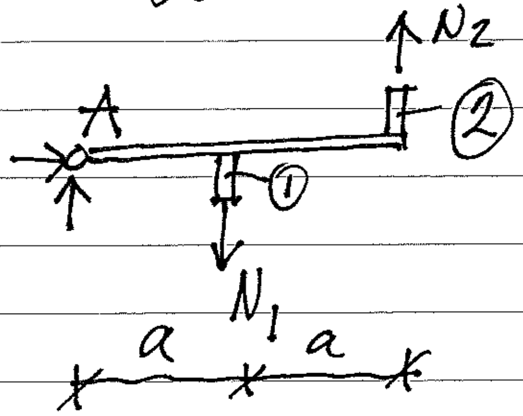
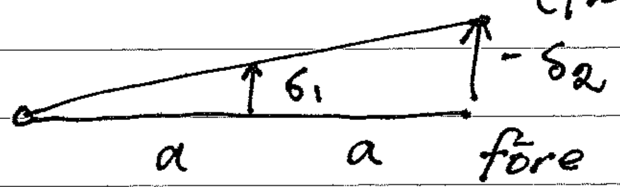
Hållf & Maskel Z2 080314

1) Frilägg den stela balken, ställ upp jämvikt



$\curvearrowleft A \quad N_2 \cdot 2a - N_1 \cdot a = 0 \quad 1)$

deformationssamband



Likformiga trianglar $\dfrac{\delta_1}{a} = \dfrac{-\delta_2}{2a} \quad 2)$

(δ positiv om stång förlängs.)

Material samband, LB (2-14) + (5-2)

$$\varepsilon_1 = \frac{\delta_1}{L} = \frac{N_1}{EA} \quad 3)$$

$$\varepsilon_2 = \frac{\delta_2}{L} = \frac{N_2}{EA} + \alpha T \quad 4)$$

3), 4) i 2) $\Rightarrow$ $\dfrac{N_2 L}{EA} + \alpha T L = -2 \dfrac{N_1 L}{EA} \quad 5)$

1) forts 1) i 5) $\Rightarrow$

$$N_2 = -\frac{1}{5} EA\alpha T \Rightarrow \text{ i } 4)$$

$$\delta_2 = \frac{4}{5} \alpha T L \quad \text{(nedåt)}$$

b/ Risk för knäckning (elastisk)
LB p 124 + (8-27)
stänger ledat infästa i båda ändar:
$\Rightarrow$ Euler 2

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$|N_1| > |N_2| \Rightarrow (N_1 = 2N_2)$$

$$\frac{2}{5} EA\alpha T = \frac{\pi^2 EI}{L^2} \Rightarrow$$

$$T = \frac{5\pi^2}{2} \frac{I}{\alpha A L^2}$$

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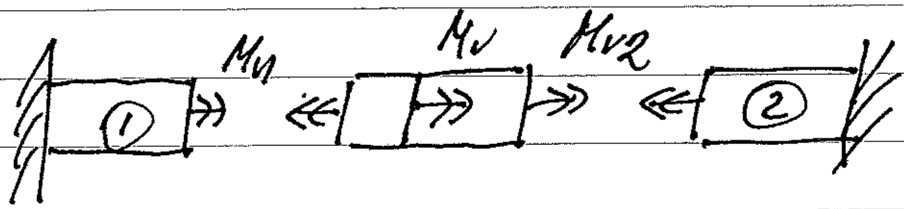
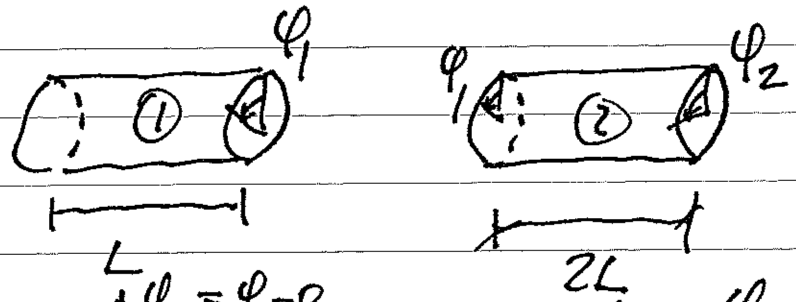
2)



Frilägg lastangreppspunkt

$\Rightarrow \quad M_v + M_{v2} - M_{v1} = 0 \qquad 1)$

Deformationssamband



$\Delta\varphi_1 = \varphi_1 - 0 \qquad \Delta\varphi_2 = \varphi_2 - \varphi_1$

$\Delta\varphi_1 + \Delta\varphi_2 = 0 \qquad 2)$

Materialsamband : LB (6-11)

$$\Delta\varphi_1 = \frac{M_{v1} L}{GK} \qquad 3)$$

$$\Delta\varphi_2 = \frac{M_{v2}\, 2L}{GK} \qquad 4)$$

$3), 4)$ i $2) \Rightarrow \quad M_{v1} + 2M_{v2} = 0 \Rightarrow$ i $1)$

$$M_{v1} = \frac{2}{3} M_v \qquad M_{v2} = -\frac{1}{3} M_v$$

Hållf. Mashel Z2 080314

2) Lösn LB p 55-56 ⇒ för tunnväggigt rör

$$\tau_v = \frac{M_v}{W_v} = \frac{M_v}{2\pi R^2 t}$$

Plasticering först i vänster del $\tau = \tau_s$

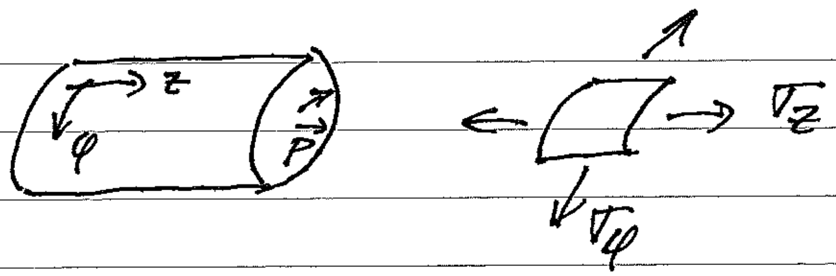
$$\tau_s = \frac{2M_v}{3 \cdot 2\pi R^2 t} \Rightarrow M_v = 3\pi R^2 t \, \tau_s$$

$$M_v = 3\pi (100)^2 \cdot 5 \cdot 100 = 47 \text{ kNm}$$

vridningsvinkeln $\Delta\varphi_1 = \dfrac{M_{v1} L}{G 2\pi R^3 t}$

då $\tau = \tau_s$ i ① $\Delta\varphi_1 = \dfrac{\tau_s L}{GR} = 0.013 \text{ rad}$

3)



LB p 181 ⇒ $\sigma_\varphi = \frac{pr}{t}$ $\sigma_r \simeq 0$

$$\sigma_z = \frac{pr}{2t}$$

Hookes lag p 206, 212 ⇒

$$\varepsilon_\varphi = \frac{1}{E}\left[\sigma_\varphi - \nu(\sigma_r + \sigma_z)\right] \Rightarrow \varepsilon_\varphi = \frac{\sigma_\varphi}{E}\left(1 - \frac{\nu}{2}\right)$$

$$\sigma_\varphi = \frac{E\varepsilon_\varphi}{1 - \frac{\nu}{2}} = \frac{2{,}1 \cdot 10^5 \cdot 0.078 \cdot 10^{-2}}{1 - \frac{0{,}3}{2}} = 193 \text{ MPa}$$

LB p 235 ⇒ då $\sigma_r, \sigma_\varphi, \sigma_z$ är huvudsp ($\sigma_r \simeq 0$)

$$\sigma_{e_{vM}} = \sqrt{\frac{1}{2}\left[(\sigma_\varphi - \sigma_z)^2 + \sigma_\varphi^2 + \sigma_z^2\right]} =$$

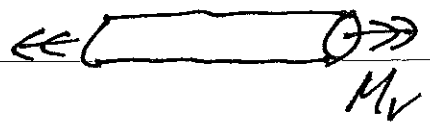
$$= \frac{\sigma_\varphi}{\sqrt{2}}\sqrt{\frac{6}{4}} = \frac{\sqrt{3}}{2}\sigma_\varphi = 167 \text{ MPa} < \sigma_s$$

Nej ingen risk för plasticering

Hållfx maskel Z2 080314

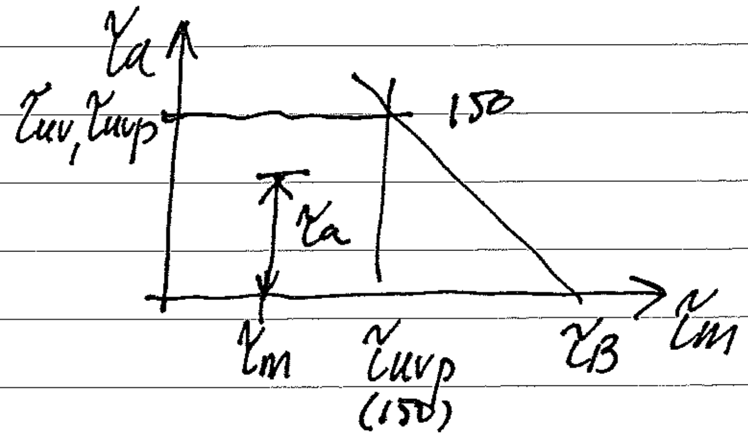
4) Staven utsätts för ett vridande moment

$$M_v = P \cdot L = (P_m \pm P_a)L$$



LB p 57 $\Rightarrow \tau = \frac{M_v}{W_v}$; $W_v = \frac{\pi d^3}{16}$

Rita in belastningen i ett Haighdiagram, LB p 248-249



$$\tau_m = \frac{P_m L}{W_v} \quad P_m = 2000\,N$$

Inverkan av spänningskonc. mm $\Rightarrow$ reducera τ_{uv}, τ_{uvp}, LB p 250 $\rightarrow$

Spår: diagram $\Rightarrow \left\{ \begin{array}{l} \frac{\rho}{d} = \frac{2}{40} = 0.05 \\ \frac{D}{d} = \frac{50}{40} = 1.25 \end{array} \right. \Rightarrow K_t = 1.7$

LB p 252 $\Rightarrow K_f = 1 + q(K_t - 1)$ Anvisningsverkan

$\left\{ \begin{array}{l} \rho = 2 \text{ mm} \\ \sigma_B = 590 \text{ MPa} \end{array} \right. \Rightarrow q \approx 0.8$

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4) Lots varmed $K_f = 1.56$

$\rightarrow$ reduktion $\dfrac{\lambda}{K_f K_d K_r} = \dfrac{0.9}{1.56} = 0.58$

Ur Haighdiagram fås:

$$\frac{\lambda}{K_f K_r K_d} \tau_{uvp} \geq \tau_a = \frac{P_a \cdot 300 \cdot 16}{\pi (40)^3}$$

$$\rightarrow P_a \leq \frac{150 \cdot 0.58 \cdot \pi (40)^3}{16 \cdot 300} = 3.6\ kN$$

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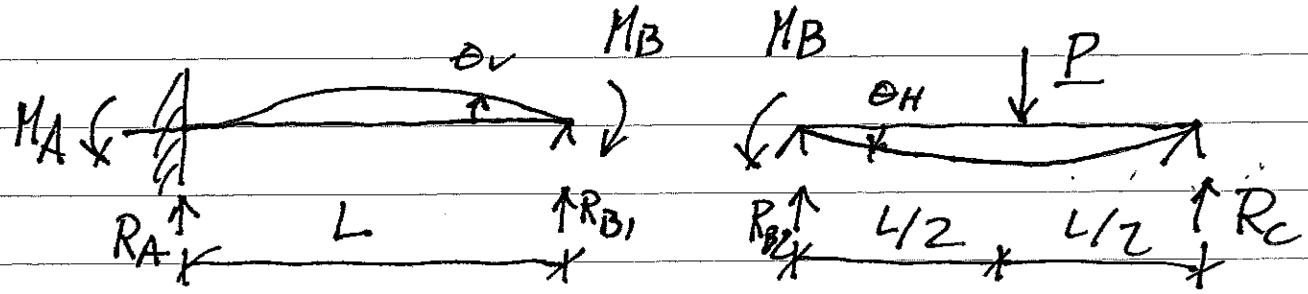
5) Gör snitt vid mittstöd inför snittmoment M_B.
Använd elementarfall FS 6.3, 6.5



kontinuitetsvillkor $\theta_V = \theta_H$

$$\theta_V = \frac{M_B L}{4EI} \qquad \theta_H = \frac{PL^2}{16EI} - \frac{M_B L}{3EI}$$

$$\Rightarrow \frac{1}{4} M_B = \frac{PL}{16} - \frac{1}{3} M_B$$

$$M_B = \frac{3}{28} PL$$

Stödreaktioner:

$$M_A = \frac{1}{2} M_B = \frac{3}{56} PL$$

$$R_A = -\frac{3}{2}\frac{M}{L} = -\frac{9}{56} P$$

$$R_{B1} = \frac{3}{2}\frac{M}{L} = \frac{9}{56} P$$

$$R_{B2} = \frac{P}{2} + \frac{M}{L} = \frac{14}{28} P + \frac{3}{28} P = \frac{17}{28} P$$

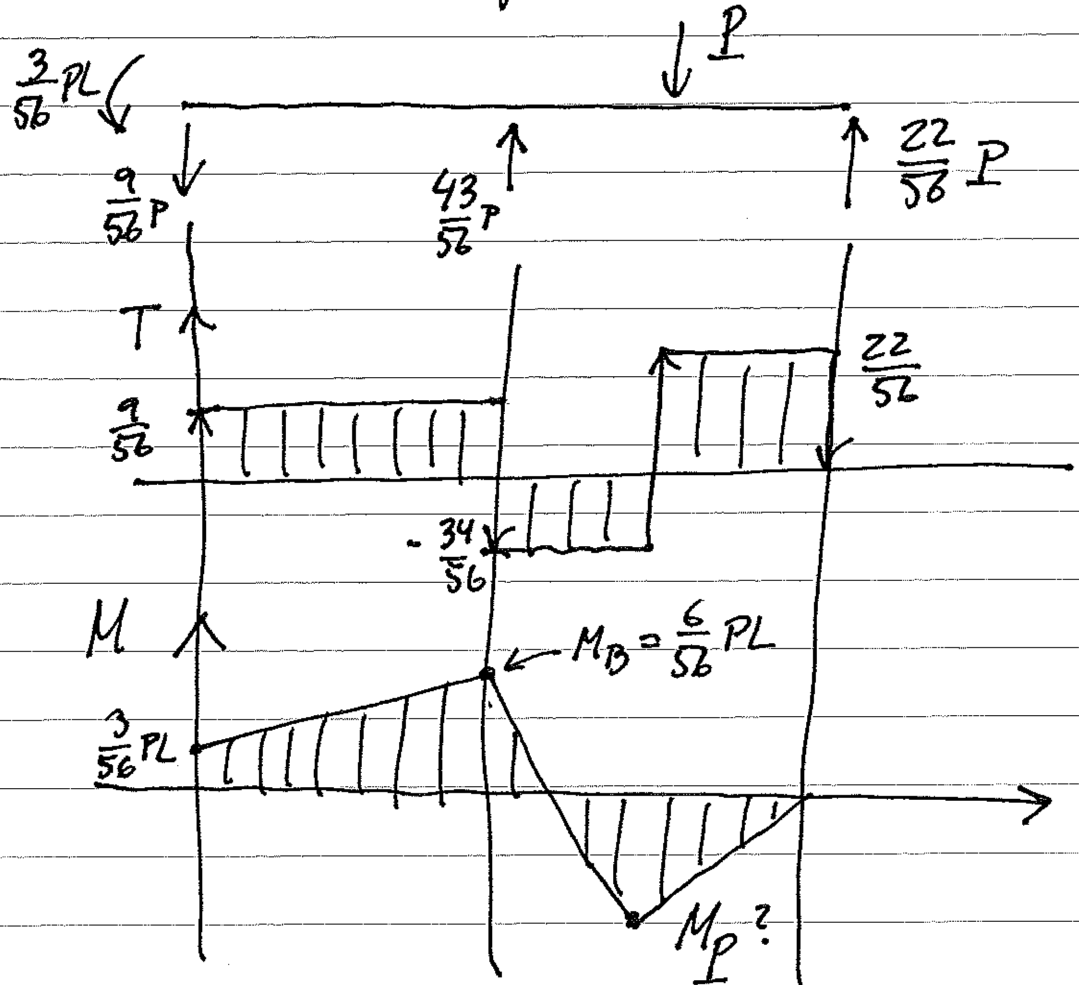
$$R_C = \frac{P}{2} - \frac{M}{L} = \frac{11}{28} P$$

5) forts

Total stödreaktion vid B

$$R_B = R_{B1} + R_{B2} = \frac{43}{56}P$$

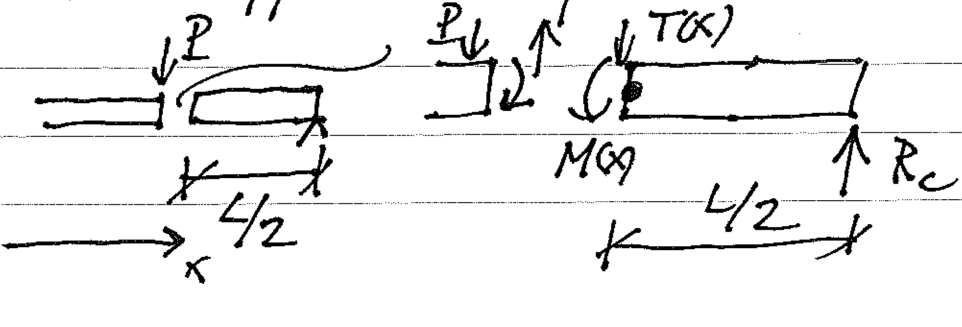
Bestäm max M (böjmoment). Rita T & M diagram:



$$\begin{cases} \frac{dT}{dx} = -q \\ \frac{dM}{dx} = T \end{cases}$$

M_P: snitta balken vid P

Ställ upp momentjämvikt:



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5/ forts

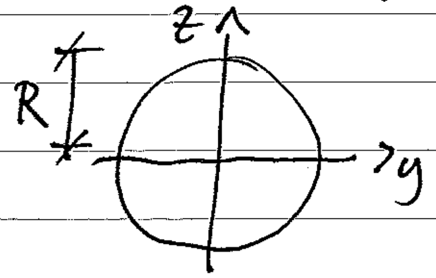
$$\curvearrowleft \; M(x) + R_c \frac{L}{2} = 0$$

$$M(x) = -R_c \cdot \frac{L}{2} = \frac{11}{56} PL$$

varmed $|M_b|_{max} = M_P = \frac{11}{56} PL$

LB p 79 & 83 $\quad \sigma = \frac{M \cdot z}{I}$

cirkulärt tvärsnitt



$$z_{max} = R$$

$$I = \frac{\pi R^4}{4}$$

$$|\sigma|_{max} \leq \sigma_{till} \Rightarrow \quad \sigma_{till} \leq \frac{11 \, PL \cdot 4}{56 \, \pi R^3}$$

$$P \leq \frac{56 \cdot \pi}{11 \cdot 4} \, \frac{\sigma_{till} \cdot R^3}{L} = \frac{56 \pi}{44} \, \frac{150 \cdot 6^3}{200}$$

$$P \leq 648 \; N$$