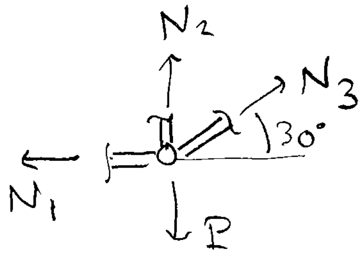
Jämvikt för knutpunkt



$$\uparrow: \; N_2 + N_3 \underbrace{\sin 30°}_{=1/2} - P = 0$$

$$\rightarrow: \; N_3 \underbrace{\cos 30°}_{=\sqrt{3}/2} - N_1 = 0$$

$$\Rightarrow N_1 = N_3 \cdot \sqrt{3}/2 \;, \quad N_2 = P - N_3/2 \qquad (1)$$

Konstitutiva samband

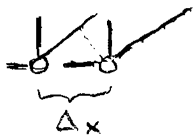
$$\delta_i = \frac{N_i L_i}{EA} \quad \text{för} \quad i = 1, 2, 3$$

$$\text{där} \quad L_1 = L \;, \quad L_2 = 2L \cdot \tan(30°) = \frac{2L}{\sqrt{3}}$$

$$L_3 = \frac{2L}{\cos(30°)} = \frac{4L}{\sqrt{3}} \qquad (2)$$

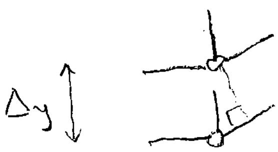
Kompatibilitet

Horisontell förflyttning av knutpunkt Δ_x



$$\Rightarrow \begin{cases} \delta_1 = \Delta_x \\ \delta_2 = 0 \\ \delta_3 = -\Delta_x \cos(30°) = -\Delta_x \sqrt{3}/2 \end{cases}$$

Vertikal förflyttn av knutpunkt Δ_y



$$\Rightarrow \begin{cases} \delta_1 = 0 \\ \delta_2 = \Delta_y \\ \delta_3 = \Delta_y \sin(30°) = \Delta_y/2 \end{cases}$$

$$\Rightarrow \delta_3 = -\delta_1 \cdot \sqrt{3}/2 + \delta_2/2 \overset{(2)}{\Rightarrow} \frac{N_3 \cdot 4}{\sqrt{3}} = -\frac{\sqrt{3}}{2} N_1 + \frac{1}{2} \frac{N_2 \cdot 2}{\sqrt{3}}$$

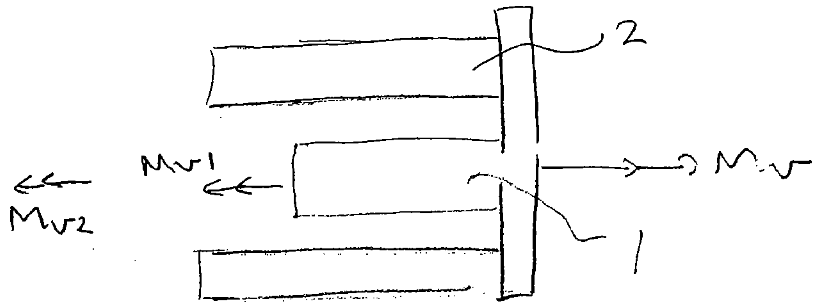
$$\overset{(1)}{\Rightarrow} N_3 \left(\frac{4}{\sqrt{3}} + \frac{\sqrt{3}}{2} \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}} \frac{1}{2} \right) = \frac{1}{\sqrt{3}} \cdot P \Rightarrow N_3 \approx 0{,}172 \cdot P$$

$$\Rightarrow N_2 \approx 0{,}914 \cdot P \;, \quad N_1 \approx 0{,}149 \cdot P$$

$\Rightarrow$ Störst spänning i stång 2:

$$\sigma_2 = \frac{N_2}{A} \leq \sigma_{till} \Rightarrow A \geq \frac{N_2}{\sigma_{till}} = 9{,}14\,[mm^2]$$

a) Jämvikt för stel skiva



$$\rightarrow: M_v - M_{v1} - M_{v2} = 0 \Rightarrow$$

$$\boxed{M_{v1} = M_v - M_{v2}} \qquad (1)$$

Deformationsvillkor

vridningen av axlarna lika stor

$$\Rightarrow \boxed{\varphi_1 = \varphi_2} \qquad (2)$$

Konstitutiva samband

$$\varphi_1 = \frac{M_{v1} L}{G K_1} \qquad \varphi_2 = \frac{M_{v2} L}{G K_2} \qquad (3)$$

(3) i (2) $\Rightarrow$ $\boxed{M_{v1} = M_{v2} \frac{K_1}{K_2}}$ insatt i (1)

$$\Rightarrow M_{v2} \frac{K_1}{K_2} = M_v - M_{v2} \Rightarrow \boxed{M_{v2} = \frac{M_v K_2}{K_1 + K_2}}$$

$$\boxed{M_{v1} = \frac{M_v K_1}{K_1 + K_2}}$$

Rotationen av skivan: $\varphi_1 = \varphi_2 = \boxed{\frac{M_v L}{G(K_1 + K_2)}}$

där $\begin{cases} K_1 = \frac{\pi}{2} b^4 \\ K_2 = \frac{\pi}{2} \left[(2b)^4 - a^4 \right] \end{cases}$

Med numeriska värden fås:

$$\varphi_1 = \varphi_2 \approx 2{,}3°$$

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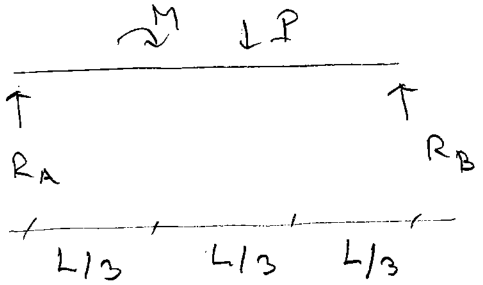
b) Största vridskjuvspänning fås enligt (6-14) i Lundh

$$\begin{cases} \tau_{max,1} = \dfrac{2 M_{v1}\, b}{\pi\, b^4} = \dfrac{2 M_{v1}}{\pi\, b^3} \approx 324 \text{ MPa} \\ \tau_{max,2} = \dfrac{2 M_{v2}\, 2b}{\pi\left((2b)^4 - a^4\right)} \approx 648 \text{ MPa} \end{cases}$$

// $M_{v1} = \dfrac{G K_1}{L} \varphi_1 = M_v \dfrac{K_1}{K_1 + K_2} \approx 509 \text{ Nm}$

$M_{v2} = M_v \dfrac{K_2}{K_1 + K_2} \approx 7091 \text{ Nm}$ //

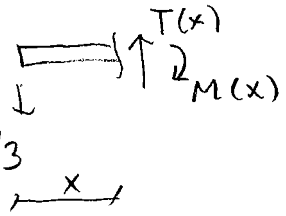
3) Jämvikt för hela balken



$$\begin{cases} \uparrow: R_A + R_B - P = 0 \\ \overset{\curvearrowright}{A}: M + 2PL/3 - R_B \cdot L = 0 \end{cases}$$

$$\Rightarrow \begin{cases} R_B = M/L + 2P/3 = P + 2P/3 = 5P/3 \\ R_A = P - R_B = -2P/3 \end{cases}$$

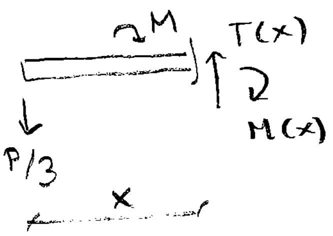
Snitta $\underline{0 \leq x \leq L/3}$



$\uparrow: T(x) - 2P/3 = 0 \Rightarrow T(x) = 2P/3$

$\overset{\curvearrowright}{x}: M(x) - 2P/3 \cdot x = 0 \Rightarrow M(x) = 2P/3 \cdot x$

$\underline{L/3 \leq x \leq 2L/3}$

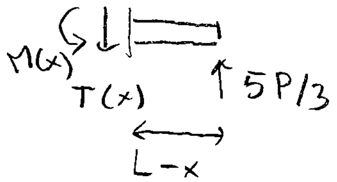


$\uparrow: T(x) - 2P/3 = 0 \Rightarrow T(x) = 2P/3$

$\overset{\curvearrowright}{x}: M + M(x) - 2P/3\, x = 0 \Rightarrow M(x) = 2P/3\, x - M$

$= P\left(\frac{2}{3}x - L\right)$

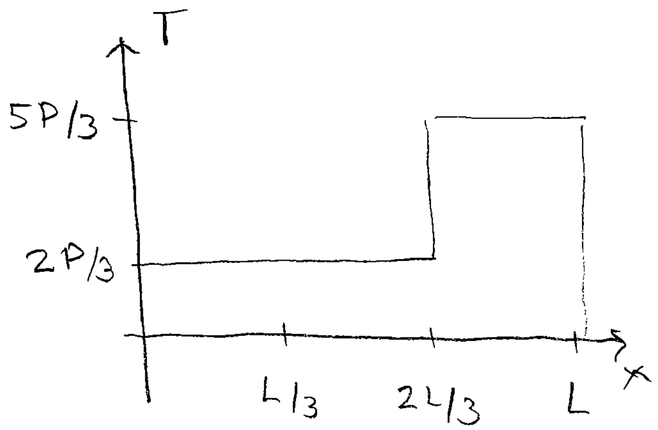
$\underline{2L/3 \leq x \leq L}$

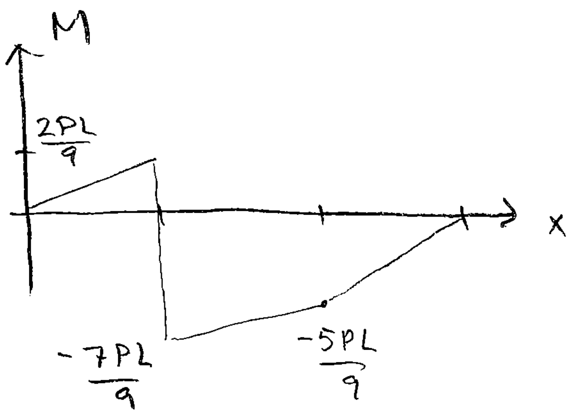


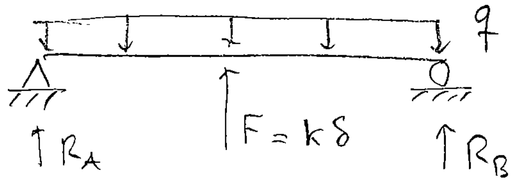
$\uparrow: -T(x) + 5P/3 = 0 \Rightarrow T(x) = 5P/3$

$\overset{\curvearrowright}{x}: -M(x) - 5P/3\,(L-x) = 0 \Rightarrow$

$M(x) = 5P/3 \cdot (x - L)$







Jämvikt:

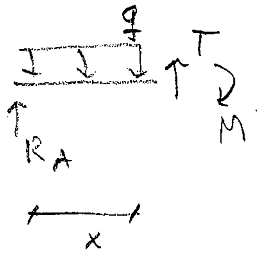
$\uparrow$: $R_A + R_B + F - q \cdot 2L = 0$

$\overset{\curvearrowright}{A}$: $q\,2L^2 - F \cdot L - R_B \cdot 2L = 0$

$\Rightarrow R_B = qL - F/2 \quad , \quad R_A = R_B$

Moment $M(x)$ (av symmetriskäl är det tillräckligt att studera $0 \le x \le L$)

$0 \le x \le L$:



$\overset{\curvearrowright}{x}$: $M(x) - qx^2/2 + R_A \cdot x = 0$

$\Rightarrow M(x) = (-qL + F/2)x + qx^2/2$

$\Rightarrow M'(x) = -qL + F/2 + qx$

Störst (till belopp) moment fås för $x = 0$, $x = L$ eller då $M'(x) = 0$

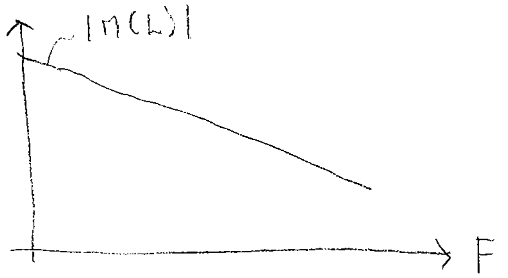
$M(0) = 0$

$M(L) = (-qL + F/2)L + qL^2/2 = -qL^2/2 + FL/2 = L/2\,(F - qL)$

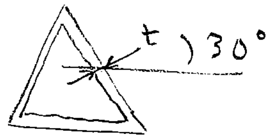
$M'(x^*) = 0 \Rightarrow x^* = L - F/(2q)$

$M(x^*) = \underbrace{(-qL + F/2)(L - F/(2q))}_{-q(L - F/(2q))^2} + \frac{q}{2}\left(L - \frac{F}{2q}\right)^2 =$

$= -q/2\left(L - \frac{F}{2q}\right)^2 = -\frac{1}{2q}\left(qL - F/2\right)^2$



Yttröghetsmoment



$$I_y = \frac{h t^3}{12} + h \cdot t \cdot \left(\frac{\sqrt{3}}{2} h \cdot \frac{1}{3} \right)^2 +$$

$$+ 2 \cdot \left[\frac{t}{\sqrt{3}/2} \left(h \cdot \sqrt{3}/2 \right)^3 \frac{1}{12} + t \cdot h \left(h \frac{\sqrt{3}}{2} \left(\frac{1}{2} - \frac{1}{3} \right) \right)^2 \right]$$

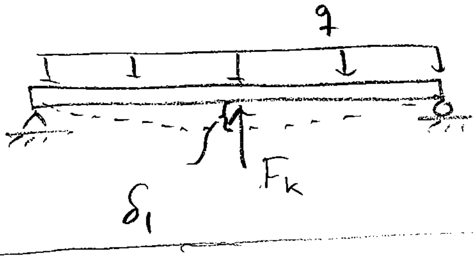
$$\approx 750225 \text{ mm}^4$$

Max böjspänning

$$\sigma_{max} = \frac{|M|_{max} \, |z|_{max}}{I_y} = \frac{7PL/9 \cdot 2/3 \, \sqrt{3}/2 \, h}{I_y} \leq 200$$

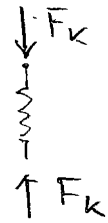
$$\Rightarrow P \leq 334 \, [N]$$

4)



nedböjn i mitten fås från FS 6.3

$$\delta_1 = -\frac{F_k(2L)^3}{48EI} + \frac{q\,5\cdot(2L)^4}{384\,EI}$$



$$F_k = k\cdot\delta = \frac{1}{10}\frac{EI}{L^3}\,\delta$$

FS 6.4

$$\delta_2 = \frac{F_k\cdot L^3}{3\,EI}$$

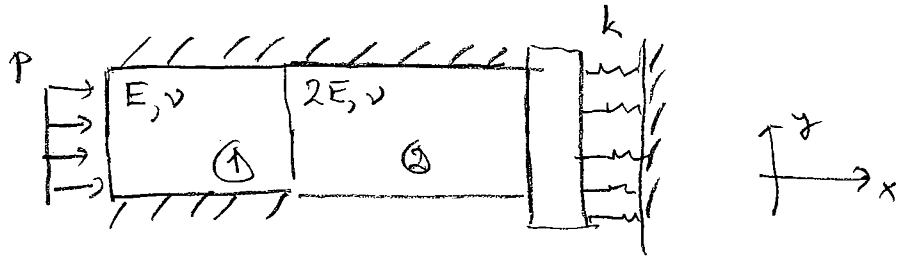
Kompatibilitet:

$$\delta = \delta_1 - \delta_2 \Rightarrow \frac{10\cancel{L^3}}{\cancel{E}I}\cdot F_k = \frac{\cancel{L^3}}{\cancel{EI}}\left[-\frac{F_k}{6} + \frac{5}{24}qL - \frac{F_k}{3}\right]$$

$$\Rightarrow F_k = \frac{5qL}{244}$$

$$\Rightarrow \delta_2 = \frac{5}{244}qL\cdot\frac{L^3}{3\,EI} = \frac{5}{732}\frac{qL^4}{EI} \approx 0{,}00683\,\frac{qL^4}{EI}$$

//

5) 

Del 1

$$\sigma_{x1} = -p, \quad \sigma_{y1} = ?, \quad \sigma_{z1} = 0$$

$$\varepsilon_{x1} = ?, \quad \varepsilon_{y1} = 0, \quad \varepsilon_{z1} = ?$$

Del 2

$$\sigma_{x2} = -p, \quad \sigma_{y2} = 0, \quad \sigma_{z2} = 0$$

$$\varepsilon_{x2} = ?, \quad \varepsilon_{y2} = ?, \quad \varepsilon_{z2} = ?$$

Hookes lag FS 8

$$\varepsilon_{y2} = \frac{1}{2E}\left[\sigma_{y2} - \nu(\sigma_{x2} + \sigma_{z2})\right] = \frac{1}{2E}\left[0 - \nu\cdot(-p)\right] =$$

$$= \frac{\nu \cdot p}{2E}$$

$$\Rightarrow \delta_{y2} = \varepsilon_{y2}\cdot L = \frac{\nu\cdot p}{2E}\cdot \cancel{L} = \cancel{L}/100 \Rightarrow$$

$$p = \frac{2E}{\nu\cdot 100} \approx 13{,}3 \quad [\text{GPa}]$$