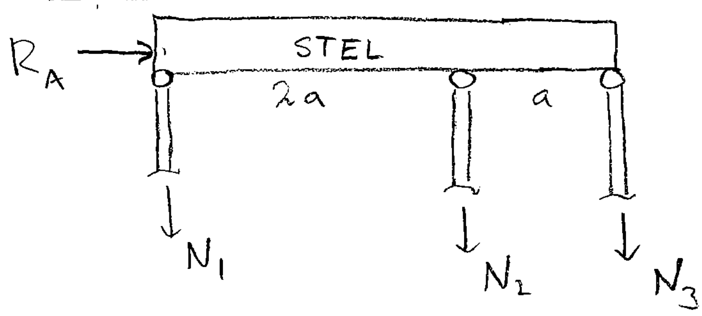
1 Jämvikt:



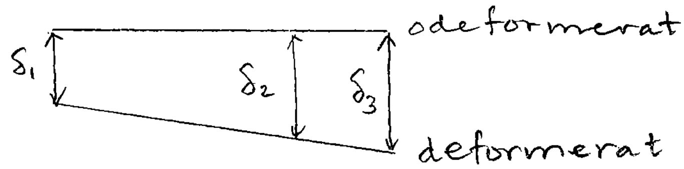

$$\begin{cases} \uparrow: -N_1 - N_2 - N_3 = 0 \\ \overset{\curvearrowright}{A}: N_2 \cdot 2a + N_3 \cdot 3a = 0 \end{cases} \Rightarrow \begin{cases} N_2 = -N_3 \cdot 3/2 \\ N_1 = -N_2 - N_3 = N_3/2 \end{cases} \qquad (1)$$

Konstitutiva samband

$$\delta_1 = \frac{N_1 L}{EA} \qquad \delta_2 = \varepsilon_2 \cdot 2L = \left(\frac{N_2}{EA} + \alpha \Delta T\right) 2L$$

$$\delta_3 = \frac{N_3 L}{EA} \qquad (2)$$

Kompatibilitet




$$\frac{\delta_3 - \delta_1}{3a} = \frac{\delta_2 - \delta_1}{2a} \Rightarrow 2\delta_3 - 2\delta_1 = 3\delta_2 - 3\delta_1 \Rightarrow$$

$$\boxed{\delta_1 - 3\delta_2 + 2\delta_3 = 0} \qquad (3)$$

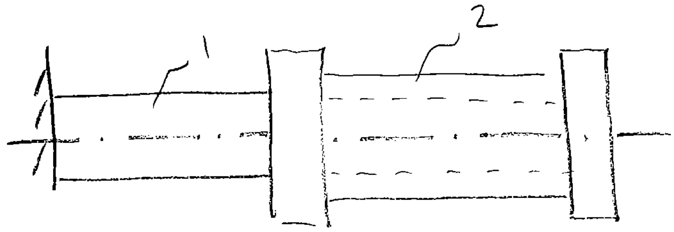
$$(2) \text{ i } (3) \Rightarrow \frac{N_1 L}{EA} - 3 \cdot \left(\frac{N_2}{EA} + \alpha \Delta T\right) 2L + \frac{2 N_3 L}{EA} = 0 \qquad (3')$$

$$(1) \text{ i } (3') \Rightarrow \frac{N_3 \cancel{L}}{2EA} - 3 \cdot \left(-\frac{N_3 \cdot 3}{2EA} + \alpha \Delta T\right) 2\cancel{L} + \frac{2 N_3 \cancel{L}}{EA} = 0$$

$$\Rightarrow \frac{N_3}{EA}\left(\frac{1}{2} + 9 + 2\right) = 6 \alpha \Delta T \Rightarrow \boxed{N_3 = \alpha \Delta T \, EA \frac{12}{23}}$$

$$\text{insatt i } (1) \Rightarrow \begin{cases} \boxed{N_1 = \alpha \Delta T \, EA \frac{6}{23}} \\ \boxed{N_2 = -\alpha \Delta T \, EA \; 18/23} \end{cases}$$

2)




a) Total vridning: $\varphi_{tot} = \varphi_1 + \varphi_2 = \dfrac{M_{v1} L}{G K_1} + \dfrac{M_{v2} L}{G K_2}$

där $M_{v1} = 2M$, $M_{v2} = M$

$$K_1 = \pi d^4/32 \qquad K_2 = \pi/32 \left( (2a)^4 - a^4 \right) = \frac{15}{32}\pi a^4$$

$$G = \frac{E}{2(1+\nu)}$$

$$\Rightarrow \varphi_{tot} = 3 \cdot \frac{\pi}{180} = 5{,}517 \cdot 10^{-4} + \frac{2648{,}34}{d^4} \Rightarrow d \approx 15 \text{ mm} //$$

b) Spänning på mantelytan

$$\begin{cases} \tau_1 = \dfrac{M_{v1} \cdot (d/2)}{K_1} = \dfrac{M \cdot d}{K_1} \approx 30{,}2 \text{ MPa} \\ \sigma_1 = \dfrac{N_1}{A_1} = \dfrac{P \; 4}{\pi d^2} \approx 56{,}6 \text{ MPa} \end{cases}$$

$$\begin{cases} \tau_2 = \dfrac{M_{v2} \cdot a}{K_2} = \dfrac{M \cdot a}{K_2} \approx 0{,}85 \text{ MPa} \\ \sigma_2 = \dfrac{N_2}{A_2} = \dfrac{P \cdot 4}{\pi \left( (2a)^2 - a^2 \right)} \approx 10{,}6 \text{ MPa} \end{cases}$$

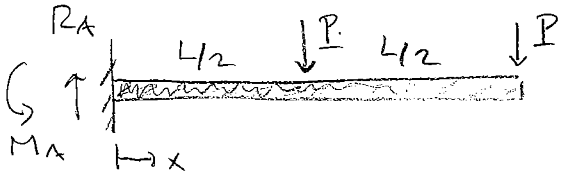
von Mises effektivspänning

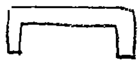
$$\sigma_{e1} = \sqrt{\sigma_1^2 + 3\tau_1^2} \approx 77 \text{ MPa}$$

$$\sigma_{e2} = \sqrt{\sigma_2^2 + 3\tau_2^2} \approx 11 \text{ MPa} \quad //$$

3

a)



$\uparrow: \; R_A - 2P = 0 \Rightarrow \boxed{R_A = 2P}$

$\circlearrowleft A: \; -M_A + PL/2 + PL = 0 \Rightarrow \boxed{M_A = 3PL/2}$

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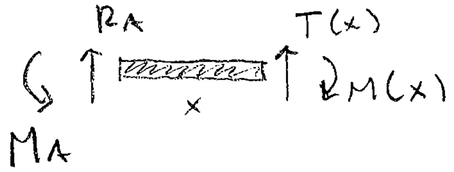
Snitta $0 < x < L/2$




$\uparrow: \; T(x) + R_A = 0 \Rightarrow T(x) = -R_A = -2P$

$\circlearrowleft x: \; M(x) + R_A x - M_A = 0 \Rightarrow$

$M(x) = M_A - R_A \cdot x =$

$= PL\left(\frac{3}{2} - \frac{2x}{L}\right)$

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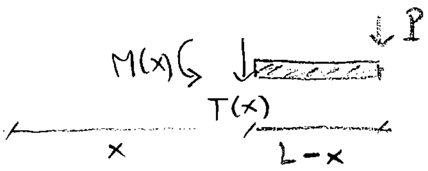
Snitta $L/2 < x < L$




$\uparrow: \; -T(x) - P = 0 \Rightarrow T(x) = -P$

$\circlearrowleft x: \; -M(x) + P(L-x) = 0 \Rightarrow$

$M(x) = P(L-x) = PL(1 - x/L)$

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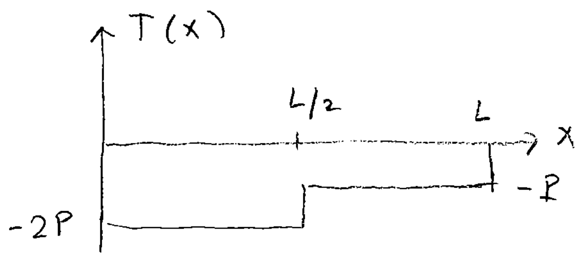
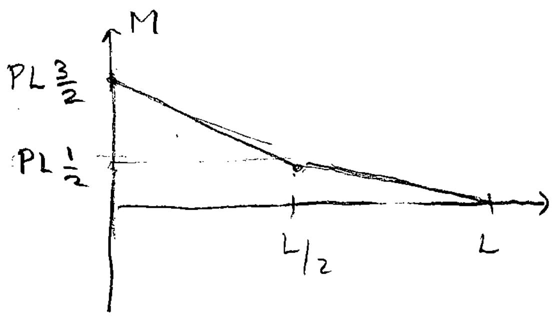
Totalt fås

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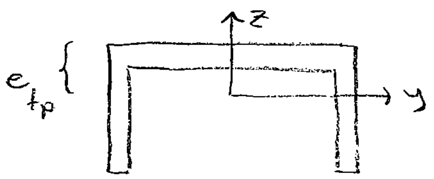
b) Tp:s läge $e_{tp}$:




$$e_{tp} = \frac{b \cdot h \cdot h/2 - (b-2t)(h-t)\left(t + \frac{h-t}{2}\right)}{b \cdot h - (b-2t)(h-t)}$$

$\approx 24{,}3$ mm

Yttröghetsmoment:

$$I = \frac{bh^3}{12} + b \cdot h \cdot (h/2 - e_{tp})^2 - \left[\frac{(b-2t)(h-t)^3}{12} + (b-2t)(h-t)\left(\frac{t}{2} - e_{tp}\right)^2\right]$$

$\approx 1{,}743 \, 10^6$ mm$^4$

forts 3)

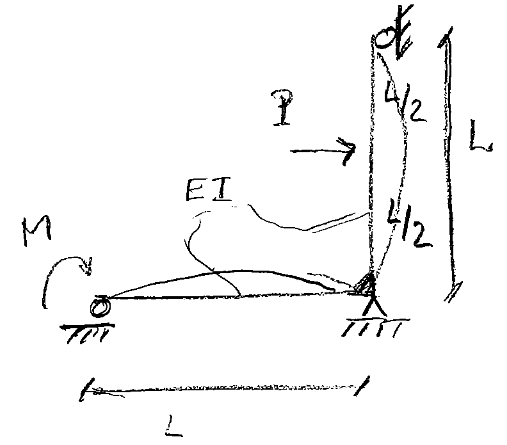
Eftersom $h - e_{tp} > e_{tp}$ fås kritisk spänning på undersidan, dvs största spänning till belopp blir:

$$\sigma_{max} = \frac{M_{max} \cdot (h - e_{tp})}{I} = \frac{3}{2} \frac{P \cdot L \cdot (h - e_{tp})}{I}$$

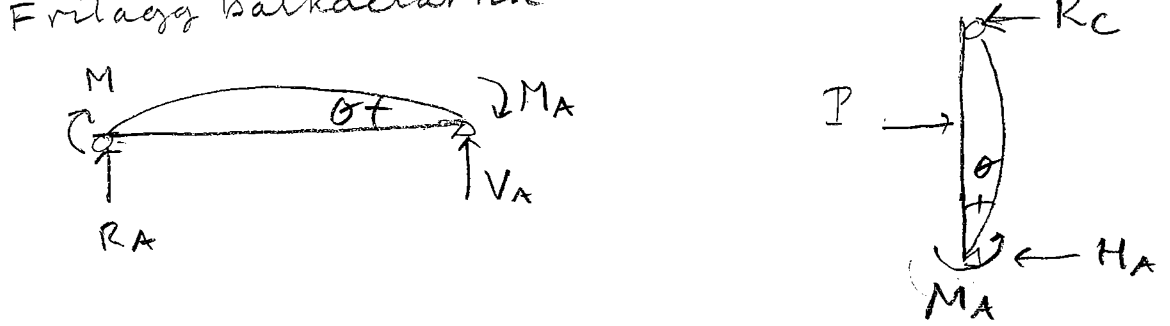
Därmed fås tillåten last

$$P_{till} = \frac{2}{3} \frac{\sigma_{till} \cdot I}{L \cdot (h - e_{tp})} \approx 208{,}6 \text{ N} \quad //$$

4)




Frilägg balkdelarna




Deformationsvillkor bestämmer $M_A$, FS 6.3

$$\theta = \begin{cases} -\dfrac{M \cdot L}{6EI} + \dfrac{M_A L}{3EI} \\ -\dfrac{M_A L}{3EI} + \dfrac{P \cdot L^2}{16\,EI} \end{cases} \Rightarrow \frac{2M_A}{3} = \frac{PL}{16} + \frac{M}{6}$$

$$\Rightarrow M_A = \frac{3PL}{32} + \frac{M}{4}$$

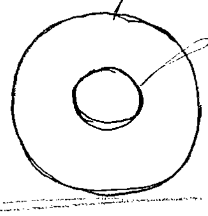
Utböjningen mitt på den vertikala balkdelen

FS 6.3

$$w_{mitt} = -\frac{M_A L^2}{16EI} + \frac{PL^3}{48\,EI} = \frac{L^2}{EI}\left[-\frac{1}{16}\left(\frac{3 \cdot PL}{32} + \frac{M}{4}\right) + \frac{PL}{48}\right]$$

villkor $w_{mitt} = 0 \Rightarrow M = \frac{23}{24} P \cdot L$ //

5)




Gl (11-14), (11-18), (11-19)

$$\begin{cases} u_r(r) = A_1 \cdot r + A_2/r \\ \sigma_r(r) = A - B/r^2, \quad \sigma_\varphi(r) = A + B/r^2 \end{cases}$$

där enl (11-17) $A = \dfrac{EA_1(1+\nu)}{1-\nu^2}$ $\quad B = \dfrac{EA_2(1-\nu)}{1-\nu^2}$ (*)

RV $\begin{cases} \sigma_r(a) = 0 \Rightarrow A - B/a^2 = 0 \Rightarrow \boxed{A = B/a^2} \quad (**) \\ u_r(b) = \Delta \Rightarrow A_1 b + A_2/b = \Delta \end{cases}$

$$\overset{(*)}{\Rightarrow} \frac{(1-\nu^2)A}{E(1+\nu)} \cdot b + \frac{(1-\nu^2)B}{E(1-\nu)} \frac{1}{b} = \Delta$$

$$\overset{(**)}{\Rightarrow} \frac{(1-\nu^2)B}{E(1+\nu)} \frac{b}{a^2} + \frac{(1-\nu^2)B}{E(1-\nu)} \frac{1}{b} = \Delta$$

$$\Rightarrow \frac{B(1-\nu^2)}{E} \left( \frac{b}{a^2} \frac{1}{1+\nu} + \frac{1}{b} \frac{1}{1-\nu} \right) = \Delta$$

$$\Rightarrow \frac{B}{bE} \left( \frac{b^2}{a^2}(1-\nu) + (1+\nu) \right) = \Delta \Rightarrow \boxed{B = \frac{\Delta \cdot b \cdot E}{\frac{b^2}{a^2}(1-\nu) + (1+\nu)}} \approx$$

Huvudspänningar vid $r = b$ är:

$\sigma_z = 0$, $\sigma_r = A - B/b^2 \approx 73{,}2$ MPa $\quad \sigma_\varphi = A + B/b^2 \approx 122$ MPa

(ty $\tau_{r\varphi} = \tau_{rz} = \tau_{\varphi z} = 0$)

Trescas effektivspänning:

$\sigma_{eT} = \sigma_\varphi - \sigma_z \approx 122$ MPa //