Discrete Event Systems

Course code: SSY165

Examination 2019-10-26

Time: 8:30-12:30, Location: Hörsalar, Hörsalsvägen

Teacher: Bengt Lennartson, phone 3722

The examination includes 25 points, where grade three requires 10 points, grade four 15 points and grade five 20 points.

The result of this examination is announced and inspection of the grading is done on Monday *November 11* and Tuesday *November 12*, 12:30-13:00 at the division.

Allowed aids at the examination:

- Standard mathematical tables such as Beta, see also formulas in the end of this examination.
- Pocket calculator.

Good luck!

Department of Electrical Engineering
Division of Systems and Control
Chalmers University of Technology

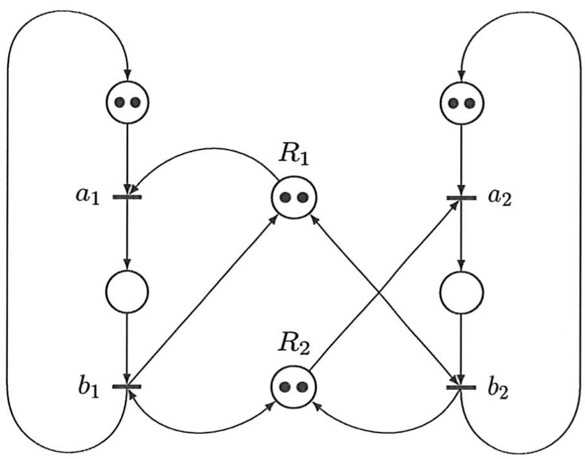
1

Prove the following implication by contradiction.

$$(p \lor q) \land (p \to r) \land (q \to r) \Rightarrow r$$

(2 p)

2



a) Generate the reachability graph of this Petri net (PN) and identify the blocking states when the initial marking vector is the marked state.

(2 p)

b) Add a minimal number of resource tokens in the places R_1 and/or R_2 to avoid the blocking states, and show ones again by a reachability graph that the modified PN has no blocking states.

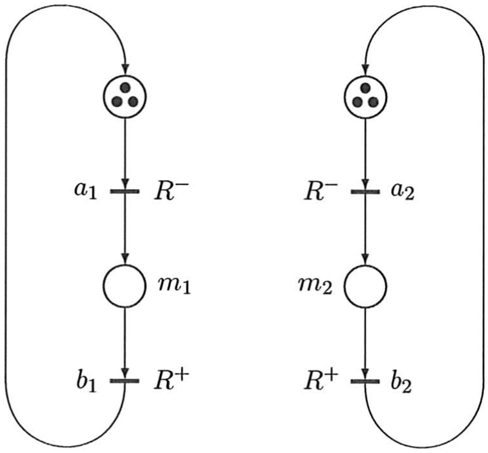
(2 p)

c) Based on the results above, determine a resource condition (without a formal proof) which guarantees a nonblocking system when the number of initial tokens is n instead of 2 in the places above the events a_1 and a_2.

(1 p)

3

In the following Petri net (PN) model, the shared variable R is increased by the operator R^+. The next value $R' = R + 1$ as long as the current value $R < R_{max}$. Thus, R^+ is formally defined as $R' = R + 1 \wedge R < R_{max}$. Corresponding decreasing operator R^- is defined as $R' = R - 1 \wedge R > 0$. This means that the variable can not be updated when the next value is outside the domain of R, in this case $\{0, 1, \ldots, R_{max}\}$.



a) Formulate an ordinary PN with an equivalent behavior as the PN above with the shared variable R. Assume that $R_{max} = 4$ is the initial value of R.

(1 p)

b) Let the two variables m_1 and m_2 in the PN above represent the number of tokens in the place beside the variable, where initially $m_1 = m_2 = 0$. Also, let e be the event variable that can take the values $\{a_1, b_1, a_2, b_2\}$. Formulate a transition predicate that includes all four transitions in the PN above, and involves the variables m_1, m_2, R, and e. Use the same formal definition of m_1 and m_2 as R above. The initial predicate is $m_1 = 0 \wedge m_2 = 0 \wedge R = 4$. A transition is admissible when the corresponding transition predicate is true. Only one transition can be true at a time, since e can only take one unique value for each individual transition.

(2 p)

c) Formulate an automaton for each variable G_{m_1}, G_{m_2}, and G_R and generate the synchronous composition $G_{m_1} \,||\, G_{m_2} \,||\, G_R$.

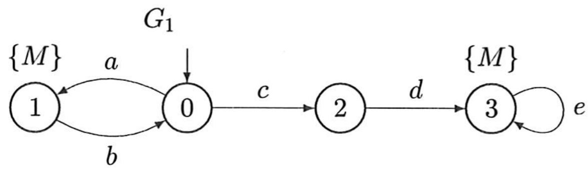
(3 p)

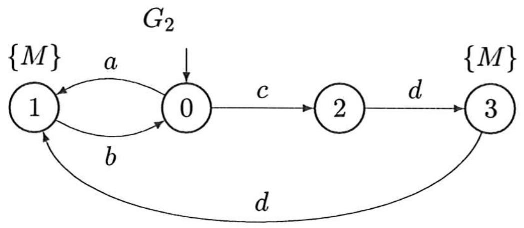
d) Which invariant condition on the three variables is valid in all states (m_1, m_2, R) of the synchronized system?

(1 p)

4

Consider the automata G_1 and G_2, where the states 1 and 3 are marked, denoted by the state label M. Since a marked state can be reached from every state in these automata, they are both nonblocking.





a) Formulate a CTL* expression which expresses that an automaton is nonblocking, and show by μ-calculus that this expression holds for both G_1 and G_2.

(3 p)

b) When all marked states must be reachable from every state, a stronger CTL* condition is required. Introduce individual state labels for the marked states, including corresponding state name as index, i.e. M_1 and M_3. Formulate a CTL* expression which guarantees that all marked states are reachable from every state, and show by μ-calculus that this condition is satisfied for G_2 but not for G_1.

(2 p)

5

Two people named A and B are playing a simple game. A number of sticks are lain out on the ground and the players take alternately one or two sticks. Note that at least one stick must be picked. The player that ends up with the last stick has lost the game. Player A is always the one that starts picking sticks.

a) Model this game by an automaton, with an initial number of n sticks, for $n = 5$. Hint: Introduce a marked state specifying that player A is to win and player B is to loose.

(2 p)

b) Generate a supervisor which guarantees that player A wins the game. Note that the set of uncontrollable events must first be decided.

(2 p)

c) Evaluate if it is possible for player A to always win the game also for $n = 6$ and $n = 7$.

(2 p)

Solution to DES exam 191026

1. $(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r) \wedge \neg r \equiv$

$(p \vee q) \wedge (\neg p \vee r) \wedge \neg r \wedge (\neg q \vee r) \wedge \neg r \equiv$

$(p \vee q) \wedge ((\neg p \wedge \neg r) \vee \underbrace{(r \wedge \neg r)}_{\mathbb{F}}) \wedge ((\neg q \wedge \neg r) \vee \underbrace{(r \wedge \neg r)}_{\mathbb{F}}) \equiv$

$(p \vee q) \wedge \neg p \wedge \neg r \wedge \neg q \wedge \neg r \equiv$

$(p \vee q) \wedge \neg (p \vee q) \wedge \neg r \equiv \mathbb{F} \wedge \neg r \equiv \mathbb{F}$

2 a)

$$\begin{array}{ccccc}
(202022) & \overset{a_2}{\underset{b_2}{\rightleftarrows}} & (201121) & \overset{a_2}{\underset{b_2}{\rightleftarrows}} & (200220) \\
a_1 \downarrow \uparrow b_1 & & a_1 \downarrow \uparrow b_1 & & a_1 \downarrow \\
(112012) & \overset{a_2}{\underset{b_2}{\rightleftarrows}} & (111111) & \overset{a_2}{\underset{b_2}{\rightleftarrows}} & (110210) \\
a_1 \downarrow \uparrow b_1 & & a_1 \downarrow \uparrow b_1 & & a_1 \downarrow \\
(022002) & \xrightarrow{a_2} & (021101) & \xrightarrow{a_2} & (020200) \leftarrow \text{blocking state}
\end{array}$$

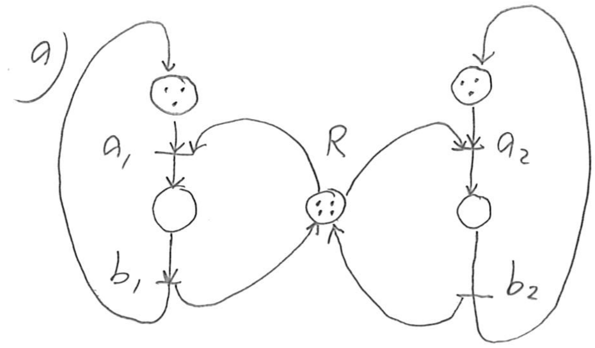
b) Add one token in the place R_1

$$\begin{array}{ccccc}
(202032) & \overset{a_2}{\underset{b_2}{\rightleftarrows}} & (201131) & \overset{a_2}{\underset{b_2}{\rightleftarrows}} & (200230) \\
a_1 \downarrow \uparrow b_1 & & a_1 \downarrow \uparrow b_1 & & a_1 \downarrow \\
(112022) & \overset{a_2}{\underset{b_2}{\rightleftarrows}} & (111121) & \overset{a_2}{\underset{b_2}{\rightleftarrows}} & (110220) \\
a_1 \downarrow \uparrow b_1 & & a_1 \downarrow \uparrow b_1 & & a_1 \downarrow \\
(022012) & \overset{a_2}{\underset{b_2}{\rightleftarrows}} & (021111) & \overset{a_2}{\underset{b_2}{\rightleftarrows}} & (020210)
\end{array}$$

No blocking states is also achieved when one token is added to R_2 instead of R_1.

c) n tokens in the initial places above the events a_1 and a_2 requires $n+1$ tokens in R_1 and n tokens in R_2 or $n+1$ in R_2 and n in R_1.

3: a)



b)

$m_i \in [0,3] \quad i = 1,2 \quad R \in [0,4]$

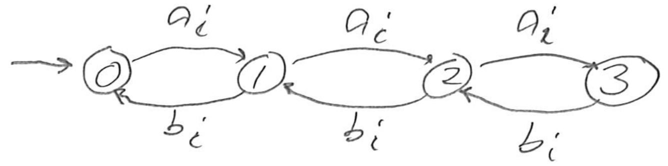
$(e = a_1 \wedge m_1' = m_1 + 1 \wedge m_1 < 3 \wedge R' = R - 1 \wedge R > 0) \vee$

$(e = b_1 \wedge m_1' = m_1 - 1 \wedge m_1 > 0 \wedge R' = R + 1 \wedge R < 4) \vee$

$(e = a_2 \wedge m_2' = m_2 + 1 \wedge m_2 < 3 \wedge R' = R - 1 \wedge R > 0) \vee$

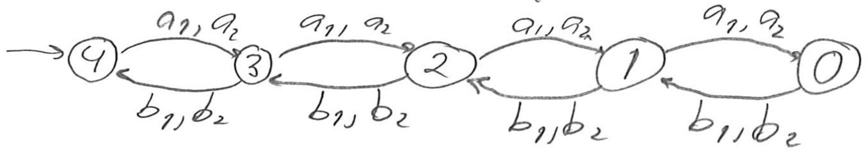
$(e = b_2 \wedge m_2' = m_2 - 1 \wedge m_2 > 0 \wedge R' = R + 1 \wedge R < 4)$

c) G_{m_i}

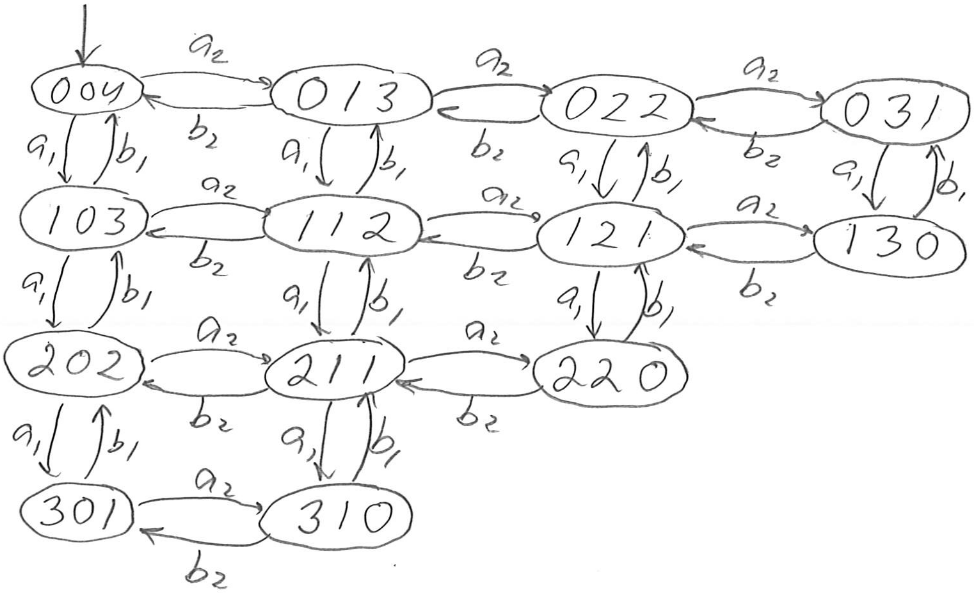


$i = 1, 2$

R



$G_{m_1} \| G_{m_2} \| G_R$



d) The sum of the state variables $m_1 + m_2 + R = 4$ in all states.

4. a) $\forall\Box\exists\Diamond M \equiv \nu y.\ \underbrace{(\mu z.\ \overbrace{M \vee \exists\bigcirc z}^{\psi_z}) \wedge \forall\bigcirc y}_{\psi}$

For both G_1 and G_2:

$\psi_z(Z) = [\![M]\!] \cup Pre^{\exists}(Z) = \{1,3\} \cup Pre^{\exists}(Z)$

$Z^1 = \{1,3\} \cup Pre^{\exists}(\emptyset) = \{1,3\}$

$Z^2 = \{1,3\} \cup Pre^{\exists}(\{1,3\}) = \{1,3\} \cup \{0,2,3\} = [0,3] = X$
$= Z^3 = Z^{\omega}$

$\psi(Y) = Z^{\omega} \cap Pre^{\forall}(Y) = [0,3] \cap Pre^{\forall}(Y)$

$Y^0 = [0,3] \quad Y^1 = [0,3] \cap Pre^{\forall}([0,3]) = [0,3] = Y^0 = Y^{\omega}$

Since $\{0\} \subseteq Y^{\omega}$ $\quad G_i \models \forall\Box\exists\Diamond M \quad i=1,2$

b) $\forall\Box(\exists\Diamond M_1 \wedge \exists\Diamond M_3) \equiv$

$\nu y.\ \underbrace{(\mu z.\ \underbrace{M_1 \vee \exists\bigcirc z}_{\psi_z} \wedge (\mu v.\ \underbrace{M_3 \vee \exists\bigcirc v}_{\psi_v}) \wedge \forall\bigcirc y}_{\psi}$

G_1: $\psi(Z) = [\![M_1]\!] \cup Pre^{\exists}(Z) = \{1\} \cup Pre^{\exists}(Z)$

$Z^1 = \{1\} \cup Pre^{\exists}(\emptyset) = \{1\},\ Z^2 = \{1\} \cup \{0\} = \{0,1\} = Z^3 = Z^{\omega}$

$\psi_3(V) = [\![M_3]\!] \cup Pre^{\exists}(V) = \{3\} \cup Pre^{\exists}(V)$

$V^1 = \{3\},\ V^2 = \{2,3\},\ V^3 = \{0,2,3\},\ V^4 [0,3] = V^5 = V^{\omega}$

$\psi(Y) = Z^{\omega} \cap V^{\omega} \cap Pre^{\forall}(Y) = \{0,1\} \cap [0,3] \cap Pre^{\forall}(Y)$
$= \{0,1\} \cap Pre^{\forall}(Y)$

$Y^0 = [0,3] \quad Y^1 = \{0,1\} \cap [0,3] = \{0,1\}$

$Y^2 = \{0,1\} \cap Pre^{\forall}(\{0,1\}) = \{0,1\} \cap \{1\} = \{1\}$

$Y^3 = \{0,1\} \cap Pre^{\forall}(\{1\}) = \{0,1\} \cap \emptyset = \emptyset = Y^4 = Y^{\omega}$

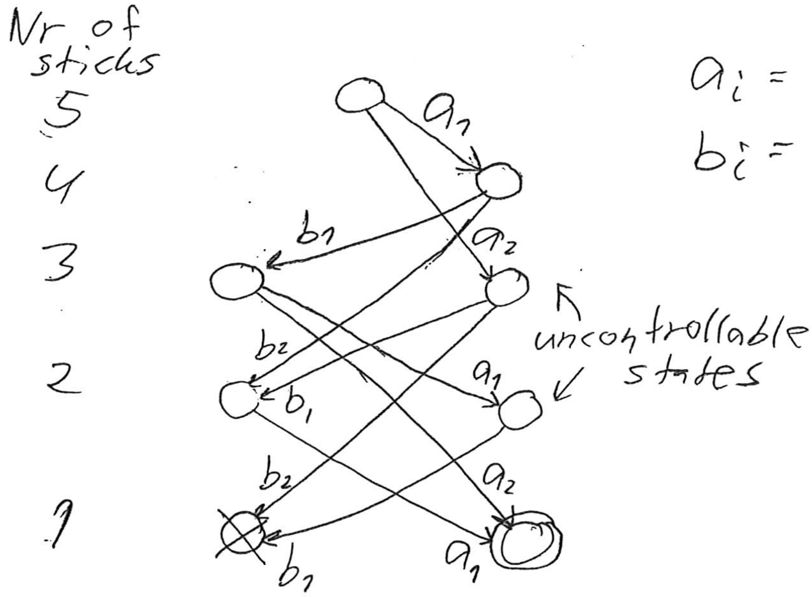
since $\{0\} \not\subseteq \emptyset \Rightarrow G_1 \not\models \forall\Box(\exists\Diamond M_1 \wedge \exists\Diamond M_2)$

G_2: $Z^{\omega} = V^{\omega} = [0,3] \Rightarrow Y^{\omega} = [0,3]$

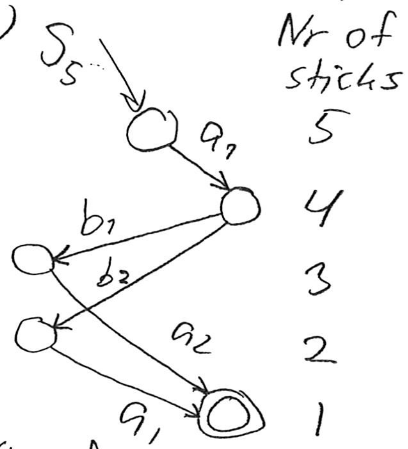
since $\{0\} \subseteq Y^{\omega}$ $\quad G_2 \models \forall\Box(\exists\Diamond M_1 \wedge \exists\Diamond M_2)$

5. a)

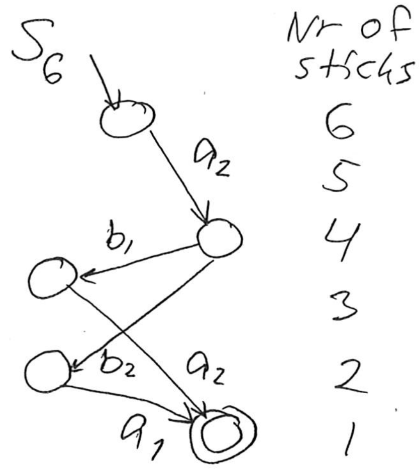
a_i = A takes i sticks,
b_i = B takes i sticks, (uncontrollable)

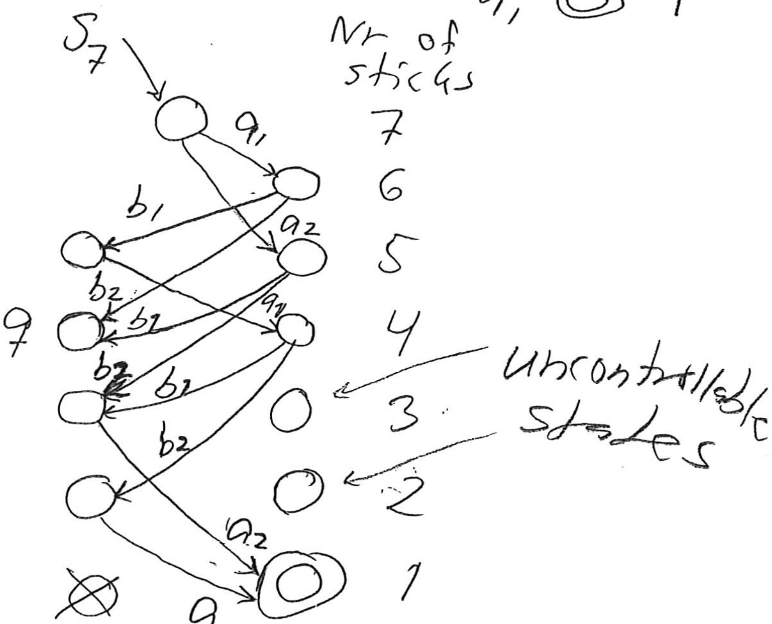


b)



c)





In state q for S_7 A must take one or two sticks ⇒ one of the uncontrollable states will be reached. Hence, A is not guaranteed to win for n=7 but for n=5 and 6.