

Exam questions for Linear Control System Design, SSY285

August 25<sup>th</sup>, 2017

## Cover page

1. Timeframe: 4 hours.
2. Examiner: Balazs Kulcsar, (☎1785, kulcsar@chalmers.se)
3. Necessary condition to obtain the exam grade is to have the course's mandatory project (all assignments and the lab) approved. Without approved mandatory project, the archived exam results are invalid.
4. 20 points can be reached in total (with 0.5 point resolution), Lic/PhD students have to reach at least 12 points to pass. For Msc students Table 1 shows the grading system.

Table 1: Grading for Msc students

Points	Grade
10 ... 11.5	3
12 ... 15.5	4
16 ... 20	5

5. During this written exam, it is *optionally* permitted to use printed materials such as:
  - *Either* of the course textbooks (only 1 book): (hardcopy or plain printed version, without notes inside!)
    - (i) *Feedback systems, an introduction for scientists and engineers* by K. J. Åström and R. M. Murray, ISBN-13: 978-0-691-13576-2, OR
    - (ii) *Reglerteknikens grunder* by Bengt Lennartson, ISBN: 91-44-02416-9, (iii) *Reglerteknik : grundläggande teori*, T Glad, L Ljung.
  - 1 piece of A4 paper, with *hand written* notes on both sides. Copied sheet can not be used.
  - Pocket calculator (non-programmable, cleared memory, without graphical plotting function).
  - Mathematical handbook Beta (without notes inside!).
6. Note that phones, tablets, computers, any other communication devices are not allowed to use during the exam session. In scheduled exam session for the course at Chalmers, teacher(s) will show up in person in the first and last 60 min.
7. Examination results will be advertised no later than 1.5 week following the examination date (via pingpong.chalmers.se). Date and place for grade inspection will be announced (pingpong).

Good luck!

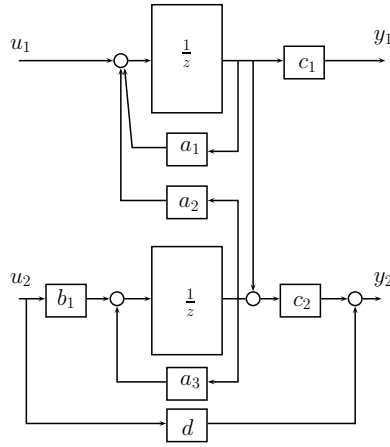


Figure 1: Open-loop block diagram

## Questions

- Briefly answer the questions with motivation (each **0.5 point**, total **2 point**).
  - What is the state-transition matrix (continuous time, LTI state space forms)? What do we use it for?
  - Your task is to diagonalize a state space model. What methodology do you recommend to use?
  - What do they stand for LQR and LQG techniques? Explain briefly how these methodology work, point out the main methodological difference.
  - What is the main conceptual difference in between the additive and multiplicative model uncertainty for LTI systems?
- Given the following system representation by means of block diagram in Figure 1, where  $a_1 = 0.1, a_2 = 0.5, a_3 = 0.1, b_1 = 0.1, c_1 = 0.2, c_2 = 0.5, d = 1$ .
  - Derive the *discrete-time* state-space representation in terms of matrix difference equation  $(A, B, C, D)$  for the depicted system in Fig. 1 (**1 point**).
  - By using the coefficient matrices  $(A, B, C, D)$ , compute the discrete time transfer function matrix  $G(z)$  (**2 point**).
  - Find the poles and the zeros of  $G(z)$ . Based on  $G(z)$ , is this a non-minimum phase representation? Is the system input-output stable? Motivate your answer (**2 point**)!
  - Is the system internally stable? Motivate your answer. (**1 point**)!
  - What is the steady state value for  $y_2(k)$  while  $k \mapsto \infty$  if  $u_1(k) = 1$  ( $U_1(z) = \frac{1}{z-1}$ ) and  $u_2(k) = 0 \forall k > 0$  is applied? (subscripts refer to the input-output channel number) (**1 point**)
- Given the following state-space representation by,

$$x(k+1) = \begin{bmatrix} -1.5 & -0.5 \\ -\alpha & 0 \end{bmatrix} x(k) + \begin{bmatrix} \frac{1}{\alpha} & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} u(k) \\ d(k) \end{bmatrix}$$

$$y(k) = \begin{bmatrix} 2 & \frac{1}{\alpha} \end{bmatrix} x(k)$$

with  $0 < |\alpha| < \infty$ , where  $u(k), d(k), x(k), y(k)$  are control input, disturbance input, state vector and measured output, respectively.

- Is the state-space representation asymptotically stable for all values of  $\alpha$ ? (**1 point**)
- Is the state-space representation observable for all  $\alpha$ ? (**1 point**)

- c) With  $\alpha = -1$  is the representation reachable and controllable? **(1 point)**
- d) With  $\alpha = 1$ , find the (similarity) state transformation matrix  $T$  that renders the state space description into reachable canonical form. With the help of  $T$  transform the system into the new form,  $(\tilde{A}, \tilde{B}, \tilde{C})!$  **(2 point)**

4. Consider the following control problem given by,

$$\begin{aligned}\dot{x}_1(t) &= -x_1(t) + u(t) \\ \dot{x}_2(t) &= x_2(t) \\ J(u) &= \frac{1}{2} \int_0^\infty (q_u \cdot u^2(t) + x_1^2(t) + x_2^2(t)) dt\end{aligned}$$

- a) With  $q_u = 0.1$  find the steady-state LQR state feedback gain  $(\tilde{K})$  that minimizes  $J(u)$  by applying the following solution matrix structure,

$$\tilde{P} = \begin{bmatrix} p_1 & p_1 \\ p_1 & p_2 \end{bmatrix}$$

What are the closed loop poles? **(2 point)**.

- b) Find the optimal cost value for the closed loop system if  $x_0 = [1 \ 1]?$  **(1 point)**

5. Given the closed loop system as shown in Figure 2.

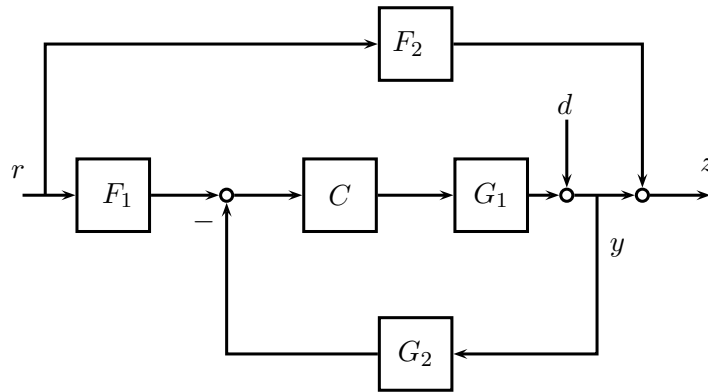


Figure 2: Closed-loop block diagram

with the plant transfer functions  $G_1$ ,  $G_2$  controller  $C$ , and pre-filter  $F_1$ ,

$$G_1 = \frac{(s+1)}{(s+2)(s+3)}, G_2 = \frac{1}{(s+1)}, C = 3, F_1 = \frac{s+2}{s+1}.$$

- a) Find the nominal transfer function matrices  $G(s)$  (with unknown  $F_2$ ) where  $\begin{bmatrix} z \\ y \end{bmatrix} = G(s) \begin{bmatrix} r \\ d \end{bmatrix}$  **(1 point)**.
- b) Set  $F_2$  such that in steady-state the transfer from  $r$  to  $z$  is 1, i.e.  $r_\infty = z_\infty$  **(1 point)?**

6. Given the continuous-time stochastic differential equations by,

$$\begin{aligned}\dot{x}_1(t) &= -2x_1(t) + x_2(t) + v_1(t) \\ \dot{x}_2(t) &= -3x_2(t) + v_2(t)\end{aligned}$$

where  $v_1$  and  $v_2$  are elements of a vector valued *correlated*, zero mean and Gaussian white noise vector process with constant intensity matrix  $V = \begin{bmatrix} 2 & 4 \\ 4 & 6 \end{bmatrix}$ . Find the steady-state state covariance matrix (while  $t \mapsto \infty$ ,  $\bar{P} = E\{(x(t) - m_x)(x(t) - m_x)^T\}$ ). **(1 point)**