CHALMERS UNIVERSITY OF TECHNOLOGY
Department of Electrical Engineering
Division of Systems and Control

EXAMINATION IN LINEAR CONTROL SYSTEM DESIGN

(Course SSY285)

Monday January 13, 2020

Time and place:	14:00 - 18:00 at SB
Teacher:	Torsten Wik (031 - 772 5146)

The total points achievable are 30. Unfinished solutions normally results in 0 points. Incorrect solutions with significant errors, unrealistic results that are not commented or solutions that are too difficult to follow also result in 0 points.

Numerically incorrect calculations that do not cause unrealistic results will normally lead to reduction of 1 point. Incorrect answers that are consequences of a previous error and do not simplify the problem will not lead to any further reduction.

The scales for grading are

Grade 3:	at least 12 points
Grade 4:	at least 18 points
Grade 5:	at least 24 points

The following aids are allowed:

1. Course text book *Control Theory* (or Swedish version *Reglerteori*) by T. Glad and L. Ljung and **one more** control textbook.

   Paper copies are accepted instead of books

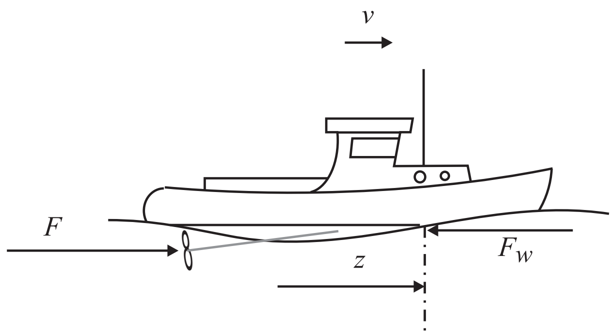
2. 1 piece of A4 paper, with *hand written* notes on both sides. Copied sheets are not allowed!

3. Mathematical and physical handbooks of tables, such as Physics handbook and Beta Mathematics Handbook.

4. Memory depleted, non-programmable pocket calculator.

Notes, mobile telephones, laptops or palmtops, are not allowed!

1. A fishing boat is to be equipped with a speed and position over ground (bottom) control based on GPS position. Here we may assume only one space dimension, i.e. the boat follows a straight line (see figure) and the position is x.



Let the boat position be $z(t)$ $[m]$, the boat speed be $v(t)$ $[m/s]$ and the propulsion force be $F(t)$ $[N]$. We may assume all friction forces together be related to the speed according to

$$F_w(t) = 400v^2(t) \quad [N]$$

The boat mass is 10 000 kg.

(a) Determine a continuous time state space model for the boat propulsion, describing the behavior from propulsion force $u = F$ to position and speed $y = \begin{bmatrix} z & v \end{bmatrix}^T$.

2 p.

(b) The normal speed is 5 m/s (approximately 10 knots). Show that the linear state space model describing the dynamic behavior around that speed is

$$\begin{aligned} \frac{d}{dt}\Delta x(t) &= \begin{bmatrix} 0 & 1 \\ 0 & -0.4 \end{bmatrix} \Delta x(t) + \begin{bmatrix} 0 \\ 10^{-4} \end{bmatrix} \Delta u(t) \\ \Delta y(t) &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(t) \end{aligned}$$

2 p.

(c) Discretize the above state space model for zero order hold and a sampling rate of 10 ms.

2 p.

2. A linear time invariant system is given by

$$\begin{aligned} \frac{d}{dt}x(t) &= \begin{bmatrix} -1 & 0.5 & 0 \\ 0.5 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x(t) \end{aligned}$$

(a) We only measure x_2 and x_3. Can x_1 be estimated?

2 p.

(b) Show that the system is not controllable?

1 p.

(c) Is the system stabilizable?

2 p.

(d) The eigenvectors of the system matrix (A) are

$$v_1 = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$$

Diagonalize the system. *You may then use*

$$\begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

3 p.

(e) What happens to the uncontrollable state, and what will the consequences be on the original states x_1, x_2, and x_3?

1 p.

(f) Use the result from (e) to give a reduced state-space model, valid after initial transients have settled and keeping the meaning and quantities from the original model.

1 p.

3. A linear time invariant state space model (A, B, C, D) has 12 states. However, if we only want to describe the system response from the control input u to the output y the number of states can be reduced. The following holds for the model

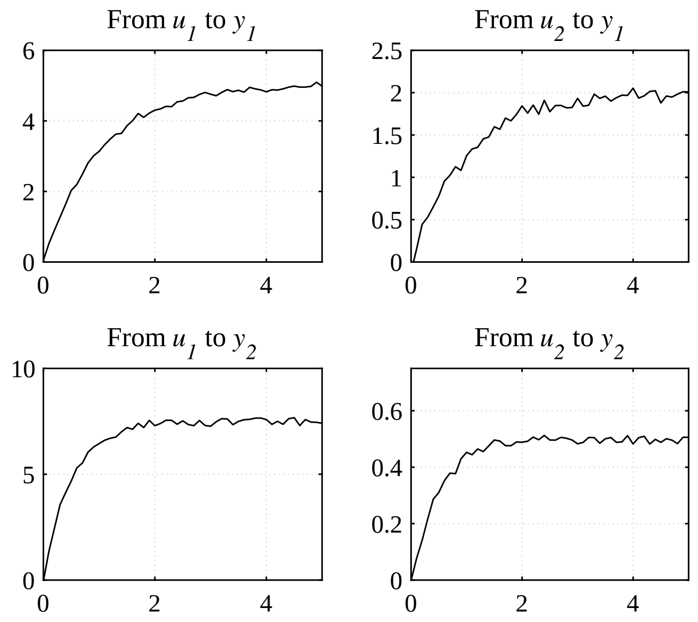
$$\begin{aligned} \operatorname{rank}\left(\begin{bmatrix} B & AB & A^2B & \ldots & A^{11}B \end{bmatrix}\right) &= 8 \\ \operatorname{rank}\left(\begin{bmatrix} C^T & A^TC^T & \ldots & (A^T)^{11}C^T \end{bmatrix}\right) &= 7 \end{aligned}$$

(a) What is the possibly lowest number of states required?

(b) What is the possibly largest number of states required?

2 p.

4. Step response experiments have been conducted on a system with two control inputs and two control outputs (see figure).



(a) State an approximate transfer function matrix for the system.

2 p.

(b) Use RGA analysis to suggest pairing in a decentralized control (suggestion can be made from figure, i.e. without answering (a). This gives 2 p.).

3 p.

5. Assume we want to estimate the states of an observable system

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) + Nw(t) \\ y(t) &= Cx(t) + v(t),\end{aligned}$$

where w and v are independent stochastic disturbances. For the Kalman filter estimate to be optimal, both w and v has to be white noise. A common situation, though, is that only the measurement noise v can be assumed white while the process disturbance w is coloured.

(a) We begin by studying a specific case, where the measurement noise has intensity $R_v = 1$, and the process and the spectrum of the process disturbance are given by

$$A = -1, \quad B = 1, \quad C = 1, \quad N = 1 \quad \text{and} \quad \Phi_w(\omega) = \frac{1}{1+\omega^2}$$

Add a model of the process disturbance to the process model and derive the optimal continuous time observer of x. State the observer on state-space form.

Unfortunately, the solution is difficult to determine by hand. However, you may use that for the optimal observer the stationary estimation error variance is $\text{Var}\{\hat{x} - x\} = 0.2$.

5 p.

(b) Now, we will study the general case. Assume that the process is stable and that the disturbance w is scalar and can be modelled using spectral factorization. Is it guaranteed that we can always estimate x optimally using a Kalman filter for such a system (motivation required)?

Hint: It may be useful to use observer canonical form to model the disturbance.

2 p.