Solution to SS4285 exam (2013/12/21)

Q1 a) $A = \begin{bmatrix} 0,1 & 0,5 \\ 0 & 0,1 \end{bmatrix}$; $B = \begin{bmatrix} 1 & 0 \\ 0 & 0,1 \end{bmatrix}$ $C = \begin{bmatrix} 0,2 & 0 \\ 0,5 & 0,5 \end{bmatrix}$ (1p) $D = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

b) $G(z) = C(zI - A)^{-1}B + D = \begin{bmatrix} \frac{2}{10z-1} & \frac{1}{(10z-1)^2} \\ \frac{5}{10z-1} & \frac{5}{2(10z-1)^2} + \frac{1}{2(10z-1)} + 1 \end{bmatrix}$ (1p)

c) poles are $z_1 = 0,1$, $z_2 = 0,1$ (0,5p) } within the unit circle ⇒ stable !

$\det G(z) = \frac{1 + 2(10z-1)}{(10z-1)^2}$

zero $z = 0,05$ (0,5p)

Q2. Minimality: contr + obs

a) $\mathcal{C}_2 = [B \ \ AB] = \begin{bmatrix} 1 & -1,5 + 2\beta \\ 2 & 2\mu - 1 \end{bmatrix} \Rightarrow \det \mathcal{C}_2 = 0 \quad 2\mu - 4\beta + 2 = 0$ (1p)

$\mathcal{O}_2 = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1,5 & \beta \end{bmatrix} \Rightarrow \det \mathcal{O}_2 \Rightarrow \beta = 0$ (1p)

b) $\mu = 1 = \beta$ $A = \begin{bmatrix} -1,5 & 1 \\ -1 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$; $C = [1 \ \ 0]$ $D = 1$ (1p)

$\text{eig}(A) = \left\{ \begin{matrix} -1 \\ 0,5 \end{matrix} \right\} \Rightarrow T^{-1} = \begin{bmatrix} -0,8944 & -0,4472 \\ -0,4472 & -0,8944 \end{bmatrix}$; $T = \begin{bmatrix} -1,49 & 0,74 \\ 0,74 & -1,49 \end{bmatrix}$ (1p)

$\tilde{A} = TAT^{-1} = \begin{bmatrix} -1 & 0 \\ 0 & +0,5 \end{bmatrix}$

$\tilde{B} = \begin{bmatrix} 0 \\ -2,23 \end{bmatrix} = TB$; $\tilde{C} = C \cdot T^{-1} = [-0,89 \ \ -0,44]$

$\tilde{D} = D = 1$ (1p)

(→ ! Eigenvalue decomp is not unique ! T might change)

Q3

a) $\bar{P}A + A^T\bar{P} - \bar{P}B\,Q_u^{-1}\,B^T\bar{P} + Q_x = 0$

$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$; $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$; $Q_u = 1$; $Q_x = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 - P_1^2 & P_1 - P_1 \cdot P_2 \\ P_1 - P_1 \cdot P_2 & 1 - P_2^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow \begin{matrix} P_1 = 1 \text{ (1p)} \\ P_2 = 1 \text{ (1p)} \end{matrix} > 0$

$* \ \bar{K} = -\bar{P}BQ_u^{-1} = -[1 \ \ 1]$;

b) $A + B\bar{K} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} =$

$= \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix} \Rightarrow \begin{matrix} \lambda_1 = -1 \\ \lambda_2 = -1 \end{matrix}$ (1p)

Q4.

a)

$A = -1$

$N = \begin{bmatrix} 1 & 2 \end{bmatrix}$

$C = 1$

$V = R_V = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $R_w = W = 1$

(1p)

$$\underbrace{\bar{P} = A\bar{P}A^T + NVN^T - A\bar{P}C^T(W + C\bar{P}C^T)^{-1}C\bar{P}A^T}$$

$\bar{p} = \bar{p} + 5 - \bar{p}^2(1+\bar{p})^{-1}$

$\bar{p}_1 = 5{,}85 \rightarrow > 0$ covariance! (1p)

$\bar{p}_2 = \cancel{-0{,}854}$

b)

observer gain

$\bar{L} = A\bar{P}C^T(W + C\bar{P}C^T)^{-1} = -0{,}8541$ (0,5p)

$A - \bar{L}C = -0{,}1459$ within the unit circle

+ block diagram (0,5p)

Q5a)

$$\begin{bmatrix} z \\ y \end{bmatrix} = \underbrace{\begin{bmatrix} F_2 + \frac{F_1 C G_1}{1 + C G_1 G_2} & \frac{1}{1 + C G_1 G_2} \\ \frac{F_1 C G_1}{1 + C G_1 G_2} & \frac{1}{1 + C G_1 G_2} \end{bmatrix}}_{G(s)} \begin{bmatrix} r \\ d \end{bmatrix}$$

$G(s) \longrightarrow$

$G_{11}(s) = F_2 + \frac{3(s+2)}{s^2+5s+9}$

$G_{12} = \frac{1}{s^2+5s+9}$; $G_{21} = \frac{3(s+2)}{s^2+5s+9}$; $G_{22} = G_{12}$; (1p)

b)

$u(s) = \frac{1}{s}$; $r_\infty = 1 = \lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s) = \lim_{s \to 0} G_{11}(s) = F_2 + 2/3$

$\Rightarrow F_2 = \frac{1}{3}$ (1p)

Q6)

It fulfills the Lyapunov equation (for steady state covariance)

$A\bar{P} + \bar{P}A^T + BVB^T = 0$ (1p)

$A = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}$ $P = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $V = ?$

$A\bar{P} + \bar{P}A^T = \begin{bmatrix} -2 & -4 \\ -4 & -6 \end{bmatrix} = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} v_1 & v_2 \\ v_2 & v_3 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow V = \begin{bmatrix} 2 & 4 \\ 4 & 6 \end{bmatrix}$ (1p)