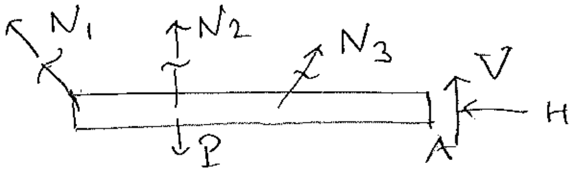
1



$\uparrow: N_1\sqrt{3}/2 + N_2 + N_3\sqrt{3}/2 - P + V = 0$

$\overset{\curvearrowleft}{A}: N_1\sqrt{3}/2 \cdot L + (N_2 - P)\,2L/3 + N_3\sqrt{3}/2\; L/3 = 0$

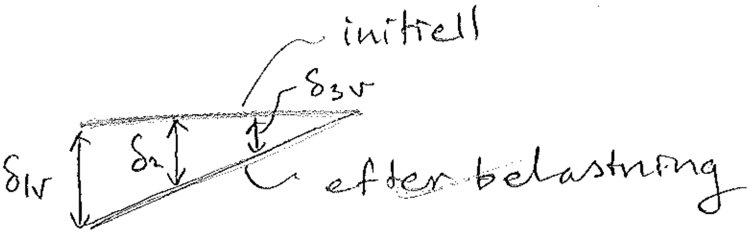
$\Rightarrow N_1 \frac{\sqrt{3}}{2} + N_2 \frac{2}{3} + N_3 \frac{1}{2\sqrt{3}} - P\cdot\frac{2}{3} = 0$

$\Rightarrow N_3 = 2\cdot\sqrt{3}\cdot\left[P\cdot\frac{2}{3} - N_2\cdot\frac{2}{3} - N_1\frac{\sqrt{3}}{2}\right] = \boxed{P\cdot\frac{4}{\sqrt{3}} - N_2\frac{4}{\sqrt{3}} - N_1\frac{1}{4}}$ (1)

Konstitutiva samband (GL 2-14)

$N_1 = \frac{EA}{H\cdot 2/\sqrt{3}}\cdot\delta_1 \qquad N_2 = \frac{EA}{H}\cdot\delta_2 \qquad N_3 = \frac{EA}{H\cdot 2/\sqrt{3}}\cdot\delta_3$ (2)

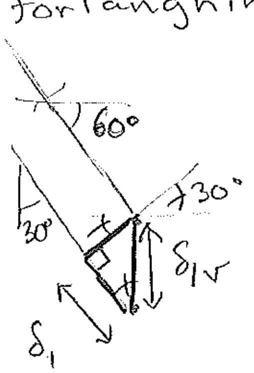
Kompatibilitet



$\frac{\delta_{1v}}{L} = \frac{\delta_2}{2L/3} = \frac{\delta_{3v}}{L/3}$

$\Rightarrow \boxed{\delta_{1v} = \frac{3}{2}\delta_2 = 3\,\delta_{3v}}$ (3)

förlängning längs stång 1 och 3



$\delta_1 = \delta_{1v}\frac{\sqrt{3}}{2}$ pss $\delta_3 = \delta_{3v}\cdot\frac{\sqrt{3}}{2}$

$\overset{(3)}{\Rightarrow} \boxed{\frac{2}{\sqrt{3}}\delta_1 = \frac{3}{2}\delta_2 = 3\cdot\frac{2}{\sqrt{3}}\cdot\delta_3 = 2\sqrt{3}\cdot\delta_3}$ (3')

(2) i (3') ⇒

$$\underbrace{\frac{3}{2}\cdot\cancel{\frac{H}{EA}}}_{\delta_1}\cdot N_2 = \underbrace{\frac{2}{\sqrt{3}}\cdot\frac{2}{\sqrt{3}}}_{4/3}\underbrace{\cancel{\frac{H}{EA}}}_{\delta_1}\cdot N_1 \Rightarrow \boxed{N_2 = \frac{2}{3}\cdot\frac{4}{3}\cdot N_1 = \frac{8}{9}N_1}$$

(2) i (3') ⇒

$$\underbrace{2\cdot\sqrt{3}\cdot\underbrace{\cancel{\frac{H\cdot 2/\sqrt{3}}{EA}}}_{\delta_3}}_{4}\cdot N_3 = \underbrace{\frac{2}{\sqrt{3}}\cdot\frac{2}{\sqrt{3}}}_{4/3}\underbrace{\cancel{\frac{H}{EA}}}_{\delta_1}N_1 \Rightarrow \boxed{N_3 = \underbrace{\frac{1}{4}\cdot\frac{4}{3}}_{1/3}\cdot N_1}$$

insatt i (1) ⇒

$$\frac{1}{3}N_1 = P\cdot\frac{4}{\sqrt{3}} - \frac{8}{9}\cdot N_1\cdot\frac{4}{\sqrt{3}} - N_1\cdot\frac{1}{4} \Rightarrow$$

$$N_1\cdot\left[\frac{1}{3}+\frac{32}{9\cdot\sqrt{3}}+\frac{1}{4}\right] = P\cdot\frac{4}{\sqrt{3}} \Rightarrow \boxed{N_1 \approx 0{,}876\cdot P}$$

$$N_2 \approx 0{,}779\cdot P$$

$$N_3 \approx 0{,}292\cdot P$$

Rotation 5° ⇒ $$\underbrace{5\cdot\frac{\pi}{180}}_{5\ \text{rad}} = \frac{\delta_2}{2L/3} \overset{(2)}{=} \frac{3}{2L}\cdot\frac{H}{EA}\cdot\underbrace{N_2}_{0{,}779\cdot P}$$

$$\Rightarrow P = 5\cdot\frac{\pi}{180}\cdot\frac{2\cdot L\cdot EA}{3\cdot H\cdot 0{,}779} \approx 87\ [N]$$ //

Spänningarna

$$\begin{cases} \sigma_1 = N_1/A = 9/8\cdot N_2/A = 9/8\cdot\sigma_2 \approx 0{,}38\,[MPa] \\ \sigma_2 = N_2/A \approx 0{,}34\ [MPa] \\ \sigma_3 = N_3/A = 1/3\cdot N_1/A = 1/3\cdot\sigma_1 \approx 0{,}127\ [MPa] \end{cases}$$

(GL 6-1)

(2.) $P = M_v \cdot \omega$ där M_v är vridande moment

$\omega = 2\pi$ rad/s

Bestäm nu M_v:

Spänning i axeldelarna (GL 6-14)

$$\tau_{max} = \frac{2 \cdot M_v \cdot d/2}{\pi \cdot (d/2)^4} = \frac{M_v \cdot d \cdot 16}{\pi \cdot d^4} = \frac{M_v \cdot 16}{\pi d^3} \leq \tau_{till,axel}$$

$\Rightarrow M_v \leq 235{,}6$ [Nm]

$\Rightarrow P = M_v \cdot 2\pi \approx 1480$ [W] //

Hylsan (GL 6-14):

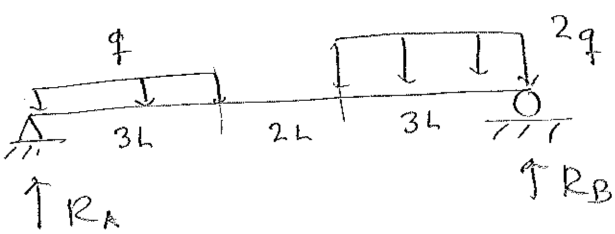
$$\tau_{max} = \frac{2 \cdot M_v \cdot D/2}{\pi \cdot ((D/2)^4 - (d/2)^4)} = \frac{M_v \cdot D \cdot 16}{\pi \cdot (D^4 - d^4)} \leq \tau_{till,hylsa}$$

insättning av numeriska värden ger:

$$\frac{235{,}6 \cdot 10^3 \cdot 25 \cdot 16}{\pi \cdot (25^4 - 20^4)} \approx 130 \text{ MPa}$$

vilket är mindre än $\tau_{till,hylsa}$

dvs JA den håller!



Stödreaktioner ur jämvikt:

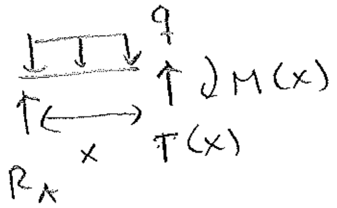
$$\uparrow: R_A + R_B - q\cdot 3L - 2q\cdot 3L = 0 \Rightarrow R_A + R_B = 9\cdot qL \quad (1)$$

$$\overset{\curvearrowright}{A}: (q\cdot 3L)\frac{3L}{2} + (2q\cdot 3L)\cdot\left(5L + \frac{3L}{2}\right) - R_B\cdot 8L = 0$$

$$\Rightarrow \boxed{R_B = \frac{1}{8}qL\cdot\left[\frac{9}{2} + 6\cdot\frac{13}{2}\right] = qL\cdot\frac{87}{16}}$$

$$\text{insatt i (1)} \Rightarrow \boxed{R_A = qL\cdot\frac{57}{16}}$$

Snitta $0 < x < 3L$



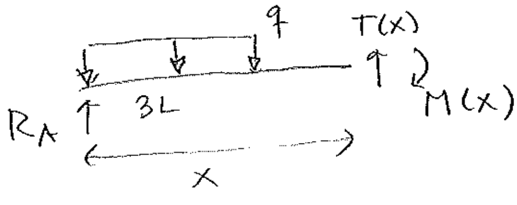
$$\uparrow: qL\cdot\frac{57}{16} - qx + T(x) = 0$$

$$\Rightarrow T(x) = q\cdot\left[-\frac{57}{16}\cdot L + x\right]$$

$$\overset{\curvearrowright}{x}: M(x) - qx\cdot\frac{x}{2} + qL\cdot\frac{57}{16}\cdot x = 0$$

$$\Rightarrow M(x) = q\cdot x\cdot\left[-\frac{57}{16}L + \frac{x}{2}\right]$$

Snitta $3L < x < 5L$:



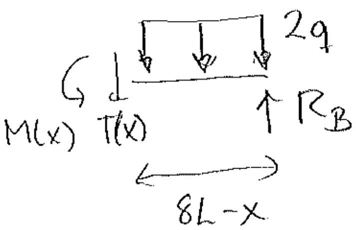
$$\uparrow: qL\cdot\frac{57}{16} - q\cdot 3L + T(x) = 0$$

$$\Rightarrow T(x) = -\frac{9}{16}\cdot qL$$

$$\overset{\curvearrowright}{x}: M(x) + qL\cdot\frac{57}{16}\cdot x - q\cdot 3L\cdot\left(x - \frac{3L}{2}\right) = 0$$

$$\Rightarrow M(x) = -\frac{9}{16}qL\cdot x - \frac{9}{2}qL^2$$

Snitta $5L < x < 8L$:



$$\uparrow: -T(x) - 2q\cdot(8L - x) + qL\cdot\frac{87}{16} = 0$$

$$\Rightarrow T(x) = 2qx - \frac{169}{16}qL$$

$$\overset{\curvearrowright}{x}: -M(x) + \underline{2q\frac{(8L-x)^2}{2}} - qL\cdot\frac{87}{16}(8L-x) = 0$$

$$\Rightarrow M(x) = qx^2 + qxL\left(-16 + \frac{87}{16}\right) + qL^2\left(64 - \frac{87}{2}\right) =$$

$$= qx^2 - \frac{169}{16} qLx + \frac{41}{2} qL^2$$

Största böjmoment fås i extrempunkter eller ändpunkter

$$\begin{cases} M(0) = 0 \\ M(3L) = q \cdot 3L \cdot \left[-\frac{57}{16} \cdot L + \frac{3L}{2}\right] = -qL^2 \frac{99}{16} \approx -6{,}19 \cdot qL^2 \\ 0 < x < 3L : M' = T = 0 \Rightarrow x = \frac{57}{16} L \text{ vilket är } > 3L \\ \text{dvs orimligt} \Rightarrow \text{ingen extrempunkt} \end{cases}$$

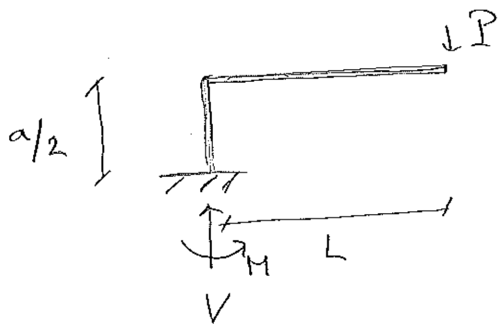
$$\begin{cases} M(3L) = -qL^2 \frac{99}{16} \\ M(5L) = -\frac{9}{16} \cdot qL \cdot 5L - \frac{9}{2} qL^2 = -\frac{117}{16} \cdot qL^2 \approx -7{,}31\, qL^2 \\ \text{ingen extrempunkt ty } M' = T \neq 0 \end{cases}$$

$$\begin{cases} M(5L) = -\frac{117}{16} qL^2 \\ M(8L) = 0 \\ M' = T = 0 \Rightarrow x = \frac{169}{32} \cdot L \approx 5{,}28 \cdot L \\ M\left(\frac{169}{32} L\right) = -\frac{7569}{1024} qL^2 \approx -7{,}39 \cdot qL^2 \end{cases}$$

Största böjmoment till belopp $= 7{,}39\, qL^2$ //

$$\sigma_{max} = \frac{M_{max} \cdot H/2}{B H^3/12} \approx 100 \; [MPa]$$

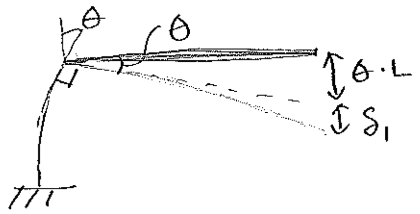
4) Pga symmetri räcker det att studera halva strukturen:

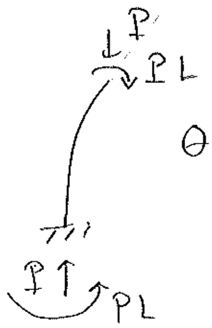


$\uparrow: \; V - P = 0 \Rightarrow V = P$

$\curvearrowright: \; P \cdot L - M = 0 \Rightarrow M = P \cdot L$

Deformationsbild:





$$\theta = \{\text{enl FS } 6.4\} = \frac{(P \cdot L) \cdot a/2}{EI} = \frac{P \cdot L \cdot a}{2 \cdot EI}$$

Nedböjningen vid lasten P fås nu som

$\delta_1 + \theta \cdot L$ där δ_1 fås från

elfall 6.4

(ty given vinkel θ)

$$\delta_1 = \frac{P \cdot L^3}{3EI}$$

Totalt fås således att ändarna närmar sig:

$$2 \cdot (\delta_1 + \theta \cdot L) = \frac{2 \cdot P L^3}{3EI} + \frac{P L^2 a}{EI} =$$

$$= \frac{P L^2}{EI} \left[\frac{2}{3} L + a \right] \quad //$$

5) Ångpanneformlerna (GL 9-105)

$$\begin{cases} \sigma_r \approx 0 & \tau_{z\varphi} = 0 \\ \sigma_z = \dfrac{pa}{2t} \\ \sigma_\varphi = \dfrac{pa}{t} \end{cases}$$

dvs biaxiellt spänningstillstånd.

FS s.13 $(x \to z,\ y \to \varphi)$

$$\begin{cases} \sigma(\varphi) = \sigma_z \cos^2(\varphi) + \sigma_\varphi \cdot \sin^2(\varphi) + \underbrace{\tau_{z\varphi}}_{=0} \cdot \sin(2\varphi) = \\ \quad = \dfrac{p \cdot a}{t}\left[\dfrac{1}{2} \cdot \cos^2\varphi + \sin^2\varphi\right] \\ \tau(\varphi) = \dfrac{\sigma_\varphi - \sigma_z}{2} \cdot \sin(2\varphi) + \underbrace{\tau_{\varphi z}}_{=0} \cdot \cos(2\varphi) \end{cases}$$

Enl. uppg. $\varphi = \pi/4 \Rightarrow \sigma = 124$ [MPa]

$$\Rightarrow \quad 124 = \frac{p \cdot 250}{12} \cdot \underbrace{\left[\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2}\right]}_{3/4} \Rightarrow p \approx 7{,}94 \text{ MPa}$$

von Mises effektivspänning, FS s.14

$$\sigma_{eM} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_z - \sigma_\varphi)^2 + (\sigma_r - \sigma_z)^2 + (\sigma_r - \sigma_\varphi)^2} \approx$$

$$= \frac{pa}{t} \frac{1}{\sqrt{2}} \sqrt{\left(\frac{1}{2} - 1\right)^2 + \left(\frac{1}{2}\right)^2 + 1^2} \approx \frac{pa}{t} \frac{\sqrt{3}}{2}$$

≈ 143 [MPa]