CHALMERS UNIVERSITY OF TECHNOLOGY
Department of Electrical Engineering
Division of Systems and Control

EXAMINATION IN LINEAR CONTROL SYSTEM DESIGN

(Course SSY285)

Monday January 8, 2024

Time and place:	14:00 - 18:00 (Johanneberg)
Teacher:	Torsten Wik (031 - 772 5146)

The total points achievable are 30. Unfinished solutions normally results in 0 points. Incorrect solutions with significant errors, unrealistic results that are not commented, or solutions that are too difficult to follow also result in 0 points.

Numerically incorrect calculations that do not cause unrealistic results will normally lead to reduction of 1 point. Incorrect answers that are consequences of a previous error and do not simplify the problem will not lead to any further reduction.

The scales for grading are

Grade 3:	at least 12 points
Grade 4:	at least 18 points
Grade 5:	at least 24 points

The following aids are allowed:

1. Course text book *Control Theory* (or Swedish version *Reglerteori*) by T. Glad and L. Ljung and **one more** control textbook.

   Paper copies are accepted instead of books.

2. 1 piece of A4 paper, with *hand written* notes on both sides. Copied sheets are not allowed!

3. Mathematical and physical handbooks of tables, such as Physics handbook and Beta Mathematics Handbook.

4. Memory depleted, non-programmable pocket calculator.

Solutions to problems and exercises are not allowed in the notes!

Mobile telephones, laptops or tablets/iPads are not allowed!

1. Many systems (and parameter variations) can be modelled by a so-called random walk, i.e.

$$\begin{aligned} x(k+1) &= x(k) + v_1(k) \\ y(k) &= x(k) + v_2(k) \end{aligned}$$

where v_1 and v_2 are zero mean uncorrelated white gaussian noise, having variances $E\{v_1^2\} = r_1$ and $E\{v_2^2\} = r_2$.

   (a) Because of the noise in the measurement of x we want to use an observer to improve the measurement. Calculate the expression for the observer that theoretically should give the lowest estimation error variance for $\hat{x}$ at time k, given data up to time k, when $r_1 = 1$ and $r_2 = 2$.

   3 p.

   (b) What will the estimation error variance be (compare with the measurement)?

   1 p.

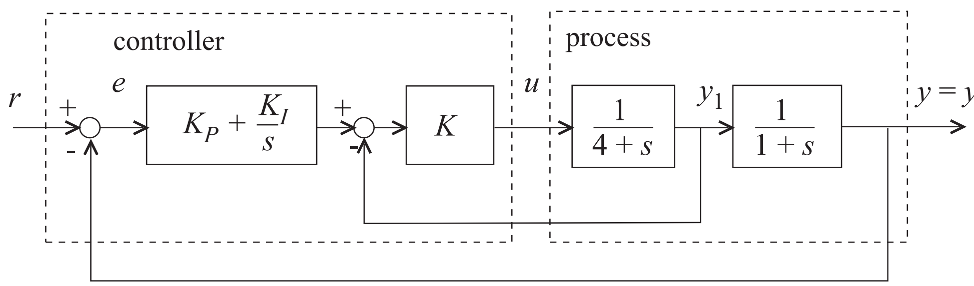
2. Suppose we have determined the spectrum of a continuous time disturbance w to be

$$\Phi_w(\omega) = \frac{1}{1+\omega^2}$$

Describing the disturbance as the output of a transfer function having white gaussian noise of intensity 1, what will the transfer function be?

1 p.

3. The idea of this problem is to show how the parameters in a cascaded control with an inner proportional controller and an outer PI controller can be determined by state feedback optimization.



Several of the questions can be solved independently of eachother!

   (a) Formulate the <u>process model</u> on a standard state space form, i.e.

$$\begin{aligned} \frac{d}{dt}x(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned}$$

   2 p.

(b) Introduce the integral state

$$x_I(t) = \int_0^t e(\tau) d\tau$$

and give an extended state space model

$$\begin{aligned} \frac{d}{dt} x_e(t) &= A_e x_e(t) + B_e u(t) + B_r r(t) \\ y(t) &= C_e x_e(t) \end{aligned}$$

including the integral state.

1 p.

(c) For the extended model we may now determine a state feedback

$$u(t) = -L x_e(t)$$

Suppose we want to minimize (we may set the setpoint $r = 0$)

$$J = \int\limits_0^\infty y^2(t) + u^2(t) + q_I x_I^2(t) dt$$

Give the necessary matrices and equations for finding the L that minimizes the above J (you need not solve them)!

2 p.

(d) Knowing L, what should K_P, K_I and K be, i.e., express them in terms of the elements of L.

2 p.

(e) Suppose the convergence towards a constant setpoint r after a load process disturbance takes a very long time, what do you recommend we should do in our design?

1 p.

4. In wastewater treatment bacteria are consuming dissolved organic substances, transforming them into carbon dioxide, water and biomass as the bacteria multiply. The bacteria can then be separated from the water by sedimentation and be reused by partial recirculation.

In such a treatment basin, the governing equations are

$$\begin{aligned} V\frac{d}{dt}S(t) &= QS_{in}(t) - QS(t) - V\frac{\mu}{Y}X(t)\frac{S(t)}{K+S(t)} \\ V\frac{d}{dt}X(t) &= QX_{in}(t) - QX(t) + V\mu X(t)\frac{S(t)}{K+S(t)} \end{aligned}$$

where the following parameters can be assumed to be constants

V	$=$	1 m^3	(the volume of the basin)
Q	$=$	$2 \text{ m}^3/\text{min}$	(flow through the basin)
μ	$=$	1 min^{-1}	(maximum growth rate of the bacteria)
Y	$=$	0.1 mol/kg	(yield coefficient)
K	$=$	1 mol/min	(saturation coefficient)

Note: you can solve (c) to (e) even if you cannot solve (a) and (b)!

(a) Let x_1 be the concentration of dissolved organics (S), x_2 the bacterial concentration (X), u the influent bacterial concentration in the feed (X_{in}) and v the influent concentration of organics to be removed (S_{in}).

Write the equations on the standard state-space model form

$$\dot{x}(t) = f(x(t), u(t), v(t))$$

1 p.

(b) Assume that we are operating around the equilibrium given by the stationary influent concentrations $\bar{S}_{in} = 6 \text{ mol/m}^3$ and a desired effluent concentration $\bar{S} = 1 \text{ mol/m}^3$.

Show that the following state-space model approximates the dynamics of the variations around this point:

$$\frac{d}{dt}\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} = \begin{bmatrix} -7 & -5 \\ 0.5 & -1.5 \end{bmatrix}\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}\Delta u + \begin{bmatrix} 2 \\ 0 \end{bmatrix}\Delta v$$

3 p.

(c) The bacterial concentration x_2 cannot be measured online. Suppose we measure the concentration of the organics (x_1), can we estimate x_2?

2 p.

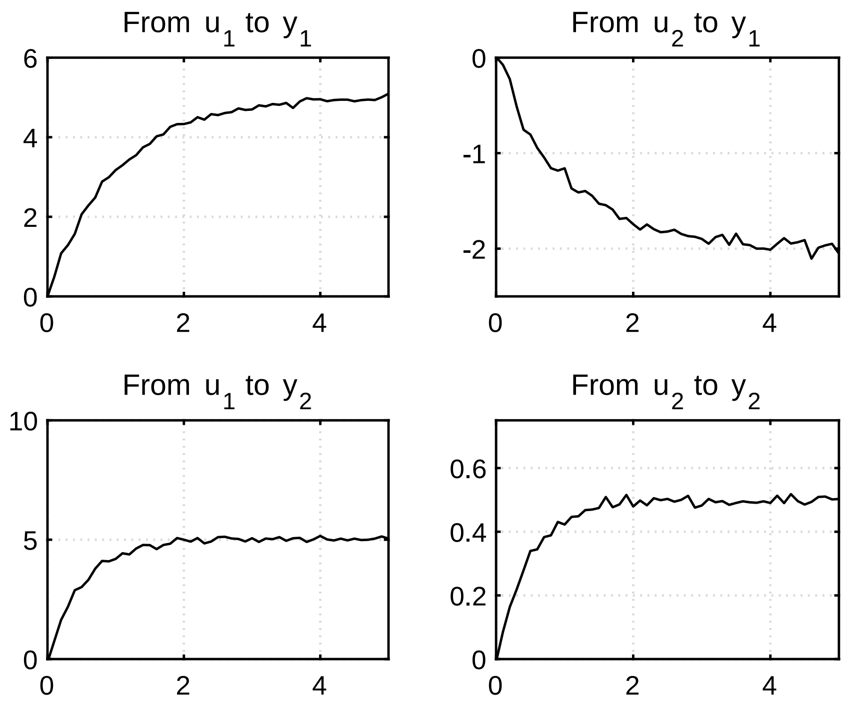
(d) Design an observer having all poles in -5.

2 p.

(e) If the observer gives estimates that reacts too strongly to measurement noise, suggest how we should move the poles (more or less negative).

1 p.

5. Step response experiments were carried out on a system with two control inputs and two control outputs (see figure). First u_1 was increased one unit and the two plots to the left were registered. Then u_2 was increased one unit and the two plots to the right were registered.



(a) State an approximate transfer function matrix $G(s)$ for the system.

2 p.

(b) Use RGA analysis to suggest pairing in a decentralized control (suggestion can be made from figure, i.e. without answering (a). This gives 2 p.).

3 p.

(c) Determine a decoupling $W_2 G W_1$ of the plant such that it can be controlled with 2 PI-controllers (you do not need to calculate the PI-parameters). What are the SISO transfer functions that the PI-controllers should be designed for?

3 p.

Good luck!