CHALMERS UNIVERSITY OF TECHNOLOGY
Department of Electrical Engineering
Division of Systems and Control

EXAMINATION IN LINEAR CONTROL SYSTEM DESIGN

(Course SSY285)

Friday August 30, 2019

Time and place:	08:30 - 12:30 at Hörsalsvägen
Teacher:	Torsten Wik (031 - 772 5146)

The total points achievable are 30. Unfinished solutions normally results in 0 points. Incorrect solutions with significant errors, unrealistic results that are not commented or solutions that are too difficult to follow also result in 0 points.

Numerically incorrect calculations that do not cause unrealistic results will normally lead to reduction of 1 point. Incorrect answers that are consequences of a previous error and do not simplify the problem will not lead to any further reduction.

The scales for grading are

Grade 3:	at least 12 points
Grade 4:	at least 18 points
Grade 5:	at least 24 points

The following aids are allowed:

1. *Feedback Systems* by Åström and Murray OR *Reglerteknikens grunder* by Lennartson OR Reglerteknik, grundlaggande teori by Glad and Ljung OR *Linear optimal control systems* by H. Kwakernaak and R. Sivan

   Paper copies are accepted instead of books
2. 1 piece of A4 paper, with *hand written* notes on both sides. Copied sheets are not allowed!
3. Mathematical and physical handbooks of tables, such as Physics handbook and Beta Mathematics Handbook.
4. Memory depleted, non-programmable pocket calculator.

Notes, mobile telephones, laptops or palmtops, are not allowed!

The results are open for review on Tuesday September 17, at 12:45 - 13:30 at the department.

Good Luck!

1. Assume we have derived a state space model

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k) \\ y(k) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(k) \end{aligned}$$

   (a) Determine L in a time invariant state feedback $u(k) = -Lx(k) + Kr(k)$ such that both closed loop poles are 0.9.

   3 p.

   (b) Determine K such that $y = r$ in steady state (provided we have a correct model and no disturbances).

   1 p.

2. Consider

$$\frac{d}{dt}x(t) = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t)$$

   By the use of a state feedback $u(t) = -Lx(t)$, can we place the poles of the closed loop system arbitrarily?

   2 p.

3. A continuous time linear system is given by

$$y(t) = \begin{bmatrix} 0 & \frac{2}{p+1} \\ 0 & \frac{1}{p} \\ \frac{1}{p+1} & \frac{2}{p+1} \end{bmatrix} u(t)$$

   Note: p is the time derivative operator (d/dt) which can be used in calculations for transfer function operators, such as the elements in the above matrix, in the same way as the Laplace variable s is used for transfer functions.

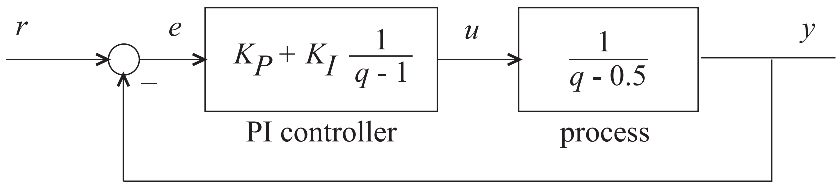
   (a) What are the poles and zeros of this multivariable system?

   2 p.

   (b) Give a state space realization of the system. Is it minimal?

   2 p.

4. A discrete time first order process is to be controlled using a discrete time PI-controller. In the illustration below we use the time shift operator q (for example: $qy(k) \equiv y(k+1)$ and $y(k) = (1/(q-0.5)u(k))$.



The idea in this problem is to show how PI controller parameters can be determined by state feedback optimization.

(a) Formulate the process model on a standard state space form, i.e.

$$
\begin{aligned}
x(k+1) &= Ax(k) + Bu(k) \\
y(k) &= Cx(k)
\end{aligned}
$$

1 p.

(b) Introduce the integral state

$$x_I(k) = \frac{1}{q-1}e(k)$$

and give an extended state space model

$$
\begin{aligned}
x_e(k+1) &= A_e x_e(k) + B_e u(k) + K_r r(k) \\
y(k) &= C_e x_e(k)
\end{aligned}
$$

including the integral state.

1 p.

(c) For the extended model we may now determine a state feedback

$$u(k) = -L_e x_e(k),$$

by minimization of (we may set $r = 0$, and note that $y^2 = x_e^T C_e C_e^T x_e$)

$$J = \sum_{k=0}^{\infty} y^2(k) + q_u u^2(k) + q_I x_I^2(k)$$

Assume we have solved the Ricatti equations giving the optimal L_e. Express K_P and K_I in terms of L_e (you may assume $r = 0$).

1 p.

(d) This method works also for higher order SISO transfer functions with no common poles and zeros. What can we say about the stability then?

1 p.

5. Consider the system

$$G(s) = \frac{1}{s}e^{-0.6s}$$

from input $u(t)$ to output $y(t)$.

This system is to be discretized for the sampling time $h = 1$ and zero order hold input. The difficulty here is that there is a time delay that is not equal to a multiple of the sampling time.

(a) Express the above model as a continuous time LTI state space model with delayed input.

1 p.

(b) Use the analytical solution to the state space model to derive the corresponding discrete time state space model on standard form

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned}$$

for which $y(k) = y(kh) = y(t)$ if $u(k) = u(t)$ for all $t = kh$.

2 p.

6. The following system is a simple delay system with additive noise:

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k) + v_1(k) \\ y(k) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(k) + v_2(k) \end{aligned}$$

where v_1 and v_2 are independent, zero mean, gaussian distributed white noise with variance $R_1 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and $R_2 = 1$.

(a) Is the system observable?

1 p.

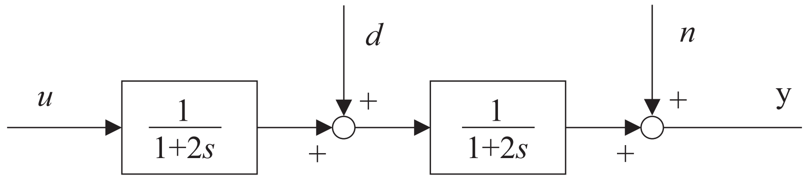
(b) Show that for a standard predictive Kalman filter, i.e. $\hat{x}_p(k) = \hat{x}(k|k-1)$, there is no feedback of the innovation $y(k) - \hat{y}(k|k-1)$ so that the estimates are based only on the model (not recommendable).

3 p.

(c) To improve the estimation we include the measurement $y(k)$ in our estimation of $\hat{x}(k)$. Determine the Kalman filter for the filter case, i.e. $\hat{x}_f(k) = \hat{x}(k|k)$. How much are the estimation errors variances reduced compared to that of $\hat{x}_p$?

3 p.

7. Consider the block scheme below.



We can assume the disturbance v is white noise with unit variance, and for d the following spectrum has been determined experimentally:

$$\Phi_d(\omega) = \frac{1}{\omega^2 + 1}$$

(a) Write the system on the state space form

$$\begin{aligned} \frac{d}{dt}x(t) &= Ax(t) + Bu(t) + Ne(t) \\ y(t) &= Cx(t) + n(t), \end{aligned}$$

where $e(t)$ is white noise with intensity $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

2 p.

(b) Assume we have determined the spectrum also for the measurement noise n as

$$\Phi_n(\omega) = \frac{\omega^2 + 4}{\omega^2 + 9}.$$

Rewrite the state space model once more on standard form (Note! Not the same matrices as in (a))

$$\begin{aligned} \frac{d}{dt}x(t) &= Ax(t) + Bu(t) + Nv_1(t) \\ y(t) &= Cx(t) + v_2(t), \end{aligned}$$

where $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ is white noise with intensity $R = \begin{bmatrix} R_1 & R_{12} \\ R_{12}^T & R_2 \end{bmatrix}$!

2 p.

(c) What will the intensity matrix R be if n, d and v are all independent?

2 p.