Discrete Event Systems

Course code: SSY165

Examination 2017-10-21

Time: 8:30-12:30, Location: M-building

Teacher: Bengt Lennartson, phone 3722

The examination includes 25 points, where grade three requires 10 points, grade four 15 points and grade five 20 points.

The result of this examination is announced and inspection of the grading is done on Wednesday *November 8* and Thursday *November 9*, 12:30-13:00 at the division.

Allowed aids at the examination:

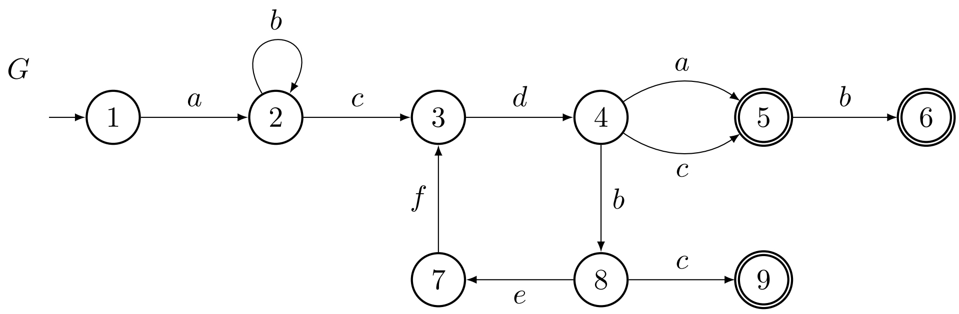
- Standard mathematical tables such as Beta, see also formulas in the end of this examination.
- Pocket calculator.

Good luck!

Department of Electrical Engineering
Division of Systems and Control
Chalmers University of Technology

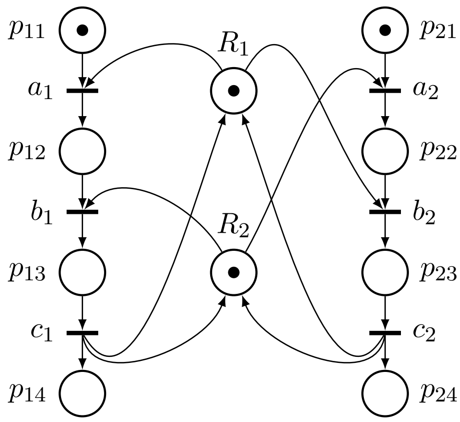
1

Formulate the language $\mathcal{L}(G)$ and the marked language $\mathcal{L}_m(G)$ for the following automaton.



(3 p)

2



a) Reformulate the Petri net model above as a number of synchronized modular automata.

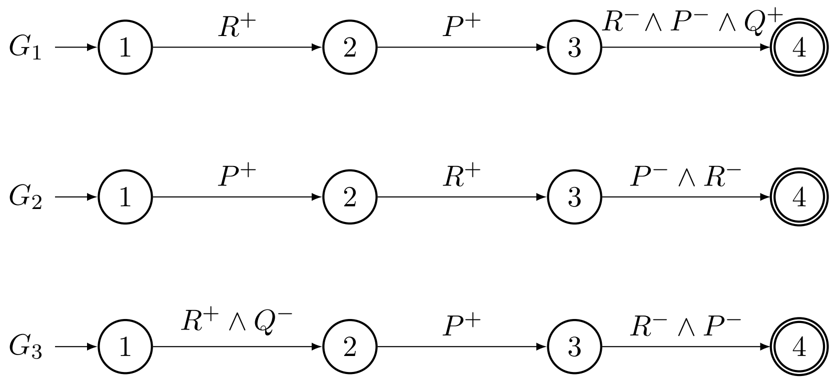
(2 p)

b) Formulate a set of synchronized modular automata when there are two tokens in place p_{11} and three tokens in place p_{21}.

(3 p)

3

Consider the following automata including variables (extended finite automata), where the logical variables R, P and Q can only take values 0 and 1. The notion $R^{\pm}$ means that the next value after the transition is $R' = R \pm 1$. The transition is however only admissible if the next value is within the domain of the variable, in this case 0 or 1. The initial values of the three variables are zero. No explicit events are included in the models, meaning that the synchronization between the automata is purely based on the shared 0/1 variables.



a) Generate the reachability graph when the three extended automata with variables are synchronized $G_1 \| G_2 \| G_3$.

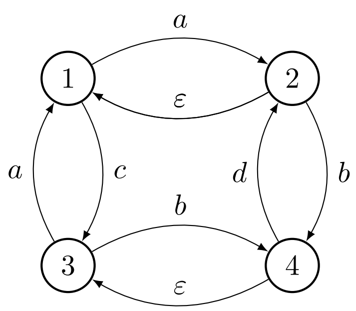
(3 p)

b) Two blocking states can be observed in the reachability graph. Add suitable guards on the variables in the local models to avoid these two blocking states.

(2 p)

4

Consider the following automaton where the events a, b, c and d are observable (e.g. by sensors), while the event ε is not observable. It means that for instance the transition from state 2 to state 1 is not detectable. Furthermore, the initial state of the system is not known.

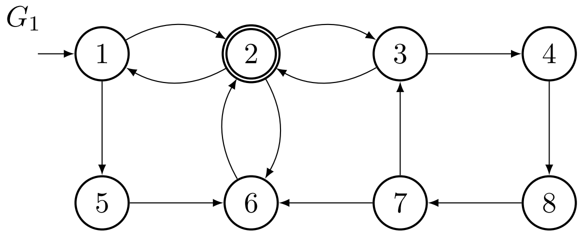


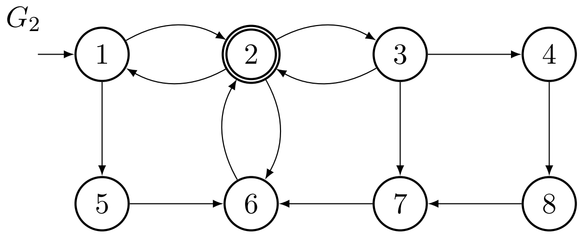
a) Generate a state observer which estimates the current state of the system, with the events a, b, c and d as inputs. Any string s including the events in the set $\{a, b, c, d\}$ is assumed to appear. The states of the observer consist of the set of possible system states that the system may occupy after a string s has been generated. Since no information about the state is initially available, the initial state of the observer is the whole set of states $\{1, 2, 3, 4\}$. Note that the observer only includes observable events and that the initial state has the state name 1234, showing that initially the observer has no~~w~~ knowledge at all about the actual state.
(3 p)

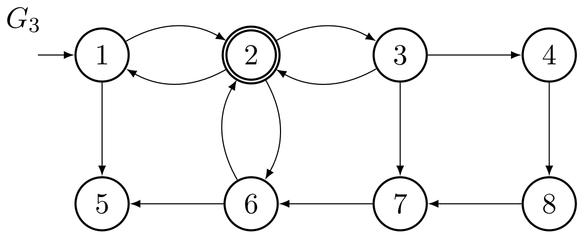
b) Assume that one specific state is a secret state that should not be possible to detect (determine) by the observer after observing any number of events. Which two states can be detected as unique states by the observer, and which two states are safe, i.e. they can not be detected by the observer as unique individual states.
(1 p)

5

Consider the following automata, where the state label of the marked state satisfies the propositional formula φ_m.







Which automata satisfy from their initial state the following temporal logic expressions. Motivate your answers.

- LTL formulas
  - $\bigcirc \varphi_m$
  - $\bigcirc\bigcirc \varphi_m$
  - $\Diamond \varphi_m$
  - $\Box\Diamond \varphi_m$
- CTL formulas
  - $\forall\Box\forall\Diamond \varphi_m$
  - $\forall\Box\exists\Diamond \varphi_m$

(3 p)

6

Consider a queueing system, modeled as a continuous Markov process with infinite buffer capacity. The arrival rate of jobs is assumed to be λ and the machine service rate is μ.

a) Show that the average number of jobs in the buffer (excluding the machine) is $\bar{N}_Q = \rho^2/(1-\rho)$, where $\rho = \lambda/\mu$ is the utilization factor.

(3 p)

b) Assume that the daily cost for the machine is 10 times higher than the place for each individual element in the buffer. Thus the total average daily cost is proportional to $10 + \bar{N}_Q$. For which utilization factors ρ of the original system, would it be more cost efficient to increase the capacity of the machine by 20% (service rate = 1.2μ) with an added machine cost of 20%.

(2 p)

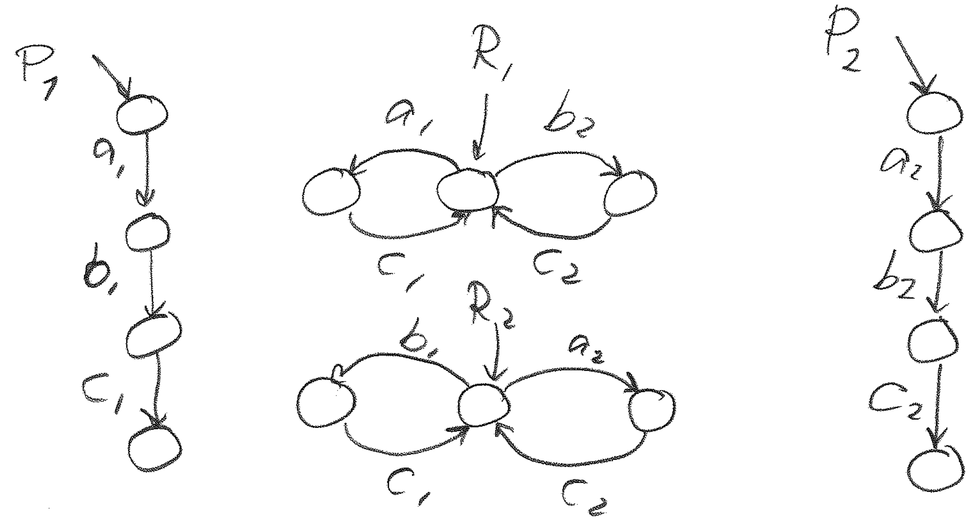
Solution Discrete Event Systems

2017-10-21 BL171021

1. $L_m(G) = ab^*c(dbef)^*d((a+c)(\varepsilon+b)+bc)$

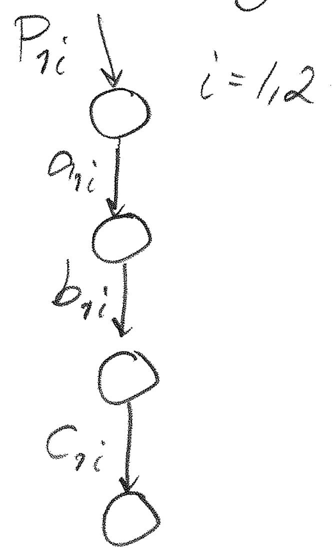
$L(G) = \overline{L_m(G)}$

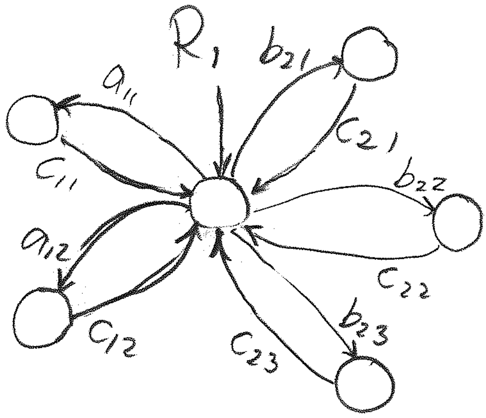
2. a)

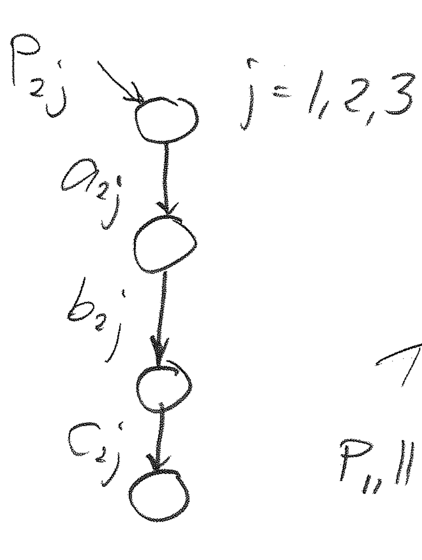


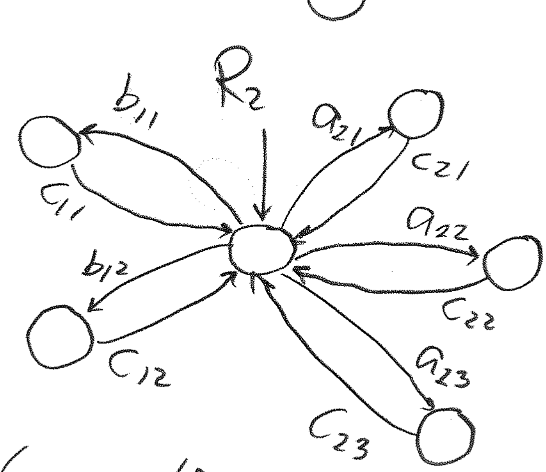
Total system $P_1 \| R_1 \| R_2 \| P_2$

b)





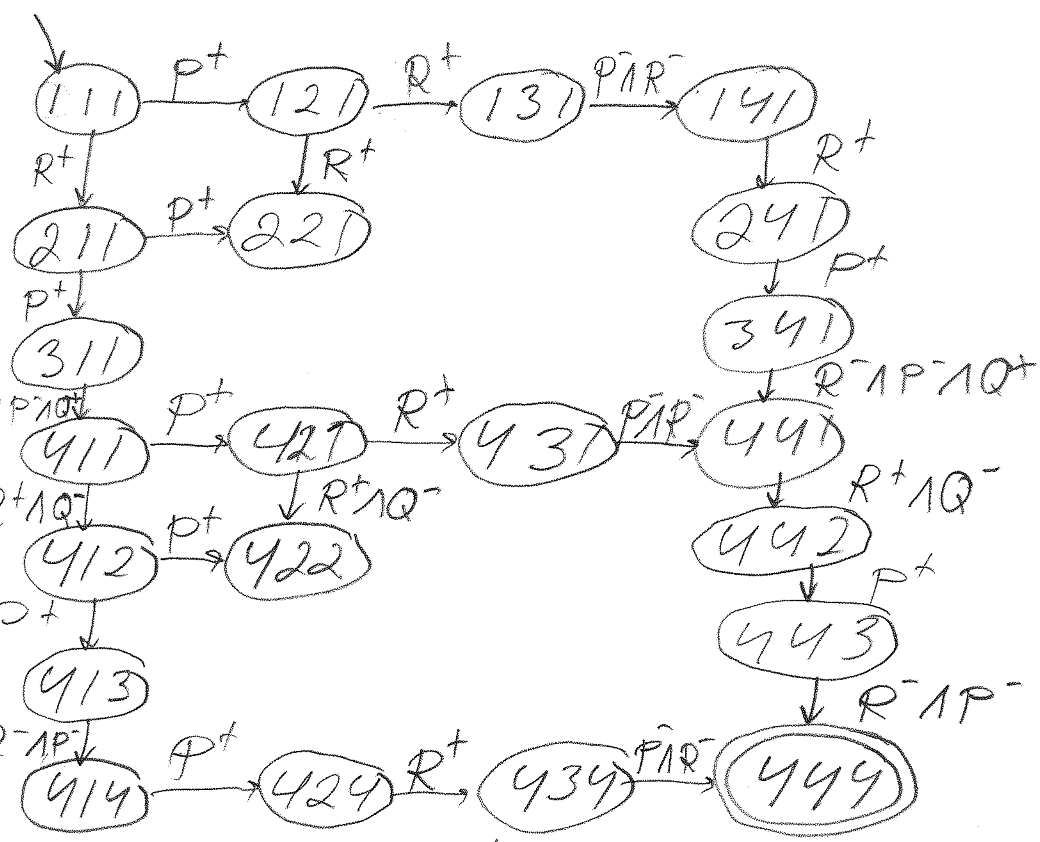




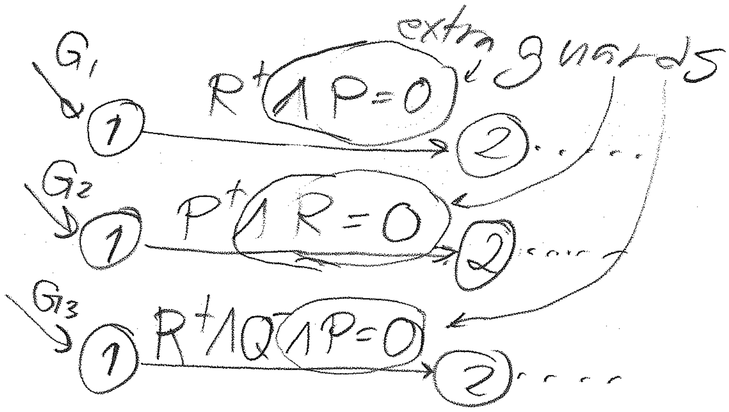
Total system

$P_{11} \| P_{12} \| R_1 \| R_2 \| P_{21} \| P_{22} \| P_{23}$

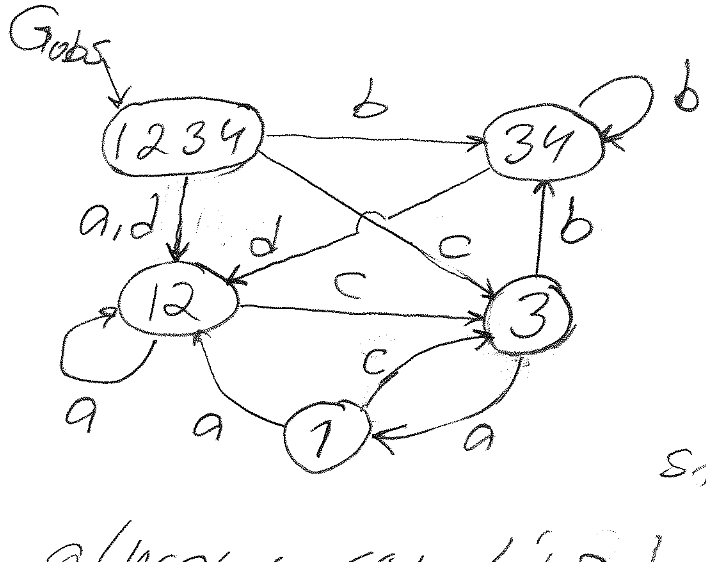
3. a)



b)



4.



4 and 2 are safe states since these states are not unique states in Gobs, but always combined with other non secret states, 2 together with 1 and 4 together with 3, while state 1 and 3 are isolated in the observer and thus detectable.

5. $\bigcirc q_m$ No model – next state is both state 2 (marked) but also state 5 (not marked)

$\bigcirc\bigcirc q_m$ No model, no path with 2 transitions to state 2

$\Diamond q_m$ G_1, G_2 – state 2 not reachable in G_3 via state 5

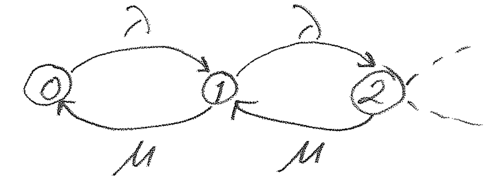
$\Box\Diamond q_m \equiv \forall\Box\forall\Diamond q_m$ G_2 – state 2 will always be eventually reached from all states in G_2

In G_1 the loop $3 \to 4 \to 8 \to 7 \to 3$ may continue for ever without passing state 2

In G_3 state 5 is a deadlock state

$\forall\Box\exists\Diamond q_m$ G_1, G_2 – from all states it is always possible to reach state 2, while G_3 has a deadlock state.

6. a)



$\lambda p_0 = \mu p_1 \Rightarrow p_1 = \frac{\lambda}{\mu} p_0 = \rho p_0$

$\lambda p_0 + \mu p_2 = \lambda p_1 + \mu p_1 = \lambda p_1 + \lambda p_0 \Rightarrow$

$p_2 = \frac{\lambda}{\mu} p_1 = \rho^2 p_0 \quad \therefore \quad p_n = \rho^n p_0$

$\sum_{i=0}^{\infty} p_i = \sum_{i=0}^{\infty} \rho^i p_0 = \frac{1}{1-\rho} p_0 = 1$

$\therefore p_0 = 1 - \rho$

$\bar{N}$ = average total number of jobs

$= \sum_{i=0}^{\infty} i p_i = \sum_{i=0}^{\infty} i \rho^i \underbrace{(1-\rho)}_{p_0} = (1-\rho) \sum_{i=0}^{\infty} i \rho^i =$

$= (1-\rho) \frac{\rho}{(1-\rho)^2} = \frac{\rho}{1-\rho}$

$\bar{N}_S$ = average number in the machine

$= 1 - p_0 = 1 - (1-\rho) = \rho$

$\bar{N}_Q$ = average number in the queue

$= \bar{N} - \bar{N}_S = \frac{\rho}{1-\rho} - \rho = \frac{\rho^2}{1-\rho} = \rho \bar{N}$

b)

$$10 + \frac{\rho^2}{1-\rho} > 12 + \frac{(\rho/1.2)^2}{1-\rho/1.2}$$

↑ 20% higher machine cost

for $\rho > 0.8074$. Note that the increased service rate 1.2μ reduces the utilization factor to $\rho/1.2$ for the same arrival rate λ.