Chalmers University of Technology
Department of Signals and Systems
Automatic Control Group

Retake exam questions for Linear Control System Design, SSY285

August 29th, 2014

Cover page

1. Timeframe: 4 hours.
2. Teacher: Balazs Kulcsar, Chalmers internal phone number: 1785
3. 20 points can be reached in total (half of a point, i.e. 0.5, can also be obtained). Note, exam grades will only be approved if all assignments and the lab have been approved by the TA too. Lic/PhD students have to reach at least 12 points to pass. For Msc students Table 1 shows the grading system.

Table 1: Grading for Msc students

Points	Grade
8 . . . 11.5	3
12 . . . 15.5	4
16 . . . 20	5

4. During this written exam, it is *optionally* permitted to use printed materials such as (teacher(s) will show up and do inspection):
   - Either of the course textbooks (hardcopy or plain printed version, without notes inside!): *Feedback systems, an introduction for scientists and engineers* by K. J. Åström and R. M. Murray, ISBN-13: 978-0-691-13576-2 OR *Reglerteknikens grunder* by Bengt Lennartson, ISBN: 91-44-02416-9.
   - 1 piece of A4 paper, with *hand written* notes on both sides.
   - Pocket calculator (non-programmable, cleared memory, without graphical plotting function). Additionally to Chalmers' rules Casio fx-991 types are allowed.
   - Mathematical handbook Beta.
5. Note that phones, tablets, computers, any other communication devices are not allowed to use during the exam session. Teacher(s) will show up in person during the examination in the first and last 60 min.
6. Examination results will be advertised no later than September 8th 2014 (pingpong.chalmers.se). Inspection of results on September 8th and 9th 2014 10-11 am, E-building floor 5, room 5414.

Good luck!

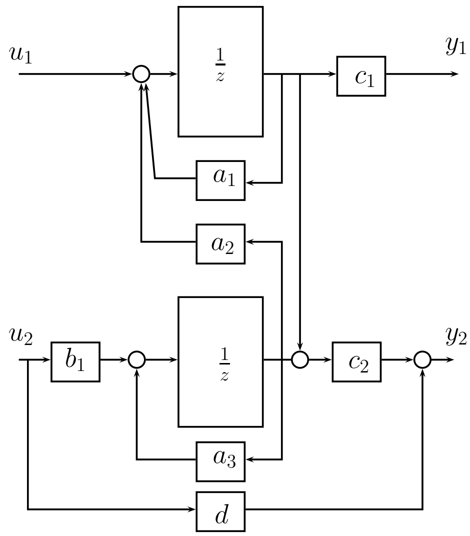

Figure 1: Open-loop block diagram

Questions

1. Given the following system representation by means of block diagram in Figure 1, where $a_2 = -0.1, a_3 = -0.2, b_1 = 1$ $c_1 = 0.1, c_2 = 1, d = 0.1$ and a_1 is unknown.

   a) Find the discrete-time state-space equations (**1 point**).

   b) Find the value of the coefficient a_1 such that the system poles become $p_1 = -0.2$ and $p_2 = 0.5$ (**1 point**).

   c) Find $G(z)$ on the basis of the coefficient matrices A, B, C, D with the previously obtained a_1. Based on $G(z)$, is this a non-minimum phase representation? Is the system input-output stable? Motivate your answer (**2 point**)!

   d) Given the initial condition $x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, determine the output value of the autonomous system $y(1)$ (**1 point**).

2. Given the following state space representation by,

$$\dot{x}(t) = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t) + (\alpha \cdot \beta) u(t)$$

   where α, β are real valued finite scalars.

   a) Find value(s) of α and β when the observability matrix of is given by $\mathcal{O} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (**1 point**).

   b) Is the system with the numerical values found previously of minimal order? (**1 point**).

   c) With the previous values for α and β, find the transformation matrix T that transforms the above state-stale representation to controller canonical form. Calculate the values $\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}$ with the help of the transformation matrix T (**3 point**).

3. Consider the following continuous time system given by a state space representation as,

$$\dot{x}_1(t) = x_2(t) + u(t)$$
$$\dot{x}_2(t) = x_1(t) + \frac{x_2(t)}{2}$$
$$J(u) = \int_0^\infty \left(x_2^2(t) + x_1^2(t) + q_u \cdot u^2(t) \right) dt.$$

where $q_u = 1$.

a) Find the optimal (steady-state LQR) feedback gain ($\bar{K}$) that minimizes $J(u)$ by applying the diagonal solution structure as $\bar{P} = \begin{bmatrix} p_1 & 4 \\ 4 & p_2 \end{bmatrix}$. What are the closed-loop optimal poles? (**3 point**)

b) Draw the block diagram for the closed loop LQR controlled system (including amplifications, integrators) (**1 point**).

c) Find the optimal cost functional value with the optimal LQR state feedback applied, i.e. $J_0(u^*)$ when $x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$(**1 point**).

4. Consider the following time discrete problem by,

$$
\begin{aligned}
x(k+1) &= -x(k) + v_1(k) + 10v_2(k), \\
y_1(k) &= x(k) + w_1(k), \\
y_2(k) &= 0.1x(k) + w_2(k),
\end{aligned}
$$

where the scalar noise intensities are $r_{w_1} = 1$, $r_{w_2} = 1$, $r_{v1} = 1$, $r_{v2} = 1$. All noises are normally distributed uncorrelated white noise sequences.

a) Find the optimal steady state Kalman filter gain $\bar{L}$ to minimize the "delayed" state error covariance, $\bar{p} = E\{\tilde{x}(k|k-1)\tilde{x}(k|k-1)^T\} > 0$(**2 point**).

b)Find the closed loop system observer's pole, i.e. thereof the $A - \bar{L}C$ with previously obtained gain $\bar{L}$. (**1 point**).

5. *(challenging exercise) Given the system dynamics and the performance index as,

$$
\begin{aligned}
\dot{x}(t) &= -1.5x(t) + \sqrt{2}u(t) + d(t) \\
y(t) &= 2x(t) \\
J(u,d) &= \int_0^\infty \left(y^2(t) + u^2(t) - \gamma^2 d^2(t)\right) dt
\end{aligned}
$$

where $d(t)$ is a 2-norm bounded but time varying disturbance signal. Find the stabilizing, state-feedback and optimal min-max solutions by the modified CARE if $\gamma = 1$. Find both $u^*(t)$ and $d^*(t)$ **(2p)**.