frac{\pi}{4}\,\frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2} + \pi$$

$$= 2\,\frac{\pi}{4} + \pi = \pi\left(\frac{1}{2} + 1\right) = \frac{3\pi}{2}$$

$$f\left(1+\frac{1}{100}\right) \approx 2 + \frac{3\pi}{2}\,\frac{1}{100} = 2 + \frac{3\pi}{200}$$

problem 6

den 15 mars 2021 14:38

$$f(x) = f_1(x) + f_2(x) + f_3(x)$$

$$f_1(x) = \frac{\sin x}{\ln x} \qquad f_1'(x) = \frac{\cos x \ln x - \sin x \frac{1}{x}}{\ln^2 x} = \frac{\cos x}{\ln x} - \frac{\sin x}{x \ln^2 x}$$

$$f_2(x) = xe^x \qquad f_2'(x) = x' e^x + x(e^x)' = 1e^x + xe^x = (1+x)e^x$$

$$f_3(x) = \sin(x^3 + \sqrt{x}) \qquad f_3'(x) = \cos(x^3+\sqrt{x}) \cdot (x^3 + \sqrt{x})'$$

$$= \cos(x^3+\sqrt{x})\left(3x^2 + \frac{1}{2\sqrt{x}}\right)$$

$$f'(x) = f_1'(x) + f_2'(x) + f_3'(x) = \frac{\cos x}{\ln x} - \frac{\sin x}{x \ln^2 x} + (1+x)e^x$$

$$+ \cos(x^3 + \sqrt{x})\left(3x^2 + \frac{1}{2\sqrt{x}}\right)$$

problem 7

den 15 mars 2021 14:42

$$32yx^3 = (x+y)^5 \qquad y' = ?, \text{ i } \quad x = y = 1$$

$$VS = HS \quad \Big| \frac{d}{dx}$$

$$VS' = HS'$$

$$(32yx^3)' = ((x+y)^5)'$$

$$32(yx^3)' = 5(x+y)^4(x+y)'$$

$$32(y'x^3 + y3x^2) = 5(x+y)^4(1+y')$$

använd $x = y = 1$

$$32(y' \cdot 1^3 + 1 \cdot 3 \cdot 1^2) = 5(1+1)^4(1+y')$$

$$32(y'+3) = 5 \cdot 2^4 \cdot (1+y') \qquad 2^4 = 4 \cdot 4 = 16$$

$2 \cdot 16$

$$32(y'+3) = 5 \cdot 16 \, (1+y')$$

$$2(y'+3) = 5(1+y')$$

$$6 + 2y' = 5 + 5y'$$

$$6-5 = (5-2)y'$$

$$1 = 3y'$$

$$\boxed{y' = 1/3}$$

problem 8

den 15 mars 2021 14:48

(a) $d(x^5) = \{ \text{GDR}: df = f' dx \} = 5x^4 dx$, TAS0

(b) $\ln[(x+dx)^3] - \ln(x^3) = d(\ln(x^3)) = \frac{1}{x^3} \cdot 3x^2 dx$

$= \frac{3}{x} dx$ TAS0

(c) $\sqrt{x+dx}\,\sin(x+dx) = \underbrace{\sqrt{x+dx}\,\sin(x+dx) - \sqrt{x}\sin x}_{d(\sqrt{x}\sin x)} + \sqrt{x}\sin x$

$= \sqrt{x}\sin x + d(\sqrt{x}\sin x)$

$= \sqrt{x}\sin x + \left(\frac{1}{2\sqrt{x}}\sin x + \sqrt{x}\cos x\right) dx$ TAS1

problem 9

den 15 mars 2021 14:53

$$\underbrace{\int f(x) \sin(x+3)\, dx}_{I_1} = \int \underbrace{\frac{(u+1)^2}{u} \sin u \, du}_{I_2}$$

Vi vet att $I_1 = I_2$. Kan vi omvandla I_1 till I_2 ?

ja, enkelt variabel byte:

$$x+3 = u, \quad dx = du, \quad x = u-3$$

$$I_1 = \int f(x) \sin(x+3)\, dx = \int f(u-3) \sin(u)\, du$$

vi ser att

$$f(u-3) = \frac{(u+1)^2}{u}$$

$$u-3 = w, \quad u = w+3$$

$$f(w) = \frac{(w+3+1)^2}{w+3} = \frac{(w+4)^2}{w+3}$$

$$\boxed{f(x) = \frac{(x+4)^2}{x+3}}$$

problem 10

den 15 mars 2021 15:47

$y(0) = -6$

$y'(0) = 1$

$y(x) = \left(\int e^x dx\right)\left(\int \cos x \, dx\right)$

$= (e^x + c_1)(\sin x + c_2)$

$y(x) = e^x \sin x + c_2 e^x + c_1 \sin x + c_1 \cdot c_2$

$-6 = y(0) = e^0 \cancel{\sin 0}^{0} + c_2 e^0 + \cancel{c_1 \cdot 0}^{0} + c_1 \cdot c_2$

$= c_2 + c_1 \cdot c_2 = c_2(1 + c_1)$

$1 = y'(0) \qquad y'(x) = e^x \sin x + e^x \cos x + c_2 e^x + c_1 \cos x + 0$

$1 = \cancel{e^0 \sin 0}^{0} + e^0 \cos 0 + c_2 e^0 + c_1 \cos 0$

$\cancel{1} = \cancel{1} + c_2 + c_1 \quad , 0 = c_2 + c_1$

vi får två elevationer med två okända variabler:

$-6 = c_2(1 + c_1)$

$0 = c_1 + c_2$

$$c_2 = -c_1 \quad ; \quad -6 = -c_1(1+c_1) = -c_1 - c_1^2$$

$$c_1^2 + c_1 - 6 = 0$$

$$(c_1-2)(c_1+3) = c_1^2 + 3c_1 - 2c_1 - 6 = c_1^2 + c_1 - 6$$

$\Rightarrow$ Lösning (a) : $c_1 = 2 \quad c_2 = -2 \quad y(x) = (e^x + 2)(\sin x - 2)$

Lösning (b) : $c_1 = -3 \quad c_2 = 3 \quad y(x) = (e^x - 3)(\sin x + 3)$

problem 11

den 15 mars 2021 16:56

$$(1+x^2)y' = 1+y^2 = (1+y^2) \quad \Big| \cdot dx$$

$$(1+x^2)y'dx = (1+y^2)\cdot dx$$

$$\frac{dy}{1+y^2} = \frac{dx}{1+x^2} \quad \Big| \int$$

$$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$$

$$\tan^{-1}(y) + c_1 = \tan^{-1}(x) + c_2$$

$$\tan^{-1}(y) = \tan^{-1}(x) + c_2 - c_1 = \tan^{-1}(x) + c$$

$$y = \tan(\tan^{-1}(x) + c)$$

$$x=0 \quad y=1 \quad \Rightarrow \quad 1 = \tan(\underbrace{\tan^{-1}(0)}_{0} + c) \quad \Rightarrow \quad 1 = \tan c$$

$$\Rightarrow \quad c = \pi/4$$

$$y = \tan\left(\tan^{-1}(x) + \frac{\pi}{4}\right) = \left\{ \tan(\alpha+\beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan\alpha \cdot \tan\beta} \right\}$$

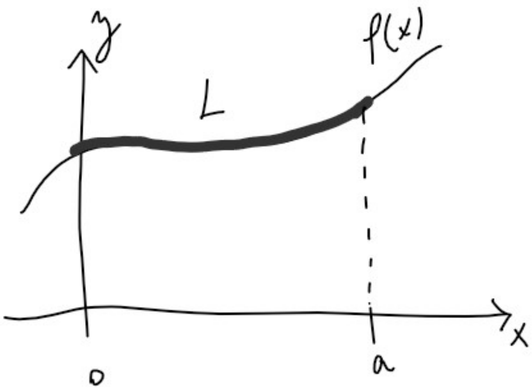
$$= \frac{\tan(\tan^{-1}(x)) + \tan(\pi/4)}{} = \frac{x+1}{} = \frac{x+1}{}$$

$$= \frac{\tan(\tan^{-1}(x)) + \tan(\pi/4)}{1 - \tan(\tan^{-1}(x)) \cdot \tan\frac{\pi}{4}} = \frac{x+1}{1 - x \cdot 1} = \frac{x+1}{1-x}$$

$$\boxed{y = \frac{1+x}{1-x}}$$

problem 12

den 15 mars 2021 17:07



$$L(a) = \int_0^a \sqrt{1 + f'(x)^2}\, dx$$

$\downarrow$

a är tidsberoende

$$L(a(t)) = \int_0^{a(t)} \sqrt{1 + f'(x)^2}\, dx$$

$$v = \frac{dL}{dt} = \frac{dL}{da} \cdot \frac{da}{dt} = \frac{da}{dt} \cdot \underbrace{\frac{d}{da} \int_0^a \sqrt{1 + f'(x)^2}\, dx}_{\text{MAHS}}$$

$$= \sqrt{1 + f'(a)^2}$$

$$v = \frac{da}{dt} \cdot \sqrt{1 + f'(a)^2}$$

$$\boxed{\frac{da}{dt} = \frac{v}{\sqrt{1 + f'(a)^2}}}$$

$t = 0 \quad a = 0 \implies$ B.V. $a(0) = 0$

(a) $f(x) = x$

$f'(x) = 1$

$$a' = \frac{v}{\sqrt{1 + 1}} \implies a(t) = \frac{vt}{\sqrt{1+1}} + c$$

$a(0) = 0 \implies c = 0$

$$\boxed{a(t) = \frac{t}{\sqrt{1 + 1^2}} = \frac{t}{\sqrt{2}} \quad \text{med } v = 1}$$

(b) $f(x) = \frac{2}{3} x^{3/2}$ $\quad a' = \frac{v}{\sqrt{\ldots}} \Big| = \frac{v}{\sqrt{\ldots}}$

(b) $f(x) = \frac{2}{3}x^{3/2}$ $\qquad a' = \left.\frac{v}{\sqrt{1+(\sqrt{x})^2}}\right|_{x=a} = \frac{v}{\sqrt{1+a}}$

$f'(x) = \cancel{\frac{2}{3}}\cancel{\frac{3}{2}}x^{1/2} = \sqrt{x}$

$v = 1 \qquad a'(t) = \frac{1}{\sqrt{1+a}}$

$\sqrt{1+a}\ da = dt \quad \Big| \int$

$\int \sqrt{1+a}\ da = t + c$

$\frac{(1+a)^{1/2+1}}{\frac{1}{2}+1} = t + c \qquad \frac{1}{2}+1 = \frac{3}{2}$

$(1+a)^{3/2} = \frac{3}{2}(t+c)$

$t=0,\ a=0 \Rightarrow \qquad 1^{3/2} = 3/2(0+c) \Rightarrow \quad c = \frac{2}{3}$

$(1+a)^{3/2} = \frac{3}{2}\left(t + \frac{2}{3}\right)$

$1 + a = \left[\frac{3}{2}\left(t + \frac{2}{3}\right)\right]^{2/3}$

$\left[\frac{3}{2}\left(t+\frac{2}{3}\right)\right]^{\frac{2}{3}} - 1$

$$\boxed{a = \left[\frac{3}{2}\left(t + \frac{2}{3}\right)\right]^{\frac{2}{3}} - 1}$$

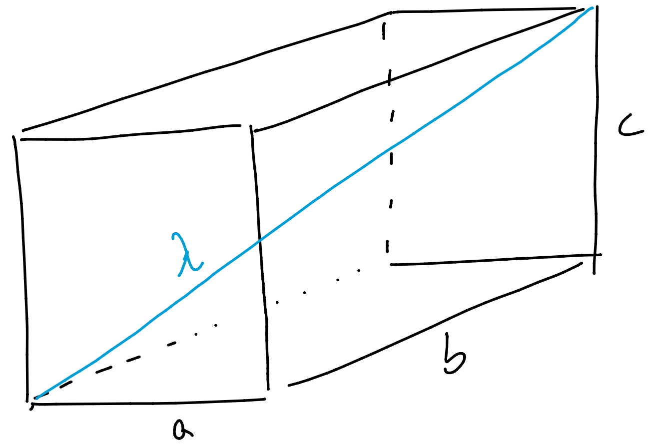
problem 13

den 15 mars 2021 21:12

$V = a \cdot b \cdot c$

$A = 2(ab + ac + bc)$

$\lambda = a^2 + b^2 + c^2$



$d\lambda = 0$

$dV = 0$

$dA = ?$

$0 = d\lambda = 2a\,da + 2b\,db + 2c\,dc$

$0 = dV = da\,bc + ac\,db + ab\,dc$

$da = \frac{1}{10}$

$a = 1, \; b = 2, \; c = 3$

insättning av siffror ger:

$0 = 2 \cdot 1 \cdot \frac{1}{10} + 2 \cdot 2 \cdot db + 2 \cdot 3 \cdot dc$

$0 = \frac{1}{10} \cdot 2 \cdot 3 + 1 \cdot 3 \cdot db + 1 \cdot 2 \cdot dc$

$0 = \frac{1}{10} + 2\,db + 3\,dc$

$0 = \frac{6}{10} + 3\,db + 2\,dc$

$2\,db + 3\,dc = -\frac{1}{10} \qquad | \cdot 2$

$3\,db + 2\,dc = -\frac{6}{10} \qquad | \cdot (-3)$

$$\left.\begin{array}{l} 4\,db + 6\,dc = -\frac{2}{10} \\ -9\,db - 6\,dc = +\frac{18}{10} \end{array}\right\} \Rightarrow -5\,db + 0 = \frac{18}{10} - \frac{2}{10} = \frac{16}{10} = \frac{8}{5}$$

$$\Rightarrow db = -\frac{1}{5}\,\frac{8}{5} = -\frac{8}{25}$$

$$\Rightarrow 2\,dc = -\frac{6}{10} - 3\,db = -\frac{6}{10} - 3(-)\frac{8}{25}$$

$$= -\frac{6}{10} + \frac{24}{25}$$

$$dc = -\frac{3}{10} + \frac{12}{25} = \frac{-3\cdot 5 + 12\cdot 2}{2\cdot 5\cdot 5} = \frac{24-15}{2\cdot 5\cdot 5}$$

$$= \frac{9}{2\cdot 5\cdot 5} = \frac{9}{50}$$