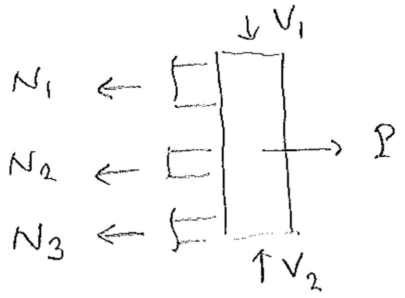
1) Friläggning av stel skiva



$\rightarrow: \; P - N_1 - N_2 - N_3 = 0 \quad (1)$

Kompatibilitet: $\delta_1 = \delta_2 = \delta_3 \quad (2)$

Konstitutiva ekv $\Rightarrow$

$$\delta_1 = \frac{N_1 L}{E \, 2A} \qquad \delta_2 = \frac{N_2 L}{E \cdot 5/4 \, A} \qquad \delta_3 = \frac{N_3 L}{E \cdot A}$$

insatt i (2) $\Rightarrow \; \dfrac{N_1}{2} = \dfrac{N_2 \cdot 4}{5} = N_3$

$$\Rightarrow \begin{cases} N_3 = N_1/2 \\ N_2 = \frac{5}{8} N_1 \end{cases} \quad \text{insatt i (1)} \Rightarrow$$

$$P - N_1 - \frac{5}{8} N_1 - \frac{N_1}{2} = 0 \Rightarrow \begin{cases} N_1 = \frac{8}{17} P \\ N_2 = \frac{5}{17} P \\ N_3 = \frac{4}{17} P \end{cases}$$

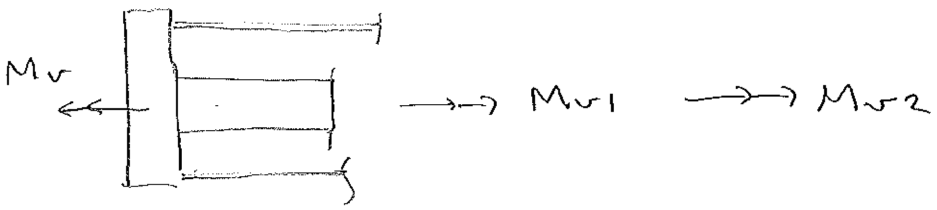
Spänningarna blir

$$\sigma_1 = \frac{4}{17} \frac{P}{A} \qquad \sigma_2 = \frac{4}{17} \frac{P}{A} \qquad \sigma_3 = \frac{4}{17} \frac{P}{A}$$

Störst tillåten spänning $\sigma_{till} = 200$ MPa

$\Rightarrow$ störst tillåten kraft $P = \dfrac{17}{4} \cdot A \cdot 200 = 170$ kN //

2) Friläggning av stel skruva



$$\rightarrow: \quad -M_v + M_{v1} + M_{v2} = 0$$

$$\Rightarrow M_{v2} = M_v - M_{v1} \qquad (1)$$

Deformationsvillkor

$$\varphi_1 = \varphi_2 \qquad (2)$$

Konstitutiva samband

$$\varphi_1 = \frac{M_{v1}\, L}{G_{stål}\, K_{stål}} \qquad \varphi_2 = \frac{M_{v2} \cdot L}{G_{AL}\, K_{AL}} \qquad (3)$$

$$(3) \text{ i } (2) \Rightarrow \quad M_{v2} = M_{v1} \frac{G_{AL} K_{AL}}{G_{stål} K_{stål}} \qquad (2')$$

$$(2') \text{ i } (1) \Rightarrow \quad M_{v1} \cdot \left(\frac{G_{stål} K_{stål} + G_{AL} K_{AL}}{G_{stål} K_{stål}} \right) = M_v$$

$$\Rightarrow \begin{cases} M_{v1} = M_v \cdot \dfrac{G_{stål} K_{stål}}{G_{stål} K_{stål} + G_{AL} K_{AL}} \\ M_{v2} = M_v \cdot \dfrac{G_{AL} \cdot K_{AL}}{G_{stål} K_{stål} + G_{AL} K_{AL}} \end{cases} \qquad (1')$$

där
$$\begin{cases} K_{stål} = \dfrac{\pi}{2} \cdot a^4 \approx 15{,}7 \cdot 10^3 \text{ mm}^4 \\ K_{AL} = 2\pi \cdot b^3 \cdot t \approx 101 \cdot 10^3 \text{ mm}^4 \end{cases}$$

$$\Rightarrow \begin{cases} M_{v1} \approx M_v \cdot 0{,}681 \\ M_{v2} \approx M_v \cdot 0{,}319 \end{cases} \qquad (1'')$$

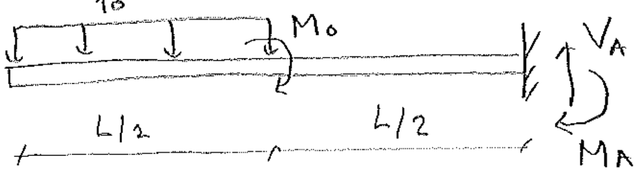
Största vridskjuvspänning (6-14), (6-4):

$$\tau_{max,1} = \frac{2 M_{v1} \cdot a}{\pi a^4} \approx 4{,}33 \cdot 10^{-4} M_v < 200 \quad [MPa]$$

$$\tau_{max,2} = \frac{M_{v2}}{2\pi b^2 t} \approx 6{,}35 \cdot 10^{-5} M_v < 100 \quad [MPa]$$

$\Rightarrow$ tillåtet vridmoment $M_v \approx \dfrac{100}{6.35 \cdot 10^{-5}} \approx \underline{\underline{1{,}57 \cdot 10^6 \; [Nmm]}}$

3a) Friläggning av balken:

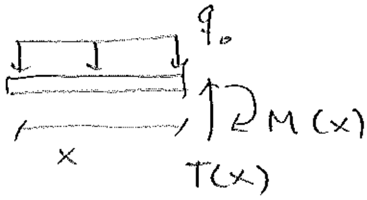


$$\uparrow: \; -q_0 L/2 + V_A = 0 \Rightarrow V_A = q_0 L/2$$

$$\overset{\curvearrowright}{A}: \; M_A + M_0 - q_0 L/2 \cdot 3L/4 = 0 \Rightarrow$$
$$M_A = q_0 L^2 \cdot 3/8 - M_0$$

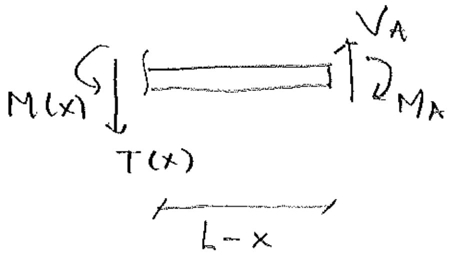
Snitta och frilägg balken

$0 < x < L/2$

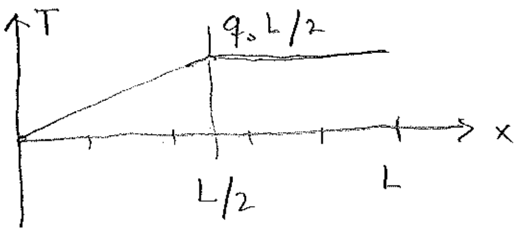


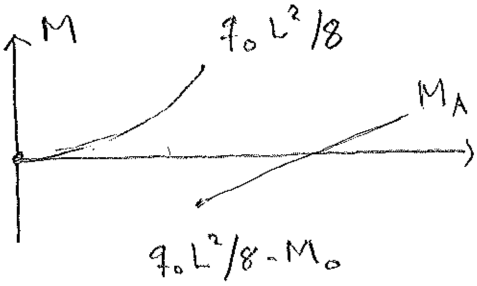
$$\uparrow: \; T(x) - q_0 x = 0 \Rightarrow T(x) = q_0 x \quad //$$
$$\overset{\curvearrowright}{x}: \; M(x) - q_0 x^2/2 = 0 \Rightarrow M(x) = q_0 x^2/2 \quad //$$

$L/2 < x < L$



$$\uparrow: \; -T(x) + V_A = 0 \Rightarrow T(x) = V_A = q_0 L/2$$
$$\overset{\curvearrowright}{x}: \; -M(x) + M_A - V_A \cdot (L-x) = 0$$
$$\Rightarrow M(x) = M_A - V_A (L-x) =$$
$$= q_0 L^2 \cdot 3/8 - M_0 - q_0 L/2 \, (L-x)$$





b) Max böjmoment

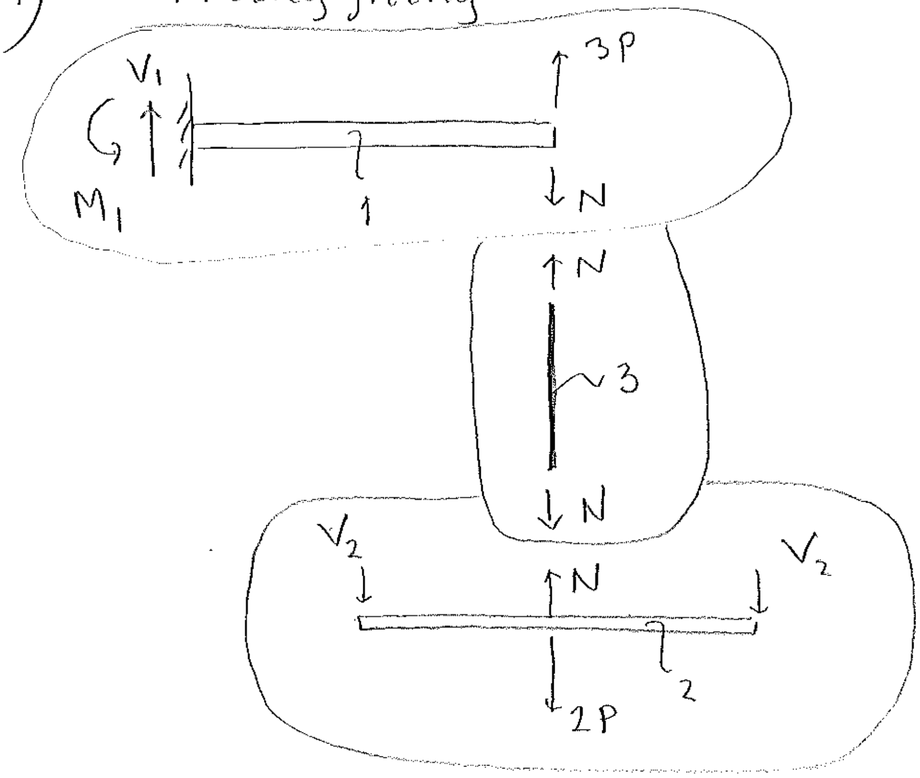
vid
$$\begin{cases} x = L/2^- \Rightarrow M = 500 \cdot 10^3 \text{ Nmm} \\ x = L/2^+ \Rightarrow M = 300 \cdot 10^3 \text{ Nmm} \\ x = L \Rightarrow M = 1{,}3 \cdot 10^6 \text{ Nmm} \end{cases}$$

dvs max böjmoment $|M|_{max} = 1{,}3 \cdot 10^6$ Nmm

Max böjnormalspänning

$$\sigma_{max} = \frac{|M|_{max} \, |z|_{max}}{I} = \frac{1{,}3 \cdot 10^6 \cdot 15}{20 \cdot 30^3/12} \approx 433 \text{ MPa}$$

4) Friläggning



Kompatibilitet: $\delta_3 = \delta_1 + \delta_2 \quad (*)$

där enl FS 6.4 $\qquad \delta_1 = \dfrac{(3P-N)(2L)^3}{3EI}$

6.3 $\qquad \delta_2 = \dfrac{(2P-N)(2L)^3}{48EI}$

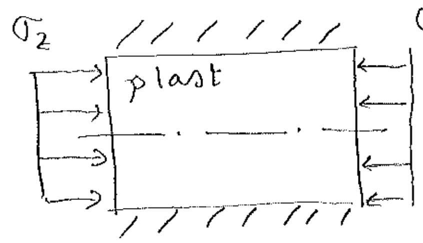
konstitutiva samband för stång: $\delta_3 = \dfrac{NL}{EA}$

Insatt i $(*) \Rightarrow \dfrac{NL}{EA} = \dfrac{8\cdot(3P-N)L^3}{3\cdot EI} + \dfrac{8}{48}\dfrac{(2P-N)L^3}{EI}$

$$\Rightarrow N\cdot\left[\frac{L}{EA} + \frac{17}{6}\cdot\frac{L^3}{EI}\right] = P\cdot\frac{25}{3}\frac{L^3}{EI} \Rightarrow$$

$$\Rightarrow \delta_3 = \frac{NL}{EA} = \frac{L}{EA}\cdot\frac{\frac{25}{3}\frac{PL^3}{EI}}{\frac{L}{EA}+\frac{17}{6}\frac{L^3}{EI}} = \frac{L}{EA}\cdot\frac{\frac{25}{3}P}{\frac{I}{A\cdot L^2}+\frac{17}{6}} = \frac{25}{3}\frac{PL}{EA}\left(\frac{1}{\frac{I}{AL^2}+\frac{17}{6}}\right)$$

$$= \frac{25}{3}\frac{PL}{EA}\cdot\frac{6\cdot AL^2}{6I+17\cdot AL^2} \quad //$$

5) 

$\sigma_z = F/\pi a^2$ (1)

Enl. (11-18), (11-19) jfr också (11-75), (11-76)

$$\sigma_r = A - \frac{B}{r^2} \qquad \sigma_\varphi = A + \frac{B}{r^2}$$

Konstanten B fås från villkoret

$|\sigma_r|, |\sigma_\varphi| < \infty$ då $r \to 0$

$\Rightarrow$ $B = 0$ $\Rightarrow$ $\sigma_r = \sigma_\varphi = A$ (2)

Randvillkor $u_r(r=a) = 0$ $\Rightarrow$ $\varepsilon_\varphi(a) = 0$

(ty $u_r = r\,\varepsilon_\varphi$)

Hookes lag

$$\varepsilon_\varphi = \frac{1}{E_{plast}}\left[\sigma_\varphi - \nu(\sigma_r + \sigma_z)\right] \overset{(1),(2)}{=}$$

$$= \frac{1}{E_{plast}}\left[A - \nu\left(A + \frac{F}{\pi a^2}\right)\right] = 0 \qquad \text{(jfr 11-81)}$$

$$\Rightarrow \quad A = \frac{\nu F}{\pi a^2(1-\nu)}$$

von Mises effektivspänning FS 9

$$\sigma_{eM} = \frac{1}{\sqrt{2}}\sqrt{\underbrace{(\sigma_r - \sigma_\varphi)^2}_{=0} + (\sigma_r - \sigma_z)^2 + (\sigma_\varphi - \sigma_z)^2} =$$

$$= \frac{1}{\sqrt{2}}\sqrt{2\cdot(\sigma_r - \sigma_z)^2} = |\sigma_r - \sigma_z| = \frac{F}{\pi a^2}\cdot\frac{1-2\nu}{1-\nu} \quad //$$

$$\sigma_{eT} = |\sigma_r - \sigma_z| = \sigma_{eM} \quad //$$