

Exam questions for Linear Control System Design, SSY285

January 12nd, 2015

Cover page

1. Timeframe: 4 hours.
2. Examiner: Balazs Kulcsar, internal phone number: 1785, kulcsar@chalmers.se
3. Necessary condition to obtain the exam grade is to have the course's mandatory project (all assignments and the lab) approved. Without approved mandatory project, the archived exam results are invalid.
4. 20 points can be reached in total (with 0.5 point resolution), Lic/PhD students have to reach at least 12 points to pass. For Msc students Table 1 shows the grading system.

Table 1: Grading for Msc students

Points	Grade
10 ... 11.5	3
12 ... 15.5	4
16 ... 20	5

5. During this written exam, it is *optionally* permitted to use printed materials such as:
 - *Either* of the course textbooks: (hardcopy or plain printed version, without notes inside!) *Feedback systems, an introduction for scientists and engineers* by K. J. Åström and R. M. Murray, ISBN-13: 978-0-691-13576-2 *OR* *Reglerteknikens grunder* by Bengt Lennartson, ISBN: 91-44-02416-9.
 - 1 piece of A4 paper, with *hand written* notes on both sides. Copied sheet can not be used.
 - Pocket calculator (non-programmable, cleared memory, without graphical plotting function).
 - Mathematical handbook Beta (without notes inside!).
6. Note that phones, tablets, computers, any other communication devices are not allowed to use during the exam session. In scheduled exam session for the course at Chalmers, teacher(s) will show up in person in the first and last 60 min.
7. Examination results will be advertised on January 22nd 2015 (pingpong.chalmers.se). Inspection of results in person, January 26th and 27th 2015 10-11 am, E-building floor 5, room 5414.

Good luck!

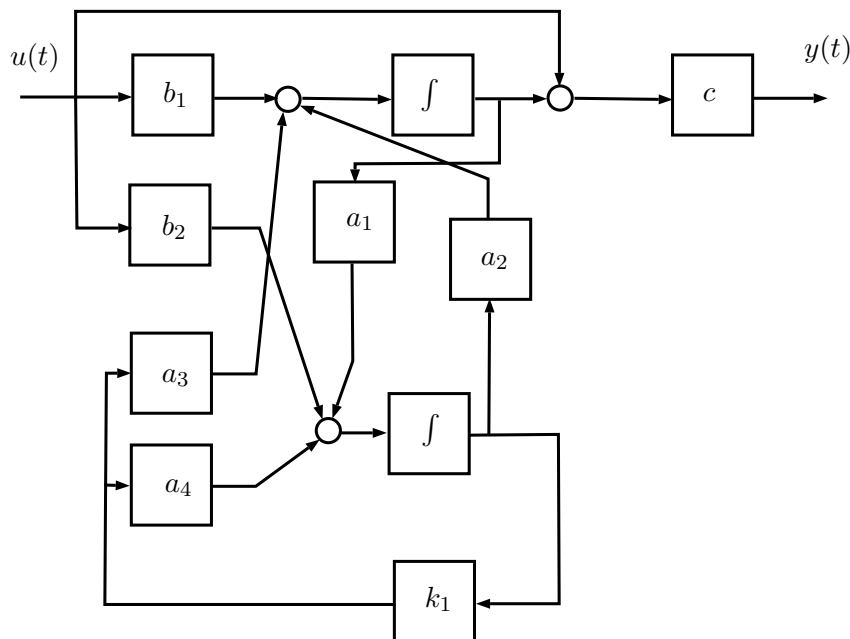


Figure 1: Block diagram

Questions

- Given the following state-space representation by,

$$\dot{x}(t) = \begin{bmatrix} -1 & 1 \\ 3 & -1 \end{bmatrix} x(t) + \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} u(t)$$

- By using the coefficient matrices (A, B, C, D) , compute the transfer function matrix $G(s)$ (**1 point**).
 - Find the poles and the zeros of $G(s)$. Based on $G(s)$, is this a non-minimum phase representation? Is the system input-output stable? Motivate your answer (**1 point**)!
- Given the following SISO system representation by means of block diagram in Figure 1, where $a_1 = 1, a_2 = 1, b_2 = -2, a_3 = 1, a_4 = 2, c = 1$.
 - Find the *continuous-time* state-space representation for the depicted system in Figure 1 in terms of matrix differential equation (A, B, C, D) parametrised in b_1 and k_1 (**1 point**).
 - Find k_1 such that the poles of the above system become $p_1 = 0$ and $p_2 = -2$ (**1 point**).
 - By using the value for k_1 found in answer b), find b_1 that makes the system of non-minimal order (**1 point**)!
 - By using the value for k_1 found in answer b) and $b_1 = 1$, find the transformation matrix T that diagonalizes the above representation. With this T , transform the above mentioned state-space representation (A, B, C, D) to a diagonal form $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})$. Calculate the values $\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}$ (**3 points**)!
 - In the diagonal state-space form, solve the homogenous differential equation ($u(t) = 0$) and find the analytic form of $y(t)$ provided that $\tilde{x}_0 = [1 \ 1]^T$. Find $y(10)$ and $y(20)$ (**1 point**)!
 - Consider the following time discrete state observer problem as

$$x(k+1) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} v(k),$$

$$y(k) = [1 \ 0] x(k) + w(k),$$

where the uncorrelated scalar noise intensities are $r_v = 1$ and $r_w = 0$ standing for the normally distributed random signals v and w , respectively.

- a) Find the optimal steady-state Kalman filter gain \bar{L} to minimize the "delayed" state error covariance $\bar{P} = E\{\tilde{x}(k|k-1)\tilde{x}(k|k-1)^T\} > 0$ (**2 points**).
- b) Find the observer's poles with the previously obtained gain \bar{L} (**1 point**).

4. Consider the following time discrete LQR design problem

$$x(k+1) = x(k) + u_1(k) + u_2(k)$$

$$J = \frac{1}{2} \sum_{j=0}^{\infty} (x^2(j) + u_1^2(j) + u_2^2(j))$$

- a) Find the optimal steady-state LQR gain \bar{K} to minimize the cost function J (**2 points**).
- b) Find the closed-loop pole with the previously obtained gain \bar{K} and draw the block diagram for the closed-loop system (**1 point**).

5. Given the closed loop system as shown in Figure 2 with the nominal plant transfer functions $G_n(s)$,

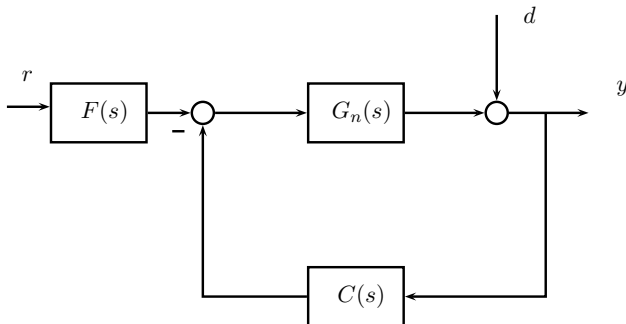


Figure 2: Closed-loop block diagram

controller $C(s)$, and pre-filter $F(s)$,

$$G_n(s) = \frac{3(s+3)}{(s+2)(s+1)}, \quad C(s) = c \cdot (s+1), \quad F(s) = \frac{1}{s+3}$$

with $c > 0$ scalar.

- a) Find the value c which provides under $d = 0$ asymptotic step reference tracking (**1 point**).
- b) Given the real systems' transfer function as $G(s) = \frac{s+1}{(s+1)^2(s+2)^2}$, find the multiplicative and additive uncertainty functions associated to the nominal plant $G_n(s)$. (**2 points**).

6. (*challenging*) Given the following state-space representation and cost functional by,

$$\dot{x}(t) = -1.5x(t) + \sqrt{2}u(t) + 0.5d(t)$$

$$J(u, d) = \int_0^{\infty} (q_x x^2(t) + u^2(t) - \gamma^2 d^2(t)) dt$$

where $d(t)$ is a 2-norm bounded but time varying disturbance signal, and $\gamma = 0.5$. We know the optimal (minimizing) state-feedback $u^*(t) = -5\sqrt{2}x(t)$ under the worst case (maximizing) disturbance $d^*(t)$. Find, by means of the modified CARE, the value for $q_x > 0$ (**2 points**).