CHALMERS UNIVERSITY OF TECHNOLOGY
Department of Electrical Engineering
Division of Systems and Control

EXAMINATION IN LINEAR CONTROL SYSTEM DESIGN

(Course SSY285)

Wednesday April 24, 2019

Time and place:	08:30 - 12:30 at Hörsalsvägen
Teacher:	Torsten Wik (031 - 772 5146)

The total points achievable are 30. Unfinished solutions normally results in 0 points. Incorrect solutions with significant errors, unrealistic results that are not commented or solutions that are too difficult to follow also result in 0 points.

Numerically incorrect calculations that do not cause unrealistic results will normally lead to reduction of 1 point. Incorrect answers that are consequences of a previous error and do not simplify the problem will not lead to any further reduction.

The scales for grading are

Grade 3:	at least 12 points
Grade 4:	at least 18 points
Grade 5:	at least 24 points

The following aids are allowed:

1. *Feedback Systems* by Åström and Murray OR *Reglerteknikens grunder* by Lennartson OR Reglerteknik, grundlaggande teori by Glad and Ljung OR *Linear optimal control systems* by H. Kwakernaak and R. Sivan

   Paper copies are accepted instead of books

2. 1 piece of A4 paper, with *hand written* notes on both sides. Copied sheets are not allowed!

3. Mathematical and physical handbooks of tables, such as Physics handbook and Beta Mathematics Handbook.

4. Memory depleted, non-programmable pocket calculator.

Notes, mobile telephones, laptops or palmtops, are not allowed!

The results are open for review on Monday May 13, at 13:00 - 14:00 at the department.

Good Luck!

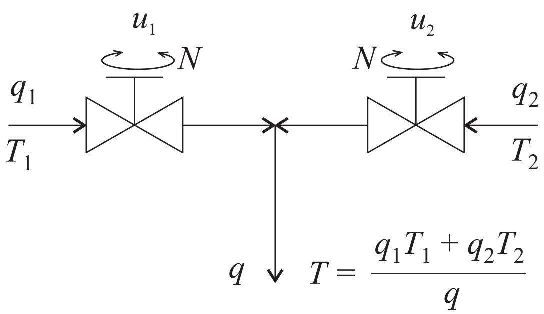
1. When using water from a tap you are interested in two properties: the flow q and the temperature T. A classical tap has two actuators, one for cold water and one for hot water. Clearly, this is a MIMO system because, for example, opening the hot water valve will increase the flow as well as the temperature.

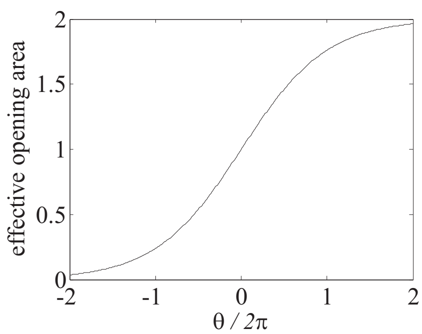
Classical two-grip tap

Modern one-grip tap

A modern one-grip tap is decoupled such that one input (vertical angle of the arm) affects the flow, and the other input (horizontal angle) affects the temperature.

Now, we have an industrial system with large flows and two *identical* valves, and we want to use multivariable control to achieve the same decoupling without redesigning the plant. The valves are motorized such that the actuation is driven by supply voltages u_1 and u_2 (see left figure).





The flow through each valve is proportional to the effective opening area (a_i), i.e.

$$q_i(t) = k_q a_i(t), \quad i = 1, 2 \tag{1}$$

where k_q is a constant. The effective opening area is a nonlinear function of the angle θ (rad) of the valve actuator (see right figure):

$$a_i(t) = 1 + \tanh\left(\frac{\theta_i(t)}{2\pi N}\right), \quad \text{where} \quad \tanh z = \frac{e^z - e^{-z}}{e^z + e^{-z}} \tag{2}$$

where N is the number of turns between closed and open valve. The friction and moment of inertia for the motor, gearbox and the valve can be ignored, and therefore

$$\frac{d}{dt}\theta_i(t) = k_\theta u_i(t), \quad i = 1, 2$$

where k_θ is a constant.

Note: This problem contains many questions. However, many of them can be solved independently of each other, so please read carefully through all questions!

(a) Assume constant temperatures T_1 and T_2 and determine a state space model for this system, with the flow q and the temperature T as outputs.

2 p.

(b) The cold temperature is $T_1 = 20°C$ and the hot temperature is $T_2 = 100°C$. Let $k_q = 1$, $k_\theta = 1$ and $2\pi N = 10$ and show that a linearized model for the operating point where $\theta_1 = \theta_2 = 0$ is

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} \Delta q \\ \Delta T \end{bmatrix} = \begin{bmatrix} 0.1 & 0.1 \\ -2 & 2 \end{bmatrix} x(t) \end{aligned}$$

Help: $(d/dz) \tanh z = 1 - \tanh^2 z$

3 p.

(c) Show that for a sampling time $h = 0.1s$ and piecewise constant control signal, the corresponding discrete time model is

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} u(k) \\ y(k) &= \begin{bmatrix} 0.1 & 0.1 \\ -2 & 2 \end{bmatrix} x(k) \end{aligned}$$

2 p.

(d) Is the discrete time system reachable?

1 p.

(e) If we measure only the temperature T, can we estimate changes in the flow?

1 p.

(f) Suppose we measure both temperature and flow. Show that the discrete time transfer function for the system is

$$H(z) = \begin{bmatrix} \frac{0.01}{z-1} & \frac{0.01}{z-1} \\ \frac{-0.2}{z-1} & \frac{0.2}{z-1} \end{bmatrix} \quad \text{(or equivalently} \quad H(q) = \begin{bmatrix} \frac{0.01}{q-1} & \frac{0.01}{q-1} \\ \frac{-0.2}{q-1} & \frac{0.2}{q-1} \end{bmatrix},$$

where q is the time shift operator).

1 p.

(g) Decouple this MIMO system completely with a compensator W such that WH is diagonal.

1 p.

(h) Next, two SISO proportional controllers, $\tilde{H}_{c1} = K_1$ and $\tilde{H}_{c2} = K_2$ are chosen for the decoupled system WH (you need not carry out this design!). Express the MIMO controller from y_1 and y_2 to u_1 and u_2 in terms of the two SISO controllers and your elements in W. Give the elements in your controller matrix *or* show in a SISO-signal block diagram.

2 p.

2. A scalar dynamic system can be described by an autoregressive (AR) process

$$x(k+1) = 0.8x(k) + v(k)$$

where v is white noise of variance 1. The state of the system is measured by two sensors having independent white Guassian noise being indenpendent of the process disturbance v. The variance of the noise of the first sensor is 1. The other sensor is less accurate and has variance 2.

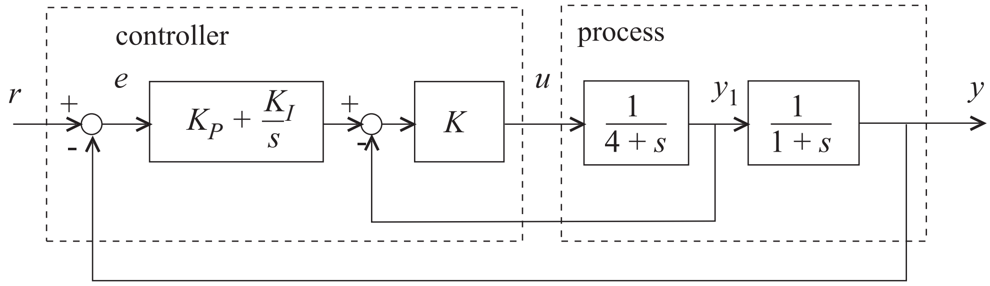
(a) Determine the optimal stationary observer (minimizing the estimation error variance) $\hat{x}(k|k-1)$ based on measurements up to time $k-1$. How large is the estimation error variance.

3 p.

(b) To improve the capacity one may calculate the estimate $\hat{x}(k)$ directly from the measurement at time k. How much can the variance be reduced then?

2 p.

3. The idea of this problem is to show how the parameters in a cascaded control with an inner proportional controller and an outer PI controller can be determined by state feedback optimization.



Note: Several of the below question can be solved independently of eachother!

(a) Formulate the process model on a standard state space form, i.e.

$$\begin{aligned} \frac{d}{dt}x(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned}$$

2 p.

(b) Introduce the integral state

$$x_I(t) = \int_0^t e(\tau)d\tau$$

and give an extended state space model

$$\begin{aligned} \frac{d}{dt}x_e(t) &= A_e x_e(t) + B_e u(t) + B_r r(t) \\ y(t) &= C_e x_e(t) \end{aligned}$$

including the integral state.

1 p.

(c) For the extended model we may now determine a state feedback

$$u(t) = -Lx_e(t),$$

Suppose we want to minimize (we may set the set point $r = 0$)

$$J = \int_0^\infty y^2(t) + u^2(t) + q_I x_I^2(t)dt$$

Give the necessary matrices and equations for finding the L that minimizes the above J (you need not solve them)!

2 p.

(d) Knowing L, what are K_P, K_I and K?

2 p.

(e) Suppose the convergence towards a constant setpoint r after a load process disturbance takes a very long time, what do you recommend we should do?

1 p.

4. Consider

$$\begin{aligned} x(k+1) &= \begin{bmatrix} -1 & 0 & -1 \\ 2 & 2 & 0 \\ -1 & 0 & 3 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} u(k) \\ y(k) &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x(k) \end{aligned}$$

(a) Show that the system is not reachable.

2 p.

(b) In fact, it is one mode (combination of the three states) that cannot be reached from the origin. Determine what combinations of the state components x_1, x_2 and x_3 that cannot be reached from the origin.

2 p.