Chalmers University of Technology
Department of Signals and Systems
Automatic Control Group

Resit exam questions for Linear Control System Design, SSY285

August 26^{th}, 2016

Cover page

1. Timeframe: 4 hours.
2. Examiner: Balazs Kulcsar, ☎1785, `kulcsar@chalmers.se`
3. Necessary condition to obtain the exam grade is to have the course's mandatory project (all assignments and the lab) approved. Without approved mandatory project, the archived exam results are invalid.
4. 20 points can be reached in total (with 0.5 point resolution), Lic/PhD students have to reach at least 12 points to pass. For Msc students Table 1 shows the grading system.

Table 1: Grading for Msc students

Points	Grade
10 . . . 11.5	3
12 . . . 15.5	4
16 . . . 20	5

5. During this written exam, it is *optionally* permitted to use printed materials such as:
   - *Either* of the course textbooks: (hardcopy or plain printed version, without notes inside!)
     Feedback systems, an introduction for scientists and engineers by K. J. Åström and R. M. Murray, ISBN-13: 978-0-691-13576-2
     OR
     Reglerteknikens grunder by Bengt Lennartson, ISBN: 91-44-02416-9.
   - 1 piece of A4 paper, with *hand written* notes on both sides. Copied sheet can not be used.
   - Pocket calculator (non-programmable, cleared memory, without graphical plotting function).
   - Mathematical handbook Beta (without notes inside!).
6. Note that phones, tablets, computers, any other communication devices are not allowed to use during the exam session. In scheduled exam session for the course at Chalmers, teacher(s) will show up in person in the first and last 60 min.
7. Examination results will be advertised not later than on September 2nd 2016 (pingpong.chalmers.se). Inspection of results in person, September 5th, 10-11 am, E-building floor 5, Ankit Gupta .

Good luck!

Questions

1. Given the following continuous-time state space realization

$$A = \begin{bmatrix} -1 & 1 \\ 0 & 0.5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

   a) Is it of minimum phase system realization? (**2 point**)

   b) Is the system IO stable? (**1 point**)

2. Given the following state-space representation by,

$$\dot{x}(t) = \begin{bmatrix} 1 & \alpha+\beta \\ 0.5 & \alpha \cdot \beta \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

   with $|\alpha| < \infty$, $|\beta| < \infty$.

   a) Is the state-space representation reachable for all finite values of α, β? (**1 point**)

   b) Is the state-space representation observable for all finite values of α, β? (**1 point**)

   c) With $\alpha = 0.5$, $\beta = 1$, find the diagonalyzing state transformation matrix T. With the help of T transform the system into a diagonal representation, $(\tilde{A}, \tilde{B}, \tilde{C})$! (**2 point**)

   d) With the above diagonal form, find $y(2)$ if $x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, and $u = 0$. (**1 point**)

   e) For the above diagonal representation, and with sampling time $T_s = 1$s, find the discrete time state-space representation matrices A_d, B_d, C_d, D_d. (**2 point**)

3. Given two discrete time state-space representations ($dim(x_1) = dim(x_2)$),

$$x_1(k+1) = A_1 x_1(k) + B_1 u(k), \quad y(k) = C_1 x_1(k)$$
$$x_2(k+1) = A_2 x_2(k) + B_2 u(k), \quad y(k) = C_2 x_2(k)$$

   and we know that $x_1(k) = T x_2(k)$, where T is a nonsingular similarity state transformation matrix. Show that the transfer function for the above representations is independent of T! (**1 point**)

4. Given the following noise corrupted linear system description as,

$$\ddot{z}(t) = u(t) + v(t)$$
$$y(t) = z(t) + w(t)$$

   where v, w are uncorrelated zero-mean and normally distributed white noise signals, with intensities $R_v = 8$, $R_w = 1$. Find the optimal dynamic output regulator minimizing (LQG controller),

$$J = \lim_{T \mapsto \infty} E\left\{ \frac{1}{2} \int_0^T \left(z^2(t) + u^2(t) \right) dt \right\}$$

   a) Find the proper state-space matrix representation of the above system (second order!). (**1 point**)

   b) By means of FARE, calculate the optimal Kalman filter gain $\bar{L}$. (**2 point**)

   c) By means of CARE, calculate the optimal LQR feedback gain $\bar{K}$. (**2 point**)

   d) What is the consequence of $R_v \mapsto \infty$ (no computation is required, only a brief discussion)? (**1 point**)

5. Given a controller $C(s) = \frac{1}{s}$, a transfer function to describe the nominal model behaviour $G_n(s) = \frac{6s}{2s+4}$. We know the upper bound of the multiplicative uncertainty as $d_m(s) = \frac{s}{0{,}25s+0{,}5}$. Check whether $C(s)$ is robustly stabilising the closed-loop system or not. (**1 point**)

6. Given the following state-space representation and cost functional by,

$$\begin{aligned} \dot{x}(t) &= 0.5x(t) + u(t) \\ J(u) &= \frac{1}{2}\int_0^\infty \left(x^T(\tau)x(\tau) + u^T(\tau)Q_u u(\tau)\right) d\tau \end{aligned}$$

The closed-loop system's pole is located at -1. Find Q_u and $\bar{P}$ (**2 point**).