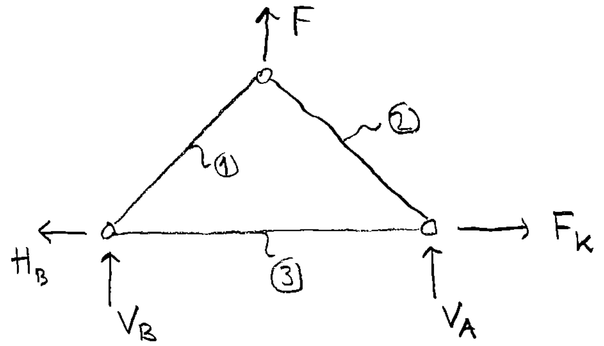
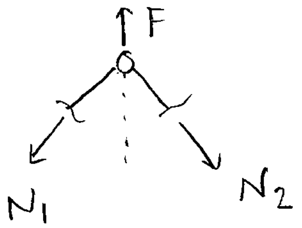
1)

Friläggning av hela stångsystemet:



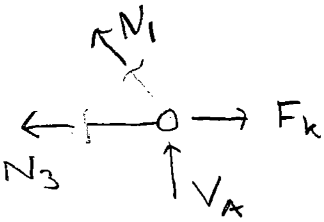
Övre knutpunkten:



$\rightarrow$: $N_2/\sqrt{2} - N_1/\sqrt{2} = 0 \Rightarrow \boxed{N_1 = N_2}$

$\uparrow$: $F - 2 \cdot N_1/\sqrt{2} = 0 \Rightarrow \boxed{N_1 = F/\sqrt{2}}$ (*)

Högra knutpunkten:



$\rightarrow$: $F_K - N_3 - N_1/\sqrt{2} = 0$

$\overset{(*)}{\Rightarrow}$ $F_K - N_3 - F/2 = 0$ (**)

Tvåfaldig säkerhet mot knäckning i horisontell,

Euler 2: $P_{kr} = \pi^2 \dfrac{EI}{L^2} = \left\{ I = \dfrac{\pi d^4}{64} \right\} = \dfrac{\pi^3 E d^4}{64 \cdot L^2}$

$\Rightarrow N_3 = -P_{kr}/2 = -\dfrac{\pi^3}{128} \dfrac{E d^4}{L^2}$

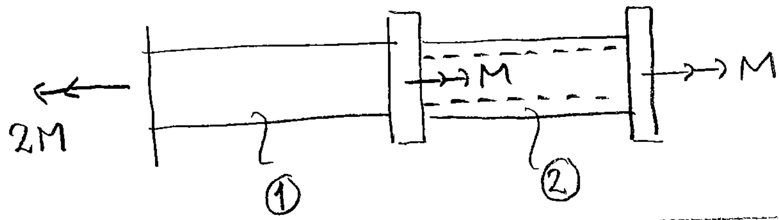
Förlängning: $\delta_3 = \dfrac{N_3 \cdot L}{EA} = -\dfrac{\pi^3 \cancel{E} d^4}{128 \cdot L} \dfrac{4}{\cancel{E} \pi d^2} = -\dfrac{\pi^2 d^2}{32 \cdot L}$

Fjäderkraft $F_K = k \cdot (-\delta_3) = \dfrac{\pi d^2 E}{40 \cdot L} \dfrac{\pi^2 d^2}{32 L} = \dfrac{\pi^3 d^4 E}{1280 L^2}$

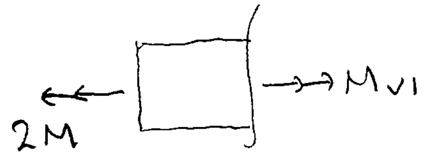
Insatt i (**) $\Rightarrow$

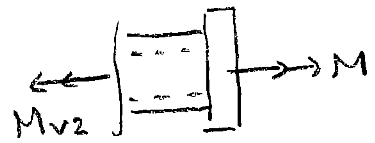
$F = 2 F_K - 2 N_3 = \dfrac{\pi^3 d^4 E}{640 L^2} + \dfrac{\pi^3 d^4 E}{64 L^2} = \dfrac{\pi^3 d^4 E}{L^2} \dfrac{11}{640}$ //

2) Friläggning av axelsystemet:



Inne vridande moment:





$\rightarrow : M_{v1} - 2M = 0$

$\Rightarrow \boxed{M_{v1} = 2M}$

$\rightarrow : M - M_{v2} = 0$

$\Rightarrow \boxed{M_{v2} = M}$

Max vridskjuvspänning (se GL 6-16)

$$\begin{cases} \tau_{max,1} = \dfrac{M_{v1}}{W_{v1}} = \dfrac{2M \cdot 2 \cdot D/2}{\pi (D/2)^4} = \dfrac{M}{D^3} \dfrac{32}{\pi} \\ \tau_{max,2} = \dfrac{M_{v2}}{W_{v2}} = \dfrac{M \cdot 2 \cdot D/2}{\pi \left((D/2)^4 - (d/2)^4 \right)} = \dfrac{M}{D^3} \dfrac{16}{\pi} \dfrac{1}{1-(d/D)^4} \end{cases}$$

Samma maxspänning $\Rightarrow \tau_{max,1} = \tau_{max,2} = \tau_s$

$$\frac{1}{1-(d/D)^4} = 2 \Rightarrow 1-(d/D)^4 = \frac{1}{2} \Rightarrow \frac{d}{D} = \left(\frac{1}{2}\right)^{1/4}$$

$\Rightarrow d \approx 0{,}84 \cdot D \approx$
$\approx 16{,}8 \text{ mm}$ //

dessutom $M = \dfrac{\pi D^3}{32} \tau_s$

Vridningsvinkeln blir $\varphi_1 + \varphi_2$, enl GL (6-11):

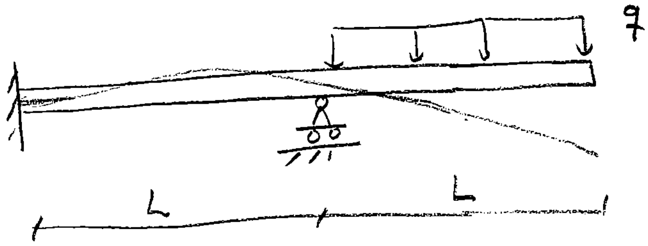
$$\varphi_1 + \varphi_2 = \frac{M_{v1} \cdot L}{G K_1} + \frac{M_{v2} \cdot L}{G K_2} = \frac{M \cdot L}{G} \left[\frac{2}{K_1} + \frac{1}{K_2} \right]$$

där $K_1 = \dfrac{\pi}{2} (D/2)^4 = \pi D^4 / 32$

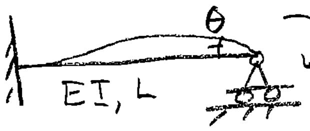
$$K_2 = \frac{\pi}{2} \left[(D/2)^4 - (d/2)^4 \right] = \frac{\pi D^4}{32} \underbrace{\left(1 - (d/D)^4 \right)}_{1/2} = \frac{\pi D^4}{64}$$

$$\Rightarrow \varphi_1 + \varphi_2 = \frac{M \cdot L}{G} \left[\frac{2 \cdot 32}{\pi D^4} + \frac{64}{\pi D^4} \right] = \frac{M \cdot L}{G \cdot \pi \cdot D^4} \cdot 128 = \frac{\tau_s \cdot L}{G D} 4 \approx 14{,}3^\circ$$ //

3)



Studera vä-del:

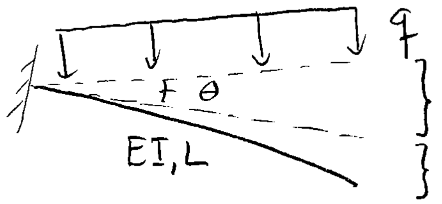


$M = q \cdot L \cdot \frac{L}{2} =$

$= qL^2/2$

enl FS 6.5 fås: $\theta = \frac{(qL^2/2)\,L}{4EI} = \frac{qL^3}{8EI}$

Studera hö-del (med fix lutning θ i sin vä ände)



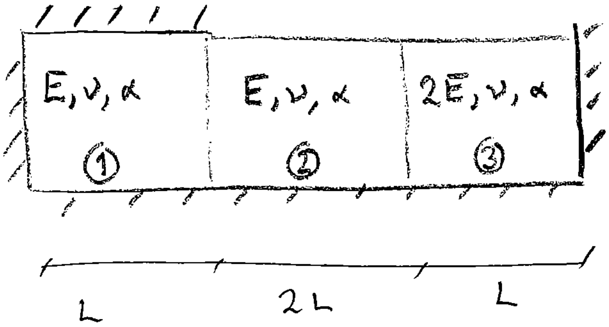
$\theta \cdot L$

enl. FS 6.4 $= \frac{q \cdot L^4}{8EI}$

Totalt fås således nedböjningen

$$\frac{q \cdot L^3}{8EI} \cdot L + \frac{qL^4}{8EI} = \frac{qL^4}{4EI}$$ //

4)



p.g.a. jämvikt
$\sigma_{x1} = \sigma_{x2} = \sigma_{x3}$

1: Hookes lag 3D, FS s.14

$$\varepsilon_{y1} = \frac{1}{E}\left[\sigma_{y1} - \nu(\sigma_{x1} + \underbrace{\sigma_{z1}}_{=0})\right] + \alpha\Delta T = 0$$

$$\Rightarrow \sigma_{y1} = \nu\sigma_{x1} - \alpha E \Delta T$$

$$\varepsilon_{x1} = \frac{1}{E}\left[\sigma_{x1} - \nu(\sigma_{y1} + \underbrace{\sigma_{z1}}_{=0})\right] + \alpha\Delta T =$$

$$= \frac{1}{E}\left[\sigma_{x1} - \nu^2\sigma_{x1} + \nu\cdot\alpha E\Delta T\right] + \alpha\Delta T =$$

$$= \frac{1}{E}(1-\nu^2)\sigma_{x1} + \alpha\Delta T(1+\nu)$$

2 $$\varepsilon_{x2} = \frac{1}{E}\left[\underbrace{\sigma_{x2}}_{\sigma_{x1}} - \nu\underbrace{(\sigma_{y2} + \sigma_{z2})}_{=0}\right] + \alpha\Delta T = \frac{\sigma_{x1}}{E} + \alpha\Delta T$$

3 $$\varepsilon_{x3} = \frac{1}{2E}\left[\underbrace{\sigma_{x3}}_{\sigma_{x1}} - \nu\underbrace{(\sigma_{y3} + \sigma_{z3})}_{=0}\right] + \alpha\Delta T = \frac{\sigma_{x1}}{2E} + \alpha\Delta T$$

Deformationsvillkor:

$$\delta_{x1} + \delta_{x2} + \delta_{x3} = 0 \Rightarrow \varepsilon_{x1}\cdot\cancel{L} + \varepsilon_{x2}\cdot 2\cancel{L} + \varepsilon_{x3}\cdot\cancel{L} = 0$$

$$\Rightarrow \frac{1}{E}(1-\nu^2)\sigma_{x1} + \alpha\Delta T(1+\nu) + 2\cdot\frac{\sigma_{x1}}{E} + 2\alpha\Delta T + \frac{\sigma_{x1}}{2E} + \alpha\Delta T = 0$$

$$\Rightarrow \frac{\sigma_{x1}}{E}\left[\underbrace{1-\nu^2+2+\frac{1}{2}}_{7/2-\nu^2}\right] + \alpha\Delta T(\underbrace{1+\nu+2+1}_{4+\nu}) = 0$$

$$\Rightarrow \sigma_{x1} = -\alpha\Delta T\cdot E\,\frac{4+\nu}{\frac{7}{2}-\nu^2} \quad //$$

5) Diff. ekv. för axialbelastad balk: GL (8-63)

$$w^{IV} + \frac{F}{EI} w'' = \frac{q}{EI} \qquad \text{ty } N = -F$$

Homogenlösn. $w_h(x) = A\sin(nx) + B\cos(nx) + Cx + D$

där $n^2 = F/EI$

Partikulärlösn. $w_p(x) = \frac{q}{F}\cdot\frac{x^2}{2}$ (ty $w_p'' = \frac{q}{F}$, $w_p^{IV} = 0$)

$$\Rightarrow w(x) = A\sin(nx) + B\cos(nx) + Cx + D + \frac{qx^2}{2F}$$

Randvillkor:

$\underline{x=0}$ $\Rightarrow \begin{cases} w(0) = 0 \Rightarrow B + D = 0 \Rightarrow \boxed{D = -B} \\ w'(0) = 0 \end{cases}$

$$w' = An\cos(nx) - Bn\sin(nx) + C + qx/F$$

$$\Rightarrow w'(0) = A\cdot n + C = 0 \Rightarrow \boxed{C = -A\cdot n}$$

$\underline{x=L}$ $\Rightarrow \begin{cases} w'(L) = 0 \Rightarrow An\cos(nL) - Bn\sin(nL) - An + \frac{qL}{F} = 0 \\ V(L) = 0 \end{cases}$

$$\Rightarrow \boxed{B = A\cdot\left(\frac{\cos(nL)}{\sin(nL)} - \frac{1}{\sin(nL)}\right) + \frac{qL}{n\cdot\sin(nL)\cdot F}} \qquad (*)$$

$$V(x) = \left\{\begin{matrix}GL\\(8\text{-}55)\end{matrix}\right\} = M'(x) + \underbrace{H}_{-F}\cdot w' = \left\{\begin{matrix}GL\\(7\text{-}65)\end{matrix}\right\} = -EIw'''(x) - Fw'(x)$$

$$w''(x) = -An^2\sin(nx) - Bn^2\cos(nx) + q/F$$

$$w'''(x) = -An^3\cos(nx) + Bn^3\sin(nx)$$

$$\Rightarrow V(L) = -EI\left(-An^3\cos(nL) + Bn^3\sin(nL)\right) - F\underbrace{w'(L)}_{=0} = 0$$

$$\Rightarrow \boxed{B = A/\tan(nL)}$$

insatt i $(*)$ $\Rightarrow$ $0 = -\frac{A}{\sin(nL)} + \frac{qL}{F\cdot n\cdot\sin(nL)} \Rightarrow \boxed{A = \frac{qL}{n\cdot F}}$

$$\Rightarrow w(x) = \frac{qL}{n \cdot F} \sin(nx) + \frac{qL}{F \cdot n \cdot \tan(nL)} \cos(nx) +$$

$$- \frac{q \cdot L \cdot x}{F} - \frac{qL}{n \tan(nL) F} + \frac{q x^2}{2F}$$