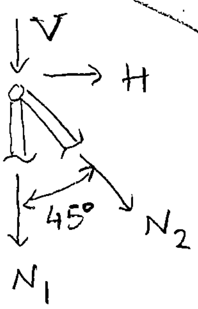
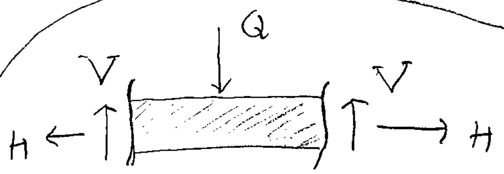
1) Friläggning:

knutpunkt





Lika stora krafter i hö och vä snitt pga symmetri

$\uparrow: \; 2V - Q = 0 \Rightarrow V = Q/2 \quad (1)$

$\uparrow: \; -V - N_1 - N_2/\sqrt{2} = 0 \quad (2)$

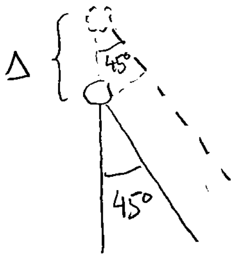
$\rightarrow: \; H + N_2/\sqrt{2} = 0 \quad (3)$

Konstitutiva samband:

$$N_1 = \frac{Ea^2}{L}\delta_1 \qquad N_2 = \frac{Ea^2}{\sqrt{2}\,L}\delta_2 \qquad (4)$$

Kompatibilitet:

knutpunkt förflyttas i vertikalled pga symmetri och stel skiva, dvs Δ neråt antas



ur figur: $\begin{cases} \delta_1 = -\Delta \\ \delta_2 = -\Delta/\sqrt{2} \end{cases} \Rightarrow \delta_1 = \delta_2\sqrt{2} \quad (5)$

(4) i $(5) \Rightarrow N_1 = \sqrt{2}\,N_2\sqrt{2} = 2N_2 \quad (5')$

$(5'), (1)$ i $(2) \Rightarrow -Q/2 - 2N_2 - N_2/\sqrt{2} = 0 \Rightarrow$

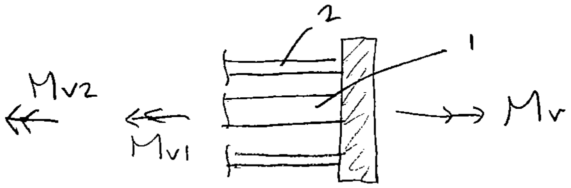
$$N_2 = \frac{-Q}{2(2 + 1/\sqrt{2})} \quad (2') \quad \overset{(5')}{\Rightarrow} \quad N_1 = \frac{-Q}{2 + 1/\sqrt{2}}$$

∴ Spänningen i de

vertikala stängerna $\sigma_1 = N_1/a^2 \approx -18{,}5$ [MPa]

diagonala —"— $\sigma_2 = N_2/a^2 \approx -9{,}2$ [MPa]

2) Friläggning av stel skiva:



$\rightarrow$: $M_v - M_{v1} - M_{v2} = 0$

$\Rightarrow \boxed{M_{v1} = M_v - M_{v2}}$ (1)

Kompatibilitet

vridningen av axlarna lika stor

$\boxed{\varphi_1 = \varphi_2}$ (2)

Konstitutiva samband

$$\varphi_1 = \frac{M_{v1} L}{G K_1} \qquad \varphi_2 = \frac{M_{v2} L}{G K_2} \qquad (3)$$

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(3) i (2) $\Rightarrow$ $M_{v1} = M_{v2} \dfrac{K_1}{K_2}$ (2')

(2') i (1) $\Rightarrow$ $M_{v2} \dfrac{K_1}{K_2} = M_v - M_{v2} \Rightarrow M_{v2} = \dfrac{M_v K_2}{K_1 + K_2}$ (1')

(1') i (1) $\Rightarrow$ $M_{v1} = \dfrac{M_v K_1}{K_1 + K_2}$ (1")

Total vridning $= \varphi_1 = \varphi_2 \overset{(3)}{=} \dfrac{M_v L}{G(K_1 + K_2)}$ (3')

där $K_1 = \dfrac{\pi}{32}\left[D^4 - (3D/5)^4\right] = \dfrac{\pi D^4 17}{625} \approx 8{,}55 \cdot 10^6$ [mm^4]

$K_2 = \dfrac{4A^2}{\oint \frac{ds}{h}} = \dfrac{4 \cdot (5D/2)^4}{\frac{1}{t} \cdot 5D/2 \cdot 4} = \dfrac{125}{8} D^3 t \approx 3.13 \cdot 10^7$ [mm^4]

$\Rightarrow \varphi_1 = \varphi_2 \approx 0{,}031$ [rad] $\approx 1.8°$ //

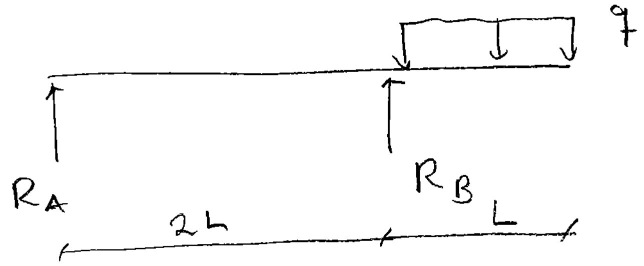
Max vridskjuvspänning

$$\tau_{max,1} = \frac{M_{v1}}{W_{v1}} \overset{(6.14)}{=} \frac{M_{v1}\, D/2}{K_1} \overset{(1'')}{=} \frac{M_v\, D}{2(K_1+K_2)} \approx 126 \text{ [MPa]}$$

$$\tau_{max,2} = \frac{M_{v2}}{W_{v2}} \overset{(16\text{-}69)}{=} \frac{M_{v2}}{2\,At} \overset{(1')}{=} \frac{M_v\, K_2}{2\,(5D/2)^2\, t \cdot (K_1+K_2)} =$$

$$\approx 314 \text{ [MPa]}$$

3) Friläggning av hela balken (statiskt bestämd)



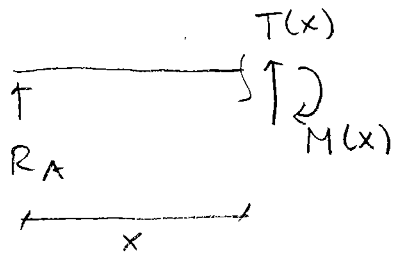
$$\uparrow: \quad R_A + R_B - qL = 0$$
$$\overset{\curvearrowright}{A}: \quad -R_B \cdot 2L + qL\,(2L + L/2) = 0$$

$R_B = 5qL/4$

$R_A = -qL/4$ (1)

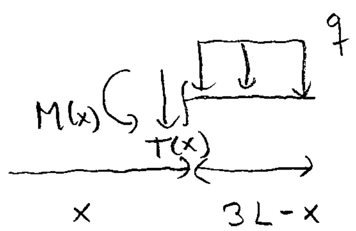
a) Snitta och frilägg

$0 < x < 2L$:

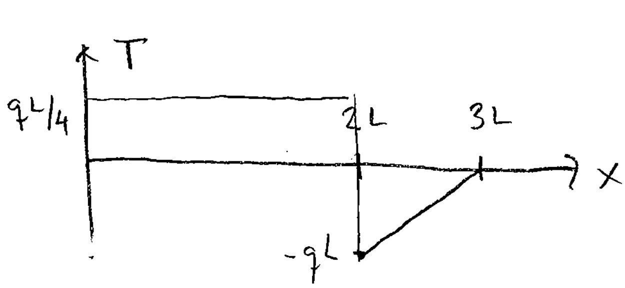


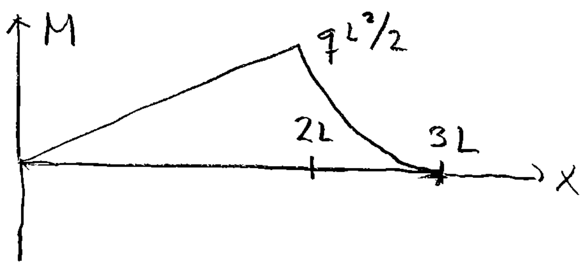
$$\uparrow: \quad T(x) + R_A = 0 \Rightarrow T(x) = -R_A$$
$$\overset{\curvearrowright}{x}: \quad M(x) + R_A x = 0 \Rightarrow M(x) = -R_A \cdot x$$

$2L < x < 3L$:



$$\uparrow: \quad -q\cdot(3L-x) - T(x) = 0$$
$$\Rightarrow T(x) = q\cdot(x-3L)$$
$$\overset{\curvearrowright}{x}: \quad -M(x) + q\cdot(3L-x)\,\frac{3L-x}{2} = 0$$
$$\Rightarrow M(x) = \frac{q}{2}\,(x-3L)^2$$





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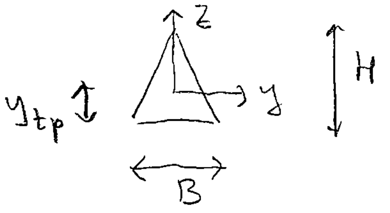
b) Max böjnormalspänning σ_{max}:



tp:s position $y_{tp} = H/3$

FS ger yttröghetsmomentet

$I_y = BH^3/36$

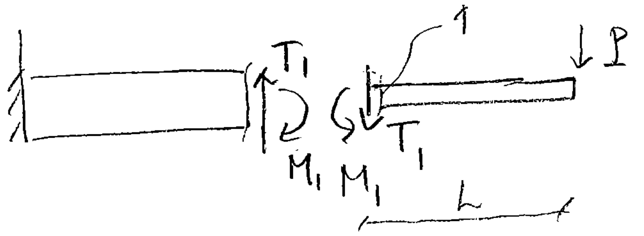
$$\sigma_{max} = \frac{\max_x(|M(x)|) \cdot \max(|z|)}{I_y} = \frac{qL^2/2 \cdot 2H/3}{BH^3/36} =$$

$$= \frac{qL^2}{BH^2} \cdot 12$$

Max tillåten spänning $\sigma_{till} = 400$ MPa

$$\Rightarrow \quad q \leq \frac{BH^2}{L^2}\frac{1}{12} \cdot \sigma_{till} \approx 0,33 \cdot 10^3 \; [N/mm]$$ //

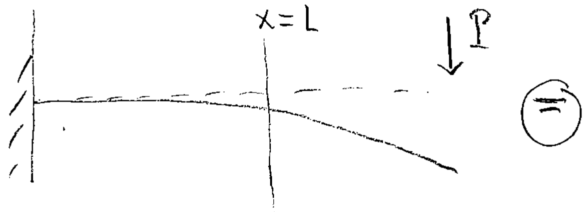
4) Snitta och frilägg



$\uparrow: -T_1 - P = 0 \Rightarrow \boxed{T_1 = -P}$

$\curvearrowleft: -M_1 + P\cdot L = 0 \Rightarrow \boxed{M_1 = P\cdot L}$

Använd superposition av elementarfall för att bestämma nedböjn. i hö ände



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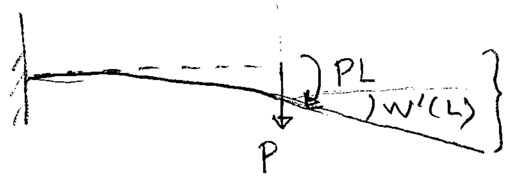
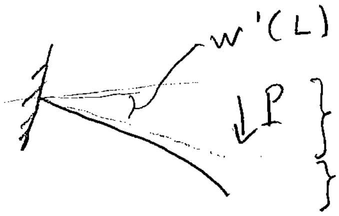


$\delta_1 = w(L) + w'(L)\,L$ från el fall 6.4

+



δ_1

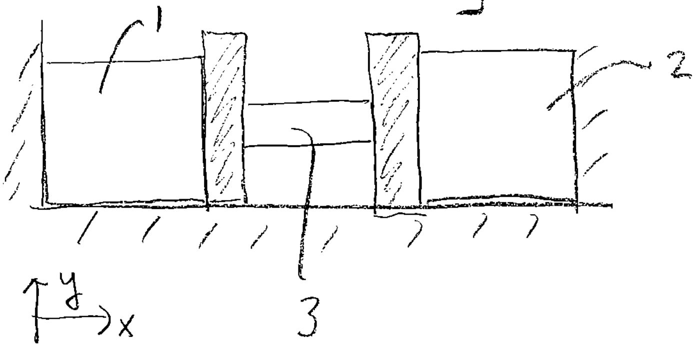
δ_2 från el. fall 6.4

Totalt blir nedböjningen $\delta_1 + \delta_2 =$

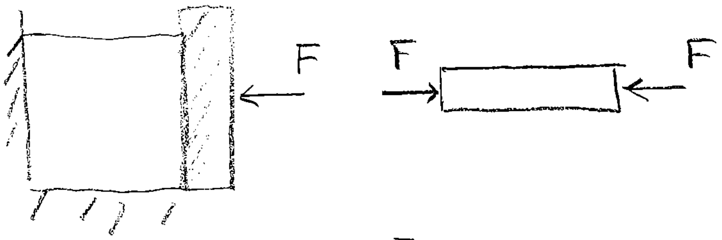
$$= \underbrace{\left(\frac{P\cdot L^3}{3\cdot(2EI)} + \frac{P\cdot L\cdot L^2}{2\cdot(2EI)}\right)}_{w(L)} + \underbrace{\left(\frac{P\cdot L^2}{2\cdot(2EI)} + \frac{P\cdot L\cdot L}{(2EI)}\right)}_{w'(L)}\cdot L + \underbrace{\frac{PL^3}{3\cdot EI}}_{\delta_2} =$$

$$= \frac{PL^3}{EI}\left(\frac{1}{6} + \underbrace{\frac{1}{4} + \frac{1}{4} + \frac{1}{2}}_{1} + \frac{1}{3}\right) = \frac{PL^3}{EI}\,\frac{3}{2} \quad //$$

5) Efter inplacering av stången



Snitta och frilägg



$$\Rightarrow \begin{cases} \sigma_{x1} = -\dfrac{F}{L^2} = \sigma_{x2} & (1) \\ \sigma_{x3} = -\dfrac{F}{\beta L^2} = \dfrac{1}{\beta}\sigma_{x1} & (2) \end{cases}$$

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Enaxlig spänning i alla tre delarna

$$\begin{cases} \varepsilon_{x1} = \varepsilon_{x2} = \dfrac{1}{E}\sigma_{x1} \overset{(1)}{=} -\dfrac{F}{E \cdot L^2} & (3) \\ \varepsilon_{x3} = \dfrac{1}{E}\sigma_{x3} \overset{(2)}{=} -\dfrac{F}{\beta E L^2} & (4) \end{cases}$$

Kompatibilitet

stångens längd efter inplacering

$a + \delta_3$

klossarnas bredd — " —

$L + \delta_1$, $L + \delta_2 = L + \delta_1$

Eftersom avståndet mellan de stela väggarna är lika före och efter inplaceringen fås:

$$\underbrace{L + L + a - \Delta}_{\text{före}} = \underbrace{L + \delta_1 + L + \delta_1 + a + \delta_3}_{\text{efter}}$$

$$\Rightarrow -\Delta = 2 \cdot \delta_1 + \delta_3 \quad (5)$$

(3), (4) i (5) $\Rightarrow \quad -\Delta = -\dfrac{2F}{EL} - \dfrac{Fa}{\beta E L^2} \Rightarrow F = \dfrac{\Delta EL}{2 + \frac{a}{\beta L}} \quad 5')$

Nu kan töjningen i y-led bestämmas mha Hookes lag:

$$\varepsilon_{y2} = \varepsilon_{y1} = -\frac{\nu}{E}\sigma_{x2} \overset{(1)}{=} \frac{\nu}{E}\frac{F}{L^2} \overset{(5')}{=} \frac{\nu}{\cancel{E}}\frac{\Delta \cancel{E} \cancel{L}}{L^{\cancel{2}}\left(2 + \frac{a}{\beta L}\right)} =$$

$$= \frac{\nu \Delta}{2L + a/\beta}$$

Höjdökningen fås nu som $\varepsilon_{y2} \cdot L = \dfrac{\nu \cdot \Delta \cdot L}{2L + a/\beta}$ //