SSY285, resit exam questions (solution manual)

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Q1) $$G(s) = C(sI-A)^{-1}B + D = \frac{1}{s^2+0{,}5s-0{,}5}\begin{bmatrix} s+0{,}5 & s^2+1{,}5s-3 \\ 2s+1{,}5 & -s-4{,}5 \end{bmatrix}$$

a), b) $\det G(s)$ → zeros $-2{,}25$, stable ⇒ min. phase

→ poles $p_1=-1$, $p_2=0{,}5$ – system unstable poles, not to stable

Q2) a) $\mathcal{C} = [B \;\; AB]$, $\det \mathcal{C} = 0{,}5(\alpha+\beta) + \alpha\cdot\beta = 0$

b) $\mathcal{O} = \begin{bmatrix} C \\ CA \end{bmatrix}$, $\det \mathcal{O} = 1$, ∀ observ.

c) eig A ⇒ $[1{,}65,\ -0{,}1514] = [p_1 \;\; p_2]$

Find some eigenvectors e.g. $T^{-1} = \begin{bmatrix} 0{,}91 & -0{,}79 \\ 0{,}39 & 0{,}6 \end{bmatrix}$

$$A_d = TAT^{-1} = \begin{bmatrix} 1{,}65 & 0 \\ 0 & -0{,}1514 \end{bmatrix} \qquad T = \begin{bmatrix} 0{,}69 & 0{,}9 \\ -0{,}45 & 1{,}05 \end{bmatrix}$$

$$B_d = TB = \begin{bmatrix} 1{,}149 \\ 0{,}07 \end{bmatrix}$$

$$C_d = CT^{-1} = [0{,}91 \quad -0{,}79]$$

eigenvectors are not unique

d) $$y(2) = C_d\cdot x(2) = C_d\left(e^{2\cdot A_d}\right)\cdot x(0) =$$

$$[0{,}91 \quad -0{,}79]\begin{bmatrix} e^{2\cdot 1{,}65} & 0 \\ 0 & e^{2\cdot -0{,}1514} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

e) Since A^{-1} exists,

$$A_{discrete} = e^{A_d\cdot T_s}, \quad B_{discrete} = A^{-1}(e^{AT_s} - I_n)B$$

$$= \begin{bmatrix} 5{,}21 & 0 \\ 0 & 0{,}85 \end{bmatrix} \qquad = \begin{bmatrix} 2{,}93 \\ 0{,}66 \end{bmatrix}$$

$C_{discrete} = C_d$; $D_{discrete} = 0$

Q3) $$G(z) = C_1(zI - A_1)^{-1}B_1 + D_1$$

$A_2 = TA_1T^{-1}$, $B_2 = \cancel{T}B$, $C_2 = CT^{-1}$

for (A_2, B_2, C_2)

$$G(z) = C_1 T^{-1}(zI - TA_1T^{-1})^{-1} T \cdot B_1 =$$

$$= C_1\underbrace{\left(T^{-1}(zI - TA_1T^{-1})T\right)^{-1}}B_1 = C_1(zI - A_1)^{-1}B_1$$

Hint: $(A\,B\,C)^{-1} = C^{-1}B^{-1}A^{-1}$

Q4) $\left.\begin{aligned} x_1 &= z \\ x_2 &= \dot z \end{aligned}\right\}$ $\begin{bmatrix}\dot x_1 \\ \dot x_2\end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u + \begin{bmatrix} 0 \\ 1 \end{bmatrix}v$

a) $y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w$

$\leftarrow z = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{M}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Cost function

$$J = \lim_{T\to\infty} E\left\{\frac{1}{2}\int_0^T x^T \underbrace{M^T M}_{Q_x} x + u^2 dt\right\} \qquad Q_u = 1$$

b) continuous time FARE $\Rightarrow \bar P_L = \begin{bmatrix} \sqrt{8} & 4 \\ 4 & 16\sqrt{8} \end{bmatrix}$; $\bar L = \begin{bmatrix} \sqrt{8} \\ 4 \end{bmatrix}$

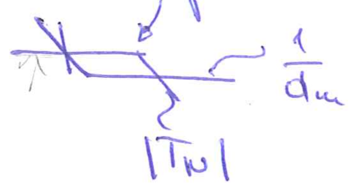
c) — " — CARE $\Rightarrow \bar P_K = \begin{bmatrix} \sqrt{2} & 1 \\ 1 & \sqrt{2} \end{bmatrix}$; $\bar K = \begin{bmatrix} 1 & \sqrt{2} \end{bmatrix}$

d) LQG LTR $R_v \to \infty$

The closed loop transfer function aims at recovering the state-feedback LQR properties

Q5) $\left| T_N = \dfrac{C \cdot G_n}{1 + C \cdot G_n} \right| < \dfrac{1}{d_m}$?, Plot Bode amplitude

$\Rightarrow$ not robustly stabilizing



Q6) $A = 0{,}5$, $B = +1$, $Q_x = 1$

$Q_u = ?$, $\bar P = ?$

$A - B\bar K = -1 = 0{,}5 - 1 \cdot Q_u^{-1}\bar P \Rightarrow \bar P = 1{,}5\,Q_u$

$\bar P A + A^T \bar P - \bar P B Q_u^{-1} B \bar P + Q_x = 0$

$\bar P + 1 - \bar P \dfrac{1{,}5}{\bar P}\bar P = 0 \Rightarrow \bar P = 2,\ Q_u = \dfrac{4}{3}$

Q6)

a) $eig(A) = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ Yes it is asymptotically stable

b) Not ~~observable~~ controllable $\mathcal{C} = [B \; AB] = \begin{bmatrix} 4 & -4 \\ 0 & 0 \end{bmatrix}$

$\det \mathcal{C} = 0$

But observable $\mathcal{O} = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -2 & 0 \end{bmatrix}$

$\det \mathcal{O} \neq 0$

c) $T_s = 1$

$A_d = e^{AT_s} = \begin{bmatrix} 0,36 & 0 \\ 0 & 0,13 \end{bmatrix}$

$B_d = A^{-1}(e^{AT_s} - I)B = \begin{bmatrix} 2,528 \\ 0 \end{bmatrix}$ $C_d = [2 \; 3]$

$D_d = 0$

Q7) Closed loop pole at $-0,5$

$A - BL = -0,5 = 0,5 - 1 \cdot R^{-1} \cdot \bar{P} \Rightarrow \bar{P} = R$

CARE: $2 \cdot 0,5 + Q - \bar{P}^2 (B R^{-1} B) = 0 \Rightarrow Q = 0$