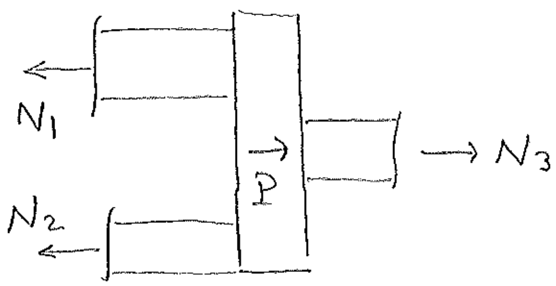
1.

Jämvikt $\rightarrow$: $P - N_1 - N_2 + N_3 = 0 \qquad (1)$



Konstitutiva samband

$$N_1 = \frac{E\cdot(2A)}{L}\,\delta_1 \qquad N_2 = \frac{E\cdot(3/2\,A)}{L}\,\delta_2 \qquad N_3 = \frac{EA}{2L/3}\,\delta_3 \qquad (2)$$

Kompatibilitet

$$\delta_1 = \delta_2 \qquad \delta_1 + \delta_3 = 0 \Rightarrow \delta_1 = \delta_2 = -\delta_3 \qquad (3)$$

(2) i (3) $\Rightarrow$ $\frac{N_1}{2} = \frac{2}{3}N_2 = -\frac{2}{3}N_3 \Rightarrow N_1 = \frac{4}{3}N_2 = -\frac{4}{3}N_3$

insatt i (1) $\Rightarrow$

$$P - \frac{4}{3}N_2 - N_2 - N_2 = 0 \Rightarrow \boxed{N_2 = \frac{3}{10}\cdot P} \quad \boxed{N_1 = \frac{2}{5}P}$$

$$\boxed{N_3 = -\frac{3}{10}P}$$

Forflyttn. av stel skiva $= \delta_1 = \delta_2 = \frac{N_1\cdot L}{E\cdot 2A} = \frac{PL}{5\cdot EA} = 1$ [mm]

(2)

Jämvikt

$$\twoheadrightarrow: M_v - M_{v1} + M_{v2} = 0 \quad (1)$$

Konstitutiva samband

$$\Delta\varphi_1 = \frac{M_{v1} \cdot L}{G\,K_1} \qquad \Delta\varphi_2 = \frac{M_{v2}\,L}{G\,K_2}$$

där $K_1 = \frac{\pi}{2} a^4 \qquad K_2 = \frac{\pi}{2}\left(a^4 - (a/2)^4\right) = \frac{\pi a^4 \cdot 15}{32}$

$$(2)$$

Kompatibilitet

$$\Delta\varphi_1 + \Delta\varphi_2 = 0 \Rightarrow \frac{M_{v1}}{K_1} + \frac{M_{v2}}{K_2} = 0 \Rightarrow M_{v2} = -\frac{K_2}{K_1} M_{v1} =$$

$$= -\frac{15/32}{1/2} M_{v1} = -\frac{15}{16} M_{v1} \qquad (3)$$

(3) i (1) $\Rightarrow M_v - M_{v1} - \frac{15}{16} M_{v1} = 0 \Rightarrow \boxed{M_{v1} = \frac{16}{31} M_v}$

$$\boxed{M_{v2} = -\frac{15}{31} M_v}$$

Största vridskjuvspänningen

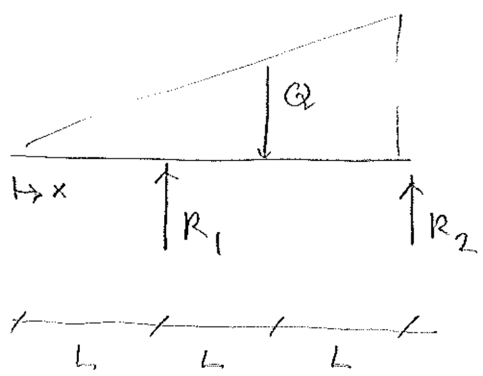
$$\tau_{max,1} = \frac{2 \cdot M_{v1} \cdot a}{\pi \cdot a^4} = \frac{32 \cdot M_v}{31 \cdot \pi \cdot a^3} \approx 65{,}7\,[\mathrm{MPa}]$$

$$\tau_{max,2} = \frac{2 M_{v2} \cdot a}{\pi \cdot \left(a^4 - (a/2)^4\right)} = -\frac{30 \cdot M_v}{31 \cdot \pi \cdot 1/16 \cdot a^3} = -\frac{480}{31} \cdot \frac{M_v}{\pi a^3} =$$

$$\approx -65{,}7\,[\mathrm{MPa}]$$

Max vridskjuvspänning $\approx 65{,}7$ MPa //

3 Jämvikt $Q = \int_0^{3L} q(x)\,dx$



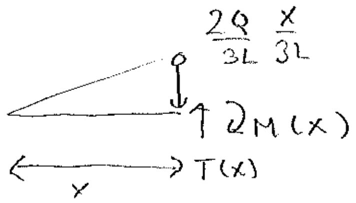
$\uparrow: R_1 + R_2 - Q = 0$

$\curvearrowleft: QL - R_2\,2L = 0$

$\Rightarrow R_1 = R_2 = Q/2$

Tvärkraft $T(x)$

$0 < x < L$

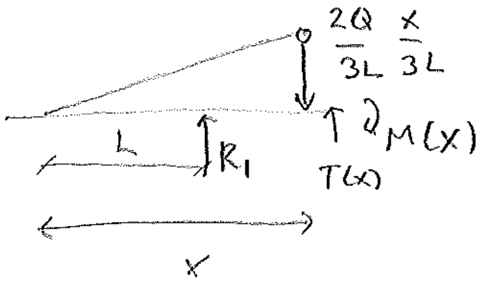


$\uparrow: T(x) - \frac{Q}{3L}\frac{x^2}{3L} = 0$

$\Rightarrow \boxed{T(x) = \frac{Qx^2}{9L^2}}$

$T(0) = 0$, $T(L^-) = \frac{Q}{9}$

$L < x < 3L$



$\uparrow: T(x) - \frac{Qx^2}{9L^2} + \frac{Q}{2} = 0$

$\Rightarrow \boxed{T(x) = \frac{Qx^2}{9L^2} - \frac{Q}{2}}$

$T(L^+) = -\frac{7}{18}Q$

$T(3L) = \frac{Q}{2}$

b) Lokala min och max av $T(x)$ där $T'(x) = 0$

$\Rightarrow x = 0 \Rightarrow T = 0$

Därför fås max i ändpunkt på intervall

dvs $T(0)$, $T(L)$ eller $T(3L)$

$\Rightarrow \max |T(x)| = \frac{Q}{2}$

Böjskjuvspänning

$$\tau = \frac{S_{A^*} \cdot T}{I \cdot b}$$

där $S_{A^*} = 5t \cdot t \cdot \left(\frac{5t}{2} + \frac{t}{2} \right) = 15\, t^3$

$$I = \frac{t (5t)^3}{12} + 2 \cdot \left[\frac{5t \cdot t^3}{12} + (5t \cdot t) \cdot \left(\frac{t}{2} + \frac{5t}{2} \right)^2 \right] =$$

$$= \frac{405}{4} \cdot t^4$$

$b = t$

$$\Rightarrow \tau_{max} = \frac{15\, t^3 \cdot \frac{1}{2} Q}{\frac{405}{4} t^4 \cdot t} = \frac{Q}{t^3} \frac{2}{27} \quad //$$

4) Elastiska linjens ekv

$$EI\,w^{IV}(x) = q(x) = q_0 \sin\left(\frac{2\pi x}{L}\right)$$

$$\Rightarrow EI\,w'''(x) = -q_0 \frac{L}{2\pi}\cos\left(\frac{2\pi x}{L}\right) + C_1 \Rightarrow$$

$$\Rightarrow EI\,w''(x) = -q_0\left(\frac{L}{2\pi}\right)^2 \sin\left(\frac{2\pi x}{L}\right) + C_1 x + C_2 \Rightarrow$$

$$\Rightarrow EI\,w'(x) = q_0\left(\frac{L}{2\pi}\right)^3 \cos\left(\frac{2\pi x}{L}\right) + C_1\frac{x^2}{2} + C_2 x + C_3 \Rightarrow$$

$$\Rightarrow EI\,w(x) = q_0\left(\frac{L}{2\pi}\right)^4 \sin\left(\frac{2\pi x}{L}\right) + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4$$

Randvillkor.

$$\begin{cases} w(0)=0 \\ w'(0)=0 \end{cases} \Rightarrow \begin{cases} \boxed{C_4 = 0} \\ q_0\left(\frac{L}{2\pi}\right)^3 + C_3 = 0 \Rightarrow \boxed{C_3 = -q_0\left(\frac{L}{2\pi}\right)^3} \end{cases}$$

$$\begin{cases} M(L)=0 \Rightarrow w''(L)=0 \Rightarrow C_1 L + C_2 = 0 \Rightarrow \boxed{C_2 = -C_1\cdot L} \\ k\,w(L) = \underbrace{-T(L)}_{-EIw'''(L)} = EI\,w'''(L) \end{cases}$$

$$\Rightarrow \underbrace{\frac{k}{EI}}_{\alpha/L^3}\left[C_1\frac{L^3}{6} - C_1 L\cdot\frac{L^2}{2} - q_0\left(\frac{L}{2\pi}\right)^3\cdot L\right] = -q_0\frac{L}{2\pi} + C_1 \Rightarrow$$

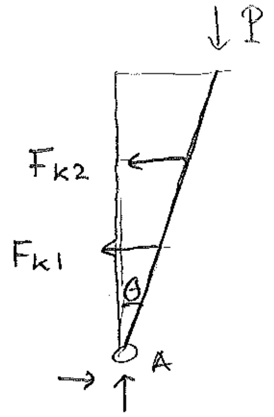
$$\Rightarrow C_1\left[-\frac{1}{3}\alpha - 1\right] = -q_0\frac{L}{2\pi} + \frac{\alpha}{(2\pi)^3} L\,q_0 \Rightarrow$$

$$\Rightarrow C_1 = \frac{q_0 L\left(\frac{1}{2\pi} - \frac{\alpha}{(2\pi)^3}\right)}{\frac{\alpha}{3}+1} = \frac{q_0\cdot L\cdot 3\cdot(4\pi^2 - \alpha)}{8\cdot\pi^3\cdot(\alpha+3)}$$

$$\Rightarrow w(x) = q_0\left(\frac{L}{2\pi}\right)^4 \sin\left(\frac{2\pi x}{L}\right) + \frac{3q_0 L}{8\pi^3}\,\frac{4\pi^2-\alpha}{\alpha+3}\left(\frac{x^3}{6} - \frac{x^2 L}{2}\right) - q_0\left(\frac{L}{2\pi}\right)^3 x$$

$$\Rightarrow \boxed{w(x) = \frac{1}{EI}\left[\frac{q_0 L 3}{2\pi^3}\,\frac{(\alpha - 4\pi^2)}{(3-\alpha)}\left[\frac{x^3}{6} - \frac{x^2 L}{2}\right] - q_0\left(\frac{L}{2\pi}\right)^3 x\right]}$$

(5.) Jämvikt i deformerat läge:



$F_{k1} = k \cdot \theta L$

$F_{k2} = k\, \theta\, 2L$

$\overset{\frown}{A}: \; P \cdot \theta\, 3L - F_{k1} \cdot L - F_{k2}\, 2L = 0$

$\Rightarrow \; P \cdot \theta \cdot 3L - k\,\theta\, L^2 - k\, 4\,\theta\, L^2 = 0$

$\Rightarrow \; \theta = 0$ eller $P_{kr} \cdot 3L - 5kL^2 = 0 \; \Rightarrow \; P_{kr} = \dfrac{5kL}{3}$ //