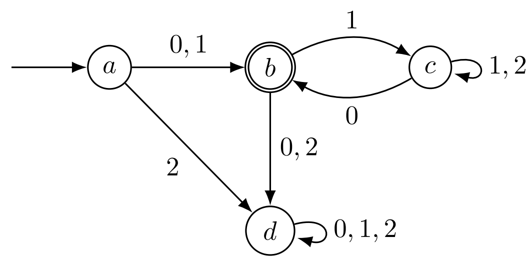
Examination,
Finite automata theory and formal languages (DIT321/TMV027)

- Date and time: 2019-03-21, 14:00–18:00.
- Author/examiner: Nils Anders Danielsson. Telephone number: 1680. Visits to the examination rooms: ∼15 and ∼17.
- Authorised aids (except for aids that are always permitted): None.
- Grade limits for Chalmers students: 3: 45, 4: 63, 5: 81.
- Grade limits for GU students: G: 45, VG: 81.
- Do not hand in solutions for several exercises on the same sheet.
- Write your examination code on each sheet.
- Solutions can be rejected if they are hard to read, unstructured, or poorly motivated.
- After correction the graded exams are available in the student office in room 4482 of the EDIT building. If you want to discuss the grading, contact the examiner no later than three weeks after the result has been reported. In this case you should not remove the exam from the student office.

1. (10p) Construct a regular expression which accepts the same language as the following DFA:



Explain your answer in such a way that the person correcting your exam can easily see that the answer is correct.

2. (10p) Give a Turing machine that accepts the language $\{ 01^n0 \mid n \in \mathbb{N} \}$ over the alphabet $\{ 0, 1 \}$.

   Explain your answer in such a way that the person correcting your exam can easily see that the answer is correct.

3. (20p) Consider the context-free grammar $G = (\{ S, A, B \}, \{ a, b \}, P, S)$, where the set of productions P is defined in the following way:

$$S \to ABA$$
$$A \to \varepsilon \mid a$$
$$B \to BbB \mid bb$$

   (a) Give a context-free grammar G' in Chomsky normal form satisfying $L(G') = L(G)$, and prove that your answer is correct.

   You do not need to provide proofs showing that algorithms covered in the course are correct. It is fine to use arguments of the following form: "Here I have used algorithm X to compute the value y, and because the result of algorithm X always satisfies property P, we have $P(y)$."

   (b) Construct the CYK table for G' and the string $abbb$.

   (c) Explain how one can use the CYK table to determine if $abbb \in L(G)$.

4. (20p) Consider the regular expression $e = ab^* + (ab)^* + bb^*$.

   (a) Give a precise description, using natural language, of the language generated by e.

   (b) Construct a DFA A over the alphabet $\{ a, b \}$ satisfying $L(A) = L(e)$. The DFA must be minimal (in terms of the number of states).

   Prove that your construction is correct. You do not need to provide proofs showing that algorithms covered in the course are correct.

   Hint: Construct an equivalent ε-NFA, convert it to a DFA, minimise the DFA.

5. (20p) Consider X, an inductively defined subset of $\{0,1\}^*$:

$$\frac{}{0 \in X} \qquad\qquad \frac{u, v \in X}{u1v \in X}$$

(a) Define a context-free grammar G satisfying $L(G) = X$.

Explain your answer in such a way that the person correcting your exam can easily see that the answer is correct. (The explanation could refer to parts (b) and (c) below.)

(b) Prove that $X \subseteq L(G)$.

(c) Prove that $L(G) \subseteq X$.

6. (20p) Consider the following languages over $\{0,1\}$:

(a) $\{\, w \in \{0,1\}^* \mid |w| \geq 7 \wedge \nexists u, v \in \{0,1\}^*.\ w = u11v \,\}$.

(b) $\{\, ww \mid w \in \{0,1\}^*, |w| \leq 7 \,\}$.

For each language, explain which of the following classes the language belongs to:

- The regular languages.
- The context-free languages.
- The languages that are not regular.
- The languages that are not context-free.

Provide proofs. You are allowed to make use of the following lemmas/facts without proving them:

- The two pumping lemmas covered in the course.
- The closure properties covered in the course.
- The fact that $\{\, ww \mid w \in \{0,1\}^* \,\}$ is not context-free.