Chalmers University of Technology
Department of Signals and Systems
Automatic Control Group

Written exam questions for Linear Control System Design, SSY285

December 17, 2012

Cover page

1. Timeframe: 4 hours.

2. Teacher: Balazs Kulcsar, Chalmers internal phone number: 1785

3. 20 points can be reached in total (half of a point, i.e. 0.5, can also be obtained). Note, exam grades will only be approved if all assignments and the lab have been approved by the TA too. Lic/PhD students have to reach at least 12 points to pass. For Msc students Table 1 shows the grading system.

Table 1: Grading for Msc students

Points	Grade
8 . . . 11.5	3
12 . . . 15.5	4
16 . . . 20	5

4. During this written exam, it is permitted to optionally use printed materials such as (teacher(s) will show up and do inspection):
   - Course textbook (hardcopy or plain printed version, without notes inside!) of *Feedback systems, an introduction for scientists and engineers* K. J. Åström and R. M. Murray, ISBN-13: 978-0-691-13576-2, Princeton University Press.
   - Available lecture notes, 6 series of slides (with the student own notes connected to the slides. Additional examples are not permitted to put on).
   - Pocket calculator (non-programmable, without graphical plotting function).
   - Mathematical tables as Beta/Physics handbook.

5. Note that phones, tablets, computers, any other communication devices are not allowed to use during the exam session. Teacher(s) will show up in person during the examination within the time intervals 2-2:30 pm and 5:30-6 pm.

6. Examination results will be advertised no later than Monday January 7th 2013 on the notice board of the Division of Automatic Control, Automation and Mechatronics, E-building floor 5. Inspection of grading is possible on January 4th 10-11 am and January 7th 1-2 pm, E-building floor 5, room 5417.

Wish you good luck and merry Christmas!

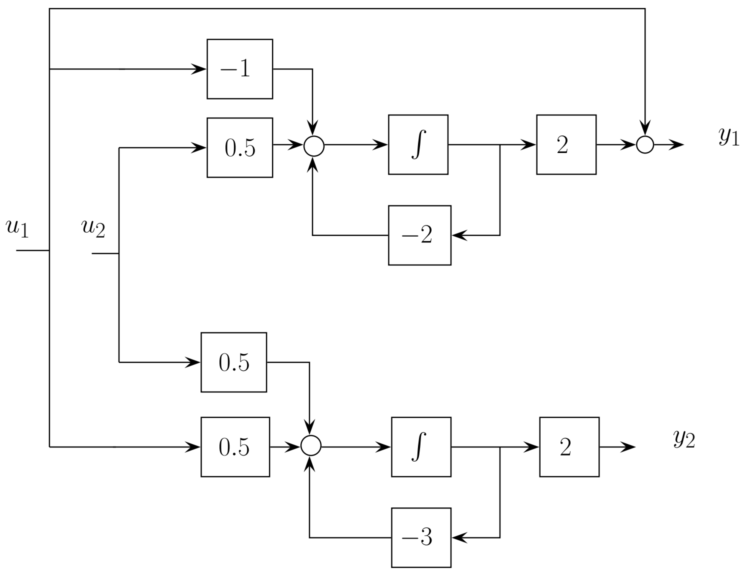

Figure 1: Open-loop block diagram

Questions

1. Given the continuous time state space representation by means of block diagram in Figure 1.

   a) Create the continuous-time diagonal state-space differential matrix equation (**1 point**).

   b) Find the transfer function $G(s)$ associated to the state-space model (**2 point**).

   c) Find the poles and the zeros of $G(s)$ (**1 point**).

   d) Based on $G(s)$ is this a non-minimum phase representation? Is it input-output stable (**1 point**)?

2. Given the following discrete-time state space representation as,

$$x(k+1) = \begin{bmatrix} -1.5 & -0.5 \\ -\alpha & 0 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(k)$$
$$y(k) = \begin{bmatrix} 2 & \frac{1}{\alpha} \end{bmatrix} x(k) + 3u(k)$$

   where $-\infty < \alpha < \infty$ and $\alpha \neq 0$.

   a) Find value(s) of α for which the system looses reachability (**1 point**).

   b) Find value(s) of α for which the system looses observability (**1 point**).

   c) Find α such that the eigenvalues of the A matrix get located at -0.5 and -1 (**1 point**).

   b) Replace the value of α found in the question part *c*) into the state space description. Find the similarity state transformation T and transform the state space representation into a controller canonical form $\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}$ (**2 point**).

3. Consider the following continuous time system given by a state space representation as,

$$\dot{x}(t) = x(t) + u_1(t) + u_2(t)$$
$$y(t) = 2x(t)$$
$$J(u) = \frac{1}{2}\int_0^{\infty} \left(\frac{x^2(t)}{10} + 100u_1^2(t) + 100u_2^2(t) \right) dt$$

   a) By using state-feedback control policy, find the optimal (steady state LQR) feedback gain ($\bar{K}$) minimizing the cost functional above. (**2 point**).

b) Find the closed loop system's eigenvalue matrix $A - B\bar{K}$ with previously obtained gain $\bar{K}$. Explain why is this a cheap or expensive control policy you applied (**1 point**)?

4. Given the noise driven continuous-time scalar dynamics as,

$$\begin{aligned} \dot{x}(t) &= 0.5x(t) + 2v(t) \\ y(t) &= 2x(t) + w(t) \end{aligned}$$

with the zero mean, white process and sensor noises having intensities, $r_v = 0.5$ and $r_w = 4$.

(a) Find the steady state Kalman filter gain to minimize the state error covariance as $\bar{P} = E\{\tilde{x}(t)\tilde{x}^T(t)\} > 0$. (**2 point**).

(b) Determine the optimal closed-loop observer pole and draw the block diagram of the observer. (**1 point**).

5. Given the closed loop system as shown in Figure 2 with the nominal plant transfer functions $G_n(s)$,

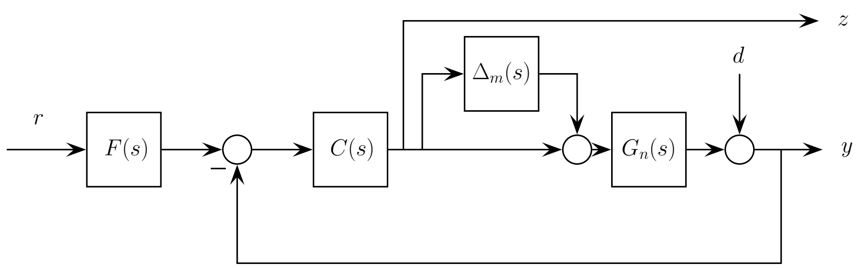

Figure 2: Closed-loop block diagram

controller $C(s)$, pre-filter $F(s)$, and multiplicative uncertainty functions,

$$G_n(s) = \frac{3(s+2)}{(s+1)^2}, \quad C(s) = s+1, \quad F(s) = \frac{1}{s+c}, \quad \Delta_m(s) = \frac{s^2}{s}$$

where $c > 0$ and finite.

(a) Find the transfer function from the reference signal to the output y. (**1 point**)

(b) Set the parameter of the pre-filter c such that $y_\infty = r_\infty$ (reach stationary output tracking). (**1 point**)

(c) Calculate the transfer functions from all inputs (r, d) to all outputs (y, z) with the previously found c value and create the transfer function matrix $\begin{bmatrix} y \\ z \end{bmatrix} = G(s) \begin{bmatrix} r \\ d \end{bmatrix}$ (**1 point**).

(d) In the above description the real system has been expressed by means of the nominal ($G_n(s)$) and the multiplicative uncertainty ($\Delta_m(s)$) transfer functions. Find the equivalent additive uncertainty block Δ_a to replace the above multiplicative uncertainty structure (**1 point**).