CHALMERS UNIVERSITY OF TECHNOLOGY
Department of Electrical Engineering
Division of Systems and Control

EXAMINATION IN LINEAR CONTROL SYSTEM DESIGN

(Course SSY285)

Friday August 30, 2024

Time and place:	08:30 - 12:30 (Johanneberg)
Teacher:	Torsten Wik (031 - 772 5146)

The total points achievable are 30. Unfinished solutions normally results in 0 points. Incorrect solutions with significant errors, unrealistic results that are not commented, or solutions that are too difficult to follow also result in 0 points.

Numerically incorrect calculations that do not cause unrealistic results will normally lead to reduction of 1 point. Incorrect answers that are consequences of a previous error and do not simplify the problem will not lead to any further reduction.

The scales for grading are

Grade 3:	at least 12 points
Grade 4:	at least 18 points
Grade 5:	at least 24 points

The following aids are allowed:

1. Course text book *Control Theory* (or Swedish version *Reglerteori*) by T. Glad and L. Ljung and **one more** control textbook.

   Paper copies are accepted instead of books.

2. 1 piece of A4 paper, with *hand written* notes on both sides. Copied sheets are not allowed!

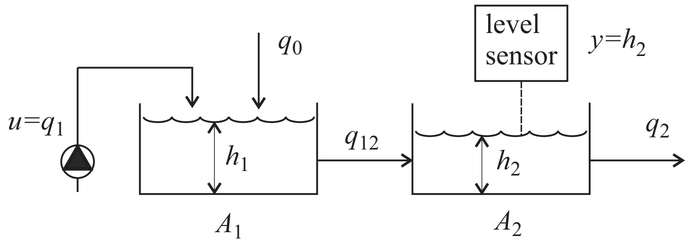
3. Mathematical and physical handbooks of tables, such as Physics handbook and Beta Mathematics Handbook.

4. Memory depleted, non-programmable pocket calculator.

Solutions to problems and exercises are not allowed in the notes!

Mobile telephones, laptops or tablets/iPads are not allowed!

1. Consider the system below with two tanks in series. To the first tank there is an uncontrollable flow q_0 of a fluid. In order to control the levels in the tanks a controller is to be installed using a level sensor, sensing the level in the second tank, and a pump delivering a flow q_1 to the first tank.



According to Bernoulli's law the flow between the two tanks is given by

$$q_{12} = k_{12}\sqrt{h_1 - h_2}$$

and the flow leaving the second tank is $q_2 = k_2\sqrt{h_2}$.

The bottom areas of the two tanks are $A_1 = 0.5$ m^2 and $A_2 = 1$ m^2, and $k_{12} = k_2 = 0.1$ m$^{5/2}$s^{-1}.

Note that all questions can be answered independantly of eachother!

(a) Linearize the system and show that around the operating point $\bar{h}_1 = 2$ m and $\bar{h}_2 = 1$ m the response from pumpflow $u = q_1$ to the measured level $y = h_2$ is given by the state space model

$$\begin{aligned}
\frac{d}{dt}\Delta h(t) &= \begin{bmatrix} -0.1 & 0.1 \\ 0.05 & -0.1 \end{bmatrix} \Delta h(t) + \begin{bmatrix} 2 \\ 0 \end{bmatrix} \Delta u(t) + \begin{bmatrix} 2 \\ 0 \end{bmatrix} \Delta q_0(t) \\
\Delta y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} \Delta h(t)
\end{aligned}$$

where Δ denotes deviations from stationary values in the operating point.

3 p.

(b) Determine the corresponding transfer function from Δu to Δy.

1 p.

(c) Even though the main purpose is to keep the level h_2 constant (to give a constant flow q_2 to the next stage in the process) it is of course important to monitor the level in tank 1 as well. To save the money though, it would be preferable if we could estimate it instead. Is that doable?

2 p.

(d) It is decided that both levels should be controlled using a stationary LQG controller minimizing

$$J = E\{\Delta h^T Q_x \Delta h + q_u \Delta u\}$$

From the beginning the tuning matrices were chosen to be $Q_x = I$ and q_u=1, but then it turned out that the first tank was sometimes flooded. What two changes would you consider first when retuning?

1 p.

(e) Another issue was found to be that the disturbances on q_0 could be low frequent, causing the level y to be off setpoint for long periods. What addition to your controller would you suggest?

Please state the matrices and equations to solve, and draw a block diagram showing the complete control structure.

3 p.

2. The process

$$G(s) = \frac{Y(s)}{U(s)} = \frac{e^{-Ts}}{s+1}$$

is to be controlled with piecewise constant control signal with a sampling interval $h = T$ s.

(a) Determine a time discrete state space model of the process.

2 p.

(b) By coincidence(!) the time delay is $T = \ln 2$ s.

Let $u(k) = K_r r(k) - u_{FB}(k)$ and determine the state feedback $u_{FB}(k) = -Lx(k)$ that gives a double pole in 0.25 and reformulate it as $u_{FB}(k) = H(q)y(k)$.

2 p.

(c) Determine the feed forward gain K_r that gives the correct stationary gain from r to y.

1 p.

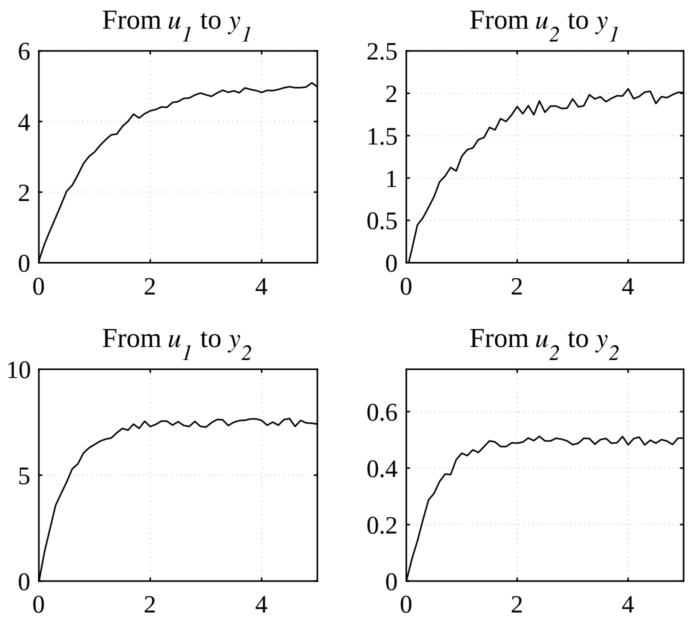
3. Consider the following MIMO-system:

$$\begin{aligned} \frac{d}{dt}x(t) &= \begin{bmatrix} -0.5 & 1 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} u(t) \end{aligned}$$

Examine the system's observability, controllability, detectability and stabilizability!

4 p.

4. Step response experiments have been conducted on a system with two control inputs and two control outputs (see figure).



(a) State an approximate transfer function matrix for the system.

2 p.

(b) Use RGA analysis to suggest pairing in a decentralized control (suggestion can be made from figure, i.e. without answering (a). This gives 2 p.).

3 p.

5. Assume we want to estimate the states of an observable system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Nw(t) \\ y(t) &= Cx(t) + v(t), \end{aligned}$$

where w and v are independent stochastic disturbances. For the Kalman filter estimate to be optimal, both w and v has to be white noise. A common situation, though, is that only the measurement noise v can be assumed white while the process disturbance w is coloured.

(a) We begin by studying a specific case, where the measurement noise has intensity $R_v = 1$, and the process and the spectrum of the process disturbance are given by

$$A = -1, \quad B = 1, \quad C = 1, \quad N = 1 \quad \text{and} \quad \Phi_w(\omega) = \frac{1}{1+\omega^2}$$

Add a model of the process disturbance to the process model and derive the optimal continuous time observer of x. State the observer on state-space form.

Unfortunately, the solution is difficult to determine by hand. However, you may use that for the optimal observer the stationary estimation error variance is $\text{Var}\{\hat{x} - x\} = 0.2$.

4 p.

(b) Now, we will study the general case. Assume that the process is stable and that the disturbance w is scalar and can be modelled using spectral factorization. Is it guaranteed that we can always estimate x optimally using a Kalman filter for such a system (motivation required)?

Hint: It may be useful to use observer canonical form to model the disturbance.

2 p.

Good luck!