Solution, re-exam SSY285

22/04/2014

1) a, block diag (1p)

$$\dot{x} = \begin{bmatrix} -1 & 10 \\ 0 & -4 \end{bmatrix} x + \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix} u \; ; \quad y = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix} x + \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} u$$ (1p)

b,

$$G(s) = \begin{bmatrix} \frac{5}{s+1} + \frac{50}{(s+1)(s+4)} + 2 & \frac{5}{s+1} + \frac{100}{(s+1)(s+4)} \\ \frac{1}{s+1} + \frac{1}{s+4} + \frac{10}{(s+1)(s+4)} & \frac{1}{s+1} - \frac{2}{s+4} - \frac{20}{(s+1)(s+4)} \end{bmatrix}$$ (1p)

c) $z = -25{,}5 \rightarrow$ „stable" zero $\Rightarrow$ No (1p)

$p_1 = -1$, $p_2 = -4 \rightarrow$ stable system (1p)

2) a)

$$\left.\begin{aligned} \det \mathcal{C} &= ab - \frac{a+b}{2} \neq 0 \\ \det \mathcal{O} &= a+b \neq 0 \end{aligned}\right\} \quad a \neq -b \text{ from } \mathcal{O} \Rightarrow a \neq 0$$ (2p)

b)

$$A = \left.\begin{bmatrix} 1 & 1{,}5 \\ 0{,}5 & 0{,}5 \end{bmatrix}\right|_{\substack{a=0{,}5 \\ b=1}} \Rightarrow T = \begin{bmatrix} 1 & 1{,}5 \\ 0 & 1 \end{bmatrix} \left[\mathcal{C}\right]^{-1} = \begin{bmatrix} 0 & 2 \\ 4 & -8 \end{bmatrix}$$

$$\hat{A} = TAT^{-1} = \begin{bmatrix} 1{,}5 & 0{,}25 \\ 1 & 0 \end{bmatrix} ; \quad \hat{B} = TB = \begin{bmatrix} 1 \\ 0 \end{bmatrix} ; \quad \hat{C} = CT^{-1} = [1, 0{,}25]$$

$\tilde{D} = D$ (3p)

3)

$$PA + A^T P - P B Q_u^{-1} B^T P + Q_x = 0 \qquad P = \begin{bmatrix} p_1 & 0 \\ 0 & p_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 - p_1^2 & p_1 - p_1 p_2 \\ p_1 - p_1 p_2 & 1 - p_2^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad p_1 = p_2 = 1 > 0$$

$$\bar{K} = -PBQ_u^{-1} = -[1 \;\; 1] ; \quad \text{b)} \; A + B\bar{K} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}$$

4) a) $A = -1$; $W = [1 \;\; 2]$; $C = [0{,}2 \;\; 0{,}1]$; $R_v = [1] = V$

$$R_w = \begin{bmatrix} 1 & 0 \\ 0 & 0{,}1 \end{bmatrix} = W \qquad \bar{P} = A\bar{P}A^T + [1 \;\; 2] V \begin{bmatrix} 1 \\ 2 \end{bmatrix} - A\bar{P}[0{,}2 \;\; 0{,}1] \left(\begin{bmatrix} 1 & \\ & 0{,}1 \end{bmatrix} + [0{,}2 \;\; 0{,}1] \bar{P} \begin{bmatrix} 0{,}2 \\ 0{,}1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0{,}2 \\ 0{,}1 \end{bmatrix} \bar{P} A$$

$\bar{P}_{1,2}$: $8{,}9$; $\cancel{-3{,}9}$ covariance ! >0 (2p)

$$\bar{L} = A\bar{P}C^T (W + C\bar{P}C^T)^{-1} = [-0{,}79 \;\; -3{,}9]$$

b) $A - \bar{L}C = -0{,}44$, <1 (1p)

(5) a)

$$y = \left[\frac{1}{G_n \cdot C} \quad \frac{G_n F}{1+G_n \cdot C} \right] \begin{bmatrix} d \\ r \end{bmatrix}$$

(1p)

b) for a step input

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} y(s) \cdot s = \lim_{s \to 0} G(s) \frac{1}{s} \cdot s$$

$$1 = \lim_{t \to \infty} 1(t) = \lim_{s \to 0} \frac{3\cancel{(s+3)}}{(s+2)(s+1)} \cdot \frac{1}{\cancel{(s+3)}} \cdot \frac{1}{1 + \frac{3c \cdot (s+3)\cancel{(s+1)}}{\cancel{(s+1)}(s+2)}} =$$

$$\lim_{s \to 0} \frac{3}{(s+1)} \cdot \frac{1}{(s+2) + \frac{3c(s+3)}{}} = \frac{3}{2+9c} = 1$$

$$3-2 = 9c \Rightarrow c = \frac{1}{9}$$

(1p)

c)

mult $G(s) = (1+\Delta_m) \cdot G_n$ $\Delta_m = \frac{G(s) - G_N(s)}{G_N}$

add $G(s) = \Delta_a + G_n$ $\Delta_a = G(s) - G_N(s)$

(1p)

(6) MCARE

$$pa + ap + c^T c - p^2 \left(b b^T - \frac{1}{\gamma^2} e \cdot e^T \right) = 0$$

$$2 \cdot (-1{,}5) p + 4 - p^2 \left(\sqrt{2} \cdot \sqrt{2} - \frac{1}{1^2} \cdot 1 \right) = 0$$

$$p^2 + 3p - 4 = 0$$

$p_1 = +1$

$\cancel{p_2 = -4}$

$$u^* = -\sqrt{2} \cdot 1 \, x(t) = -\sqrt{2} \, x(t)$$

$$d^* = \frac{1}{\gamma^2} \cdot 1 \cdot 1 \, x(t) = x(t)$$

(2p)