Examination,
Finite automata theory and formal languages (DIT321/TMV027)

- Date and time: 2019-08-21, 8:30–12:30.
- Author/examiner: Nils Anders Danielsson. Telephone number: 1680. Visits to the examination rooms: ∼9:30 and ∼11:30.
- Authorised aids (except for aids that are always permitted): None.
- Grade limits for Chalmers students: 3: 45, 4: 63, 5: 81.
- Grade limits for GU students: G: 45, VG: 81.
- Do not hand in solutions for several exercises on the same sheet.
- Write your examination code on each sheet.
- Solutions can be rejected if they are hard to read, unstructured, or poorly motivated.
- After correction the graded exams are available in the student office in room 4482 of the EDIT building. If you want to discuss the grading, contact the examiner no later than three weeks after the result has been reported. In this case you should not remove the exam from the student office.

1. (10p) Consider the context-free grammar $G = (\{ S, A, B \}, \{ a, b \}, P, S)$, where the set of productions P is defined in the following way:

$$\begin{aligned} S &\to A \mid BA \\ A &\to a \mid bB \\ B &\to B \end{aligned}$$

   (a) Which of the nonterminals of G are nullable?

   (b) Which of the nonterminals of G are generating?

   (c) List all strings in $L(G)$.

2. (10p) Give a Turing machine M such that $L(M) = \{ 0^{2n} \mid n \in \mathbb{N} \}$. The machine's input alphabet should be $\{ 0 \}$.

   Explain your answer in such a way that the person correcting your exam can easily see that the answer is correct.

3. (20p) Consider the ε-NFA A given by the following transition table:

	a	b	ε
$\to s_0$	$\{ s_0, s_2 \}$	$\{ s_1 \}$	$\{ s_1 \}$
s_1	$\{ s_2 \}$	$\emptyset$	$\emptyset$
$*s_2$	$\emptyset$	$\{ s_1 \}$	$\emptyset$

   (a) Construct a regular expression e over the alphabet $\{ a, b \}$ satisfying $L(e) = L(A)$.

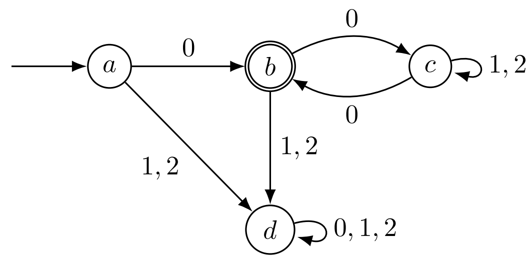
   Prove that your construction is correct. You do not need to provide proofs showing that algorithms covered in the course are correct. It is fine to use arguments of the following form: "Here I have used algorithm X to compute the value y, and because the result of algorithm X always satisfies property P, we have $P(y)$."

   (b) Construct a minimal DFA B over the alphabet $\{ a, b \}$ satisfying $L(B) = L(A)$.

   Prove that your construction is correct. You do not need to provide proofs showing that algorithms covered in the course are correct.

   (c) Give a precise description, using natural language, of the language $L(A)$. Explain your answer in such a way that the person correcting your exam can easily see that the description is correct.

4. (20p) Consider the regular expression $e = 0 + 00(00 + 1 + 2)^*0$ and the DFA A, which is defined by the following transition diagram:



Either prove that $L(e) = L(A)$ or that $L(e) \neq L(A)$.

5. (20p) Consider X, an inductively defined subset of $\{ 0, 1 \}^*$:

$$\frac{}{\varepsilon \in X} \qquad \frac{w \in X}{01w11 \in X}$$

   (a) Define a context-free grammar G satisfying $L(G) = X$.
   Explain your answer in such a way that the person correcting your exam can easily see that the answer is correct. (The explanation could refer to parts (b) and (c) below.)

   (b) Prove that $X \subseteq L(G)$.

   (c) Prove that $L(G) \subseteq X$.

6. (20p) Consider the following languages over $\{ 0, 1 \}$:

   (a) $\{ ww^{\mathrm{R}} \mid w \in \{ 0, 1 \}^*, |w| \geq 7 \}$, where w^{R} is w reversed.

   (b) $\{ ww^{\mathrm{R}} \mid w \in \{ 0 \}^*, |w| \geq 7 \}$.

   For each language, answer the following two questions:

   - Is the language regular?
   - Is the language context-free?

   Provide proofs. You are allowed to make use of the following lemmas/facts without proving them:

   - The two pumping lemmas covered in the course.
   - The closure properties covered in the course.
   - The fact that $\{ ww^{\mathrm{R}} \mid w \in \{ 0, 1 \}^* \}$ is context-free and not regular.