**Chalmers** | Göteborgs Universitet 2010-12-14

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# Functional Programming DIT 141 / TDA 451

2010-12-14 14.00 – 18.00 VV ("Väg och Vatten")

David Sands , [Answering questions on the day:David Sands 0737 207663]

- There are four Questions (with 11 + 11 + 18 + 12 = 52 points); a total of 25 points definitely guarantees a pass.
- Results: latest January.
- **Permitted materials:**
  - Dictionary
- **Please read the following guidelines carefully:**
  - Read through all Questions before you start working on the answers.
  - Begin each Question on a new sheet.
  - Write clearly; unreadable = wrong!
  - Full points are given to solutions which are short, elegant, and correct. Fewer points may be given to solutions which are unnecessarily complicated or unstructured, or unnecessarily inefficient.
  - For each part Question, if your solution consists of more than a few lines of Haskell code, use your common sense to decide whether to include a short comment to explain your solution.
  - You can use any of the standard Haskell functions *listed at the back of this exam document*, plus any functions of the QuickCheck library.
  - You are encouraged to use the solution to an earlier part of a Question to help solve a later part — even if you did not succeed in solving the earlier part.

*How many programmers does it take to change a light bulb? None Its a hardware problem.*

**Q 1.** (a) *(2 points)* Give the type of the following function:

```
q1 []  _           = []
q1 ((x:xs):xss) y = [x == y] : q1 xss y
```

(b) *(3 points)* Redefine `q1` without using recursion (but you may use any recursive functions defined in the Prelude).

(c) *(2 points)* Simplify the following function definition as much as possible:

```
q1b :: Bool -> Int -> String
q1b x y
  | x == False          = "True"
  | x == True && even y = "False"
  | otherwise           = "True"
```

(d) *(4 points)* Define a function `minmax` (including its type) which given a non-empty list returns a pair of the smallest and the largest element in the list. Your definition should use a single tail-recursive helper function which computes the pair, and no other recursive functions.

**Q 2.** This question is about representing and writing a type checker for a tiny language of Haskell-like expressions.

The subset of Haskell expressions, `Hexp`, has expressions of just the following kinds: variables (identifiers) such as `x`, `y` and `z`, integer literals such as `42` and `-1`, boolean literals `True` and `False`, equality expressions of the form $e_1$ `==` $e_2$, and conditionals of the form `if` $e_1$ `then` $e_2$ `else` $e_3$, where $e_1$, $e_2$, and $e_3$ stand for any `Hexp` expressions.

(a) *(3 points)* Define a datatype to represent the above language of `Hexp` expressions. You should allow any expressions to be built, not just type correct ones. For simplicity you may assume that variables can be any string.

(b) *(2 points)* Give definitions for `example1`, and `example2` which should represent the following two `Hexp` expressions (one of which is badly typed!):

```
if x == False then 2 else 3
```

```
example1 = Hif  (Var "x" `Heq` HB False) (HI 2) (HI 3)
```

(c) *(6 points)* To determine whether a given `Hexp` expression is type correct we need to know the type of the variables it contains. The following types can be used to represent these things:

```
data HType = HBool | HInt deriving (Eq,Show) -- the type of an Hexp
```

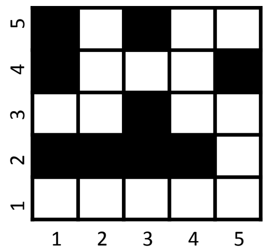
Define a function

```
hType :: TEnv -> Hexp -> Maybe HType
```

For example, `hType [("x",HBool)] example1` should give `Just HInt` but both `hType [("x",HBool)] example2` and `hType [("x",HInt)] example2` should give `Nothing`.

You may decide for yourself what your function does in the case that the type environment does not have types for all variables in the expression.

**Q 3.** A maze consists of an $n$ x $n$ grid in which some squares are black. Here is an example of a 5 x 5 maze:




A maze is represented by its size and a list of the positions of its black squares:

```
type Position = (Int,Int)
type Maze = (Int, [Position])
```

For example the maze above could be represented by

```
maze :: Maze
maze = (5,[(1,2),(1,4),(1,5),(2,2),(3,2),(3,3),(3,5),(4,2),(5,4)])
```

A *path* through a maze is a sequence of positions of white squares:

```
type Path = [Position]
```

The first position represents the *end* of the path and the last position represents its *start*. In a path any two consecutive positions are either side-by-side or one above the other. No position can occur more than once in a path.

An example path from the south-east corner `(5,1)` to the north-east corner `(5,5)` of `maze` is

```
path = [(5,5),(4,5),(4,4),(4,3),(5,3),(5,2),(5,1)]
```

This question is about defining a function which can find a path from a given start position to a given end position.

(a) *(6 points)* Define a function

```
neighbour :: Maze -> Position -> [Position]
```

where `neighbour m p` provides a list of all the white squares which are either to the left or right, or above or below `p`. For example `neighbour maze (4,3)` could give `[(4,4),(5,3)]`.

(b) *(3 points)* Define a function

```
extend :: Maze -> Path -> [Path]
```

which gives all the possible ways to extend the given set of non-empty paths in the given maze with one square at the beginning. For example `extend maze [(4,4),(4,3)]` could give `[[(3,4),(4,4),(4,3)],[(4,5),(4,4),(4,3)]]`

(c) *(6 points)* Define a function

```
allpaths :: Maze -> Position -> [Path]
```

which computes the list of all paths in the given maze starting at the given position.

Hints: You might consider first defining a recursive function which computes all paths of length $k$ from a given start position. An alternative approach is to make use of the function `iterate`.

(d) *(3 points)* Define a function

```
fromto :: Maze -> Position -> Position -> [Path]
```

where `fromto m p q` computes all paths from a start position `p` to an end position `q` in a maze `m`.

**Q 4.** (continuation from Question 3)

(a) *(6 points)* In order to make a maze an instance of class `Arbitrary` we need to make a new data type thus:

```
data TestMaze = M Maze deriving (Eq,Show)
```

```
instance Arbitrary TestMaze where
  arbitrary = ...
```

Provide a definition for `arbitrary`, ensuring that mazes are well formed: all squares are within the given dimensions of the maze, and no square appears more than once in the list of positions of black squares.

Hint: functions from Test.QuickCheck such as `choose :: (Int,Int) -> Gen Int` (for generating a number in a given range) and `listOf :: Gen a -> Gen [a]` (for converting a generator of things into a generator of lists of things) could be useful.

(b) *(6 points)* Define a quickCheck property which tests a useful relationship between all the paths in `fromto m start end` and those in `fromto m end start`. Since such a test requires taking `start` and `end` to be white squares within `m`, in your property you should take `start` to be the lowest numbered white square in `m` and `end` to be the highest numbered (the exact ordering you use for squares is not important).