Solution manual for re-retake exam 08/29/2014

Q1) a) $A = \begin{bmatrix} a_1 & -0,1 \\ 0 & -0,2 \end{bmatrix} \Rightarrow$ eig(A) $\Rightarrow a_1 = 0,5$ (1p)

b) $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $C = \begin{bmatrix} 0,1 & 0 \\ 1 & 1 \end{bmatrix}$; $D = \begin{bmatrix} 0 & 0 \\ 0 & 0,1 \end{bmatrix}$ (1p)

c) $G(z) = \begin{bmatrix} \frac{0,1}{z-0,5} & \left(\frac{1}{z-0,5}\right) \\ \left(\frac{-0,01}{z^2-0,3z-0,1}\right) & \frac{0,1z^2+0,07z-0,07}{z^2-0,3z-0,1} \end{bmatrix}$ (1p)

$\Rightarrow \forall |p_i| < 1 \rightarrow$ stable

non minimum phase (1p)

$z_1 = -10,2$ outside the unit circle

d) $y(1) = \begin{bmatrix} 0,1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0,5 & -0,1 \\ 0 & -0,2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0,04 \\ +0,2 \end{bmatrix}$ (1p)

Q2 a) $\mathcal{O} = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \alpha & \beta \end{bmatrix} \Rightarrow \alpha = 1$, $\beta = 0$ (1p)

b) $\mathcal{C} = [B \quad AB] = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix} \Rightarrow \det(\mathcal{C}) \neq 0$, $\det(\mathcal{O}) \neq 0$ (1p)

$\rightarrow$ minimal order system

c) $T^{-1} = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$; $T = \begin{bmatrix} 0 & 1 \\ 0,5 & -0,5 \end{bmatrix}$ (1p)

$\bar{A} = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$; $\bar{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\bar{C} = [1 \quad 0]$ (2p)

Q3) a) continuous time CARE: $PA + A^TP - PBQ_u^{-1}B^TP + Q_x = 0$

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 9 - p_1^2 & p_2 - 3p_1 + 2 \\ * & p_2 - 7 \end{bmatrix}$ (1p)

$p_1 = 3$, $p_2 = 7$

$\bar{K}^T = -PBQ_u^{-1} = [3 \quad 4]$ (1p)

$P_1 = \begin{bmatrix} -1,5 \\ -1 \end{bmatrix}$ (1p)

b) Block diagram. (1p)

$J_0 = [1 \quad 1]\begin{bmatrix} 3 & 4 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 18$ (1p)

Q4) $A = -1$ $N = [1 \;\; 10]$ $C = [1 \;\; 0,1]^T$ $R_v = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$R_w = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \bar{P} \Rightarrow$

$\bar{P} - A\bar{P}A^T = NRvN^T - A\bar{P}C^T([R_w] + [C\bar{P}C^T])^{-1} \cdot C\bar{P}A$

$\bar{P} \begin{cases} 101,98 > 0 \\ -0,98 \end{cases}$ (1p)

$\bar{L} = A\bar{P}C^T(R_w + C\bar{P}C^T)^{-1} = [-0,98 \;\; -9,098]$ (1p)

$A - \bar{L}C = -9,0096$ (1p)

Q5) MCARE

$pa + ap + c^Tc - p^2\left(bb^T - \frac{1}{\mu^2}ll^T\right) = 0$

$2 \cdot (-1,5)p + 4 - p^2\left(\sqrt{2}\cdot\sqrt{2} - \frac{1}{1^2}\cdot 1\right) = 0$

$p^2 + 3p - 4 = 0$ $\quad p_1 = +1 \rightarrow$ (1p) $\quad p_2 = -4$

$u^* = -\sqrt{2}\,x(t)$

$d^* = \frac{1}{\mu}\sqrt{2}\cdot 1\cdot 1 = x(t)$ (1p)