Discrete Event Systems

Course code: SSY165

Examination 2012-10-23

Time: 14:00-18:00, Location: H-building

Teacher: Bengt Lennartson, phone 3722

The examination includes 25 points, where grade three requires 10 points, grade four 15 points and grade five 20 points.

The result of this examination will be announced latest on Tuesday *November 6* on the notice board of the division, at the entrance in the south east corner on floor 5 of the E-building. *Inspection* of the grading is done on Tuesday *November 6* and Wednesday *November 7* at 12:30-13:00.

Allowed aids at the examination:

- Standard mathematical tables such as Beta, see also formulas in the end of this examination.
- Pocket calculator.

Good luck!

Department of Signals and Systems
Division of Automatic Control, Automation and Mechatronics
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1

Show the following implication by a contradiction.

$$(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r) \Rightarrow r$$

(2 p)

2

Prove that

$$\exists x[p(x) \vee q(x)] \Leftrightarrow \exists x[p(x)] \vee \exists x[q(x)]$$

by assuming a universal set Ω with a finite number of arbitrary elements

$$\Omega = \{a_1, a_2, \cdots, a_n\}$$

(1 p)

3

Two discrete event subsystems G_1 and G_2 are modeled by the following formal languages

$$L(G_1) = \overline{(ab)^*}$$
$$L(G_2) = \overline{(ac)^*}$$

a) Introduce a state variable x_i and corresponding state values $x_i \in \{0, 1\}$ for G_i, $i = 1, 2$, and generate automata for G_1 and G_2.

(1 p)

b) By introducing an additional variable $\acute{x}_i$ for the next state, and an event variable e which takes values from the event set $\{a, b, c\}$, possible transitions in G_i can be expressed by a predicate (boolean function)

$$P_i(x_i, e, \acute{x}_i)$$

This boolean function is true when the values of the current state variable x_i, the next state variable $\acute{x}_i$ and corresponding event variable e match a transition in the automaton G_i. Formulate the transition predicates for G_1 and G_2.

(1 p)

c) Based on the automata and variables in a), generate an automaton for the synchronized system $G_1||G_2$ including state variable values, as well as the corresponding transition predicate for $G_1\|G_2$.

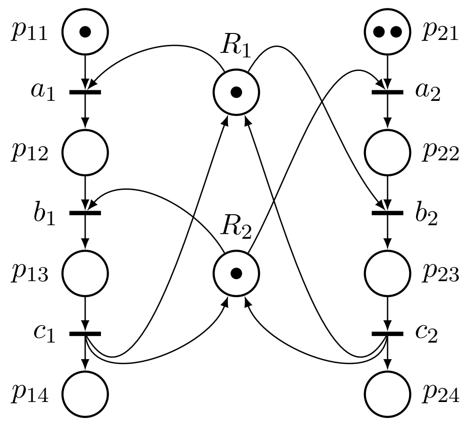
(2 p)

d) Formulate a rule on how predicates for synchronized automata can be generated based on the predicates of the individual automata. Separate between shared events (in this example event a) and local events (b in G_1 and c in G_2).

(2 p)

4

Consider the following Petri net where two shared resources R_1 and R_2 are used by two tasks. The left one, including events a_1, b_1, and c_1, only involves one object (one initial token), while the right task, including events a_2, b_2, and c_2, involves two objects (two initial tokens).



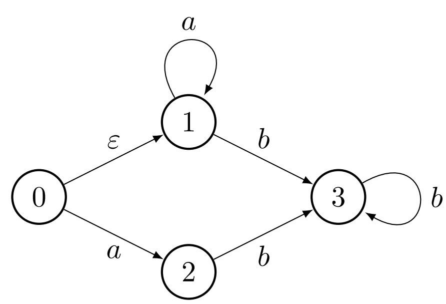
a) Generate a corresponding automaton (reachability graph) for the Petri net.
(2 p)

b) Synthesize a controllable and nonblocking supervisor when the events c_1 and c_2 are uncontrollable. The only specification is that all initial tokens in the two tasks must reach corresponding final places p_{14} and p_{24}.
(2 p)

c) Add an extra control place and appropriate arcs to the original Petri net to obtain a controllable and nonblocking supervisor. Motivate that this solution can handle an arbitrary number of initial tokens in p_{11} and p_{21}.
(2 p)

5

Consider the following automaton where the events a and b are observable (e.g. by sensors), while the event ε is not observable. It means that the transition from state 0 to state 1 is not detectable. Furthermore, the initial state of the system is not known.



Generate a state observer which estimates the current state of the system, with the events a and b as inputs. Any string $s \in \{a, b\}^*$ is assumed to appear. The states of the observer consist of the set of possible system states that the system may occupy after a string s has been generated. Since no information about the state is initially available, the initial state of the observer is the whole set of states $\{0, 1, 2, 3\}$. To indicate the complexity of the task, the observer involves totally the same number of states and self loops as the system itself, but only observable events and set of states as observer states.

(2 p)

6

Consider ones again the resource booking system in Task 4. When there is only one token in each initial place p_{11} and p_{21}, the Petri net can alternatively be represented as extended finite automata (EFAs).

a) Formulate one EFA for each task, E_1 and E_2 and include resource variables R_1 and R_2 to handle the mutual exclusion between the two resources.

(2 p)

b) Generate a corresponding automaton for the synchronized system $E_1 \| E_2$ (a simplified version of Task 4a)).

(1 p)

c) Assume that all events are controllable, and generate one additional guard in each original EFA E_1 and E_2, such that the deadlock state that appears in the synchronized system is avoided.

(2 p)

7

For a server system with an unlimited buffer the utilization factor is assumed to be equal ρ. Then the probability that the system has k number of parts is $\rho^k(1-\rho)$. How many parts are there on average in the server system?

(3 p)

Solution to Exam: Discrete Event Systems

2012-10-23

BL 121030

1. $p \Rightarrow q$ if and only if $p \wedge \neg q \Rightarrow \mathbb{F}$.

$$(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r) \wedge \neg r \Leftrightarrow$$

$$(p \vee q) \wedge (\neg p \vee r) \wedge (\neg q \vee r) \wedge \neg r \Leftrightarrow$$

$$(p \vee q) \wedge ((\neg p \wedge \neg q) \vee r) \wedge \neg r \Leftrightarrow$$

$$(p \vee q) \wedge (\neg(p \vee q) \vee r) \wedge \neg r \Leftrightarrow$$

$$(p \vee q) \wedge (\neg(p \vee q) \wedge \neg r) \vee \underbrace{(r \wedge \neg r)}_{\mathbb{F}}) \Leftrightarrow$$

$$(p \vee q) \wedge \neg(p \vee q) \wedge \neg r \Leftrightarrow \mathbb{F} \wedge \neg r \Leftrightarrow \mathbb{F}$$

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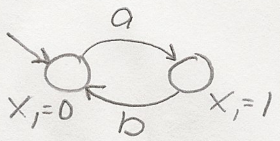
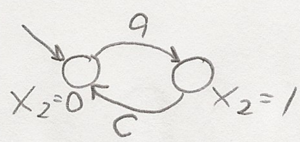
2. $$\exists x[p(x) \vee q(x)] \Leftrightarrow p(a_1) \vee q(a_1) \vee \ldots \vee p(a_n) \vee q(a_n)$$

$$\Leftrightarrow p(a_1) \vee \ldots \vee p(a_n) \vee q(a_1) \vee \ldots \vee q(a_n) \Leftrightarrow \exists x[p(x)] \vee \exists x[q(x)]$$

3. a) G_1



G_2

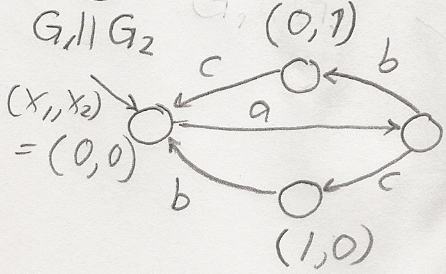


b) $P_1(x_1, e, x_1')$: $((x_1=0) \wedge (e=a) \wedge (x_1'=1)) \vee ((x_1=1) \wedge (e=b) \wedge (x_1'=0))$

$P_2(x_2, e, x_2')$: $((x_2=0) \wedge (e=a) \wedge (x_2'=1)) \vee ((x_2=1) \wedge (e=c) \wedge (x_2'=0))$

c)

$G_1 \| G_2$



$P_{12}((x_1, x_2), e, (x_1', x_2'))$:

$$((x_1=0) \wedge (x_2=0) \wedge (e=a) \wedge (x_1'=1) \wedge (x_2'=1))$$

$$\vee ((x_1=1) \wedge (x_2=1) \wedge (e=b) \wedge (x_1'=0) \wedge (x_2'=1))$$

$$\vee (x_1=1) \wedge (x_2=0) \wedge (e=b) \wedge (x_1'=0) \wedge (x_2'=0)$$

$$\vee (x_1=1) \wedge (x_2=1) \wedge (e=c) \wedge (x_1'=1) \wedge (x_2'=0)$$

$$\vee (x_1=0) \wedge (x_2=1) \wedge (e=c) \wedge (x_1'=0) \wedge (x_2'=0)$$

d) Introduce the notations

$P_i^a(x_i, e, x_i'): (x_i = 0) \wedge (e = a) \wedge (x_i' = 1) \qquad i = 1, 2$

$P_1^b(x_1, e, x_1'): (x_1 = 1) \wedge (e = b) \wedge (x_1' = 0)$

$P_2^c(x_2, e, x_2'): (x_2 = 1) \wedge (e = c) \wedge (x_2' = 0)$

$$\Rightarrow \quad P_1(x_1, e, x_1'): \; P_1^a(x_1, e, x_1') \vee P_1^b(x_1, e, x_1')$$

$$P_2(x_2, e, x_2'): \; P_2^a(x_2, e, x_2') \vee P_2^c(x_2, e, x_2')$$

$$P_{12}((x_1,x_2), e, (x_1', x_2')): \left(P_1^a(x_1, e, x_1') \wedge P_2^a(x_2, e, x_2')\right)$$

$$\vee \left(P_1^b(x_1, e, x_1') \wedge (x_2 = 1) \wedge (x_2' = 1)\right)$$

$$\vee \left(P_1^b(x_1, e, x_1') \wedge (x_2 = 0) \wedge (x_2' = 0)\right)$$

$$\vee \left(P_2^c(x_2, e, x_2') \wedge (x_1 = 1) \wedge (x_1' = 1)\right)$$

$$\vee \left(P_2^c(x_2, e, x_2') \wedge (x_1 = 0) \wedge (x_1' = 0)\right) \Leftrightarrow$$

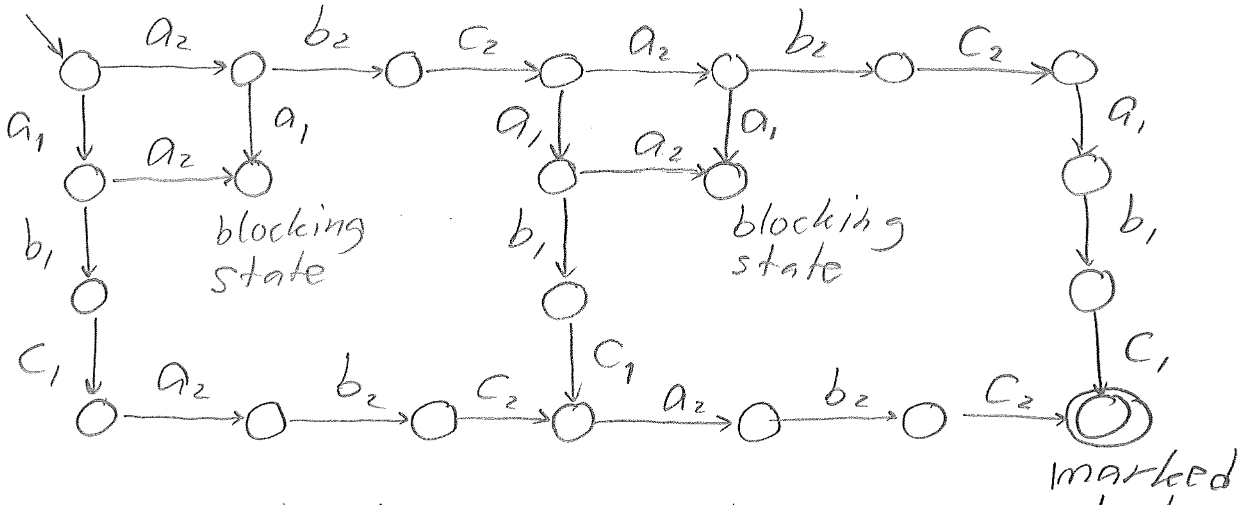
$$\left(P_1^a(x_1, e, x_1') \wedge P_2^a(x_2, e, x_2')\right) \vee$$

$$\left(P_1^b(x_1, e, x_1') \wedge (x_2' = x_2)\right) \vee$$

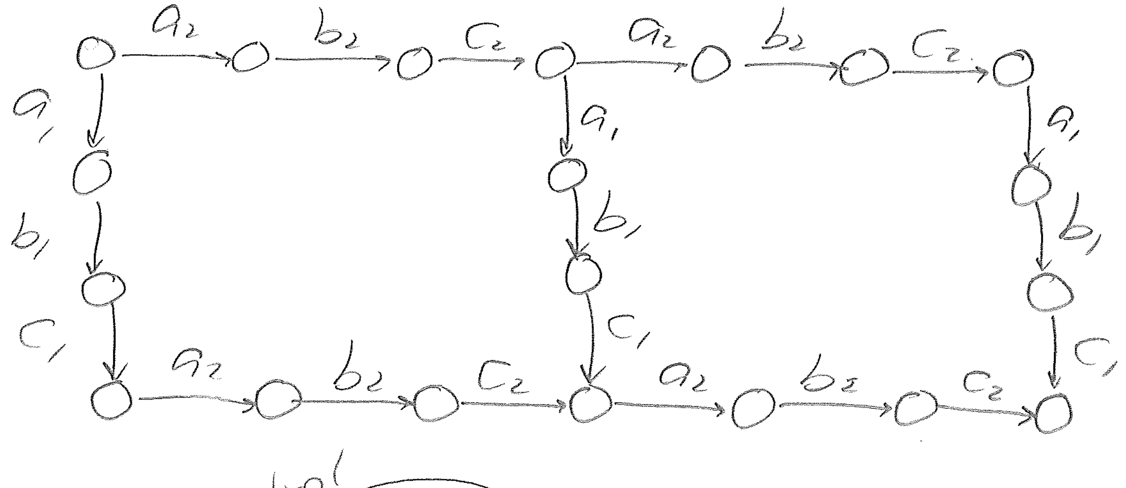
$$\left(P_2^c(x_2, e, x_2') \wedge (x_1' = x_1)\right)$$

∵ For a shared event $e = a$ the individual predicates P_1^a and P_2^a are conjuncted $P_1^a \wedge P_2^a$. For a nonshared event $e = b$ for G_1 the synchronized predicate becomes $P_1^b \wedge (x_2' = x_2)$, i.e. keep current value for the model in G_2 without event b.

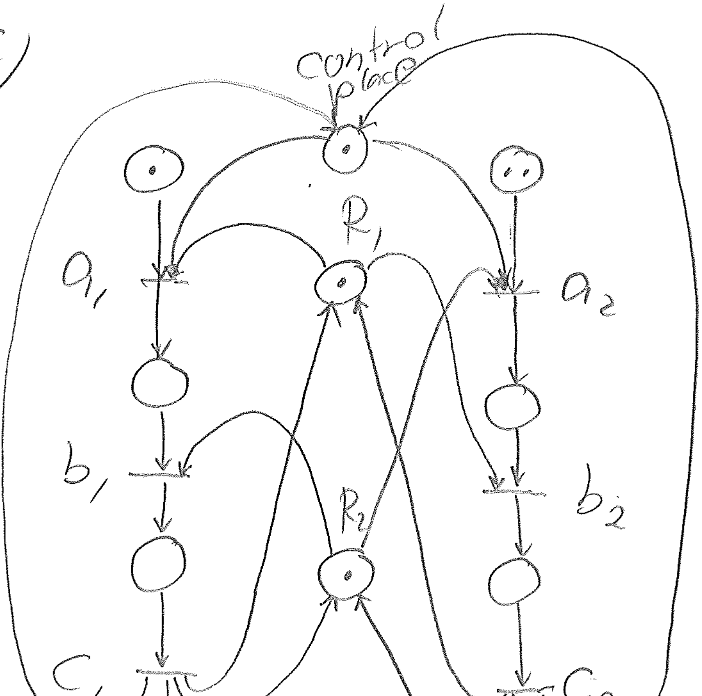
4. a)



b) To reach the marked state the two blocking states are removed. This results in the supervisor

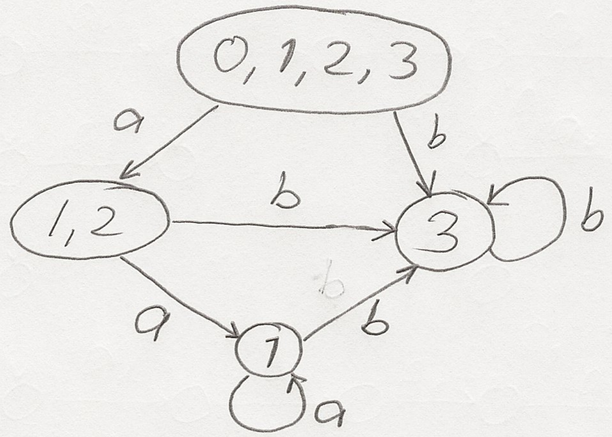


c)



The control place with the added arcs guaratees that only one of the two tasks is executed at a time. This implies that the nonblocking states are avoided, independently of the number of initial tokens in p_{11} and p_{21}. Deadlock occurs when task 1 allocates R_1 and task 2 allocates R_2.

5.



7.

$$\bar{N} = \sum_{k=0}^{\infty} k\, p_k =$$

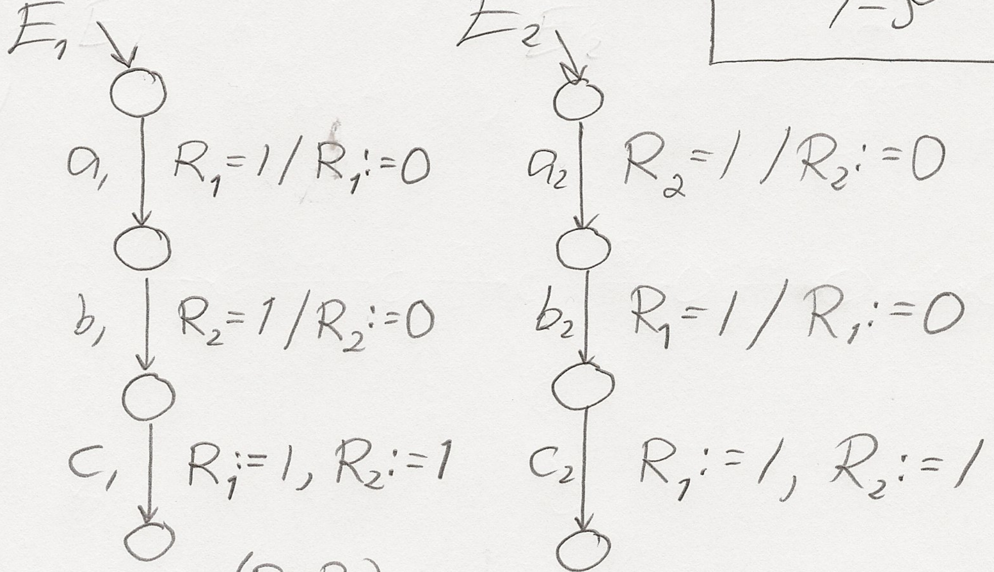
$$= \sum_{k=0}^{\infty} k \rho^k (1-\rho) =$$

$$= (1-\rho) \sum_{k=0}^{\infty} k \rho^k = (1-\rho) \frac{\rho}{(1-\rho)^2}$$

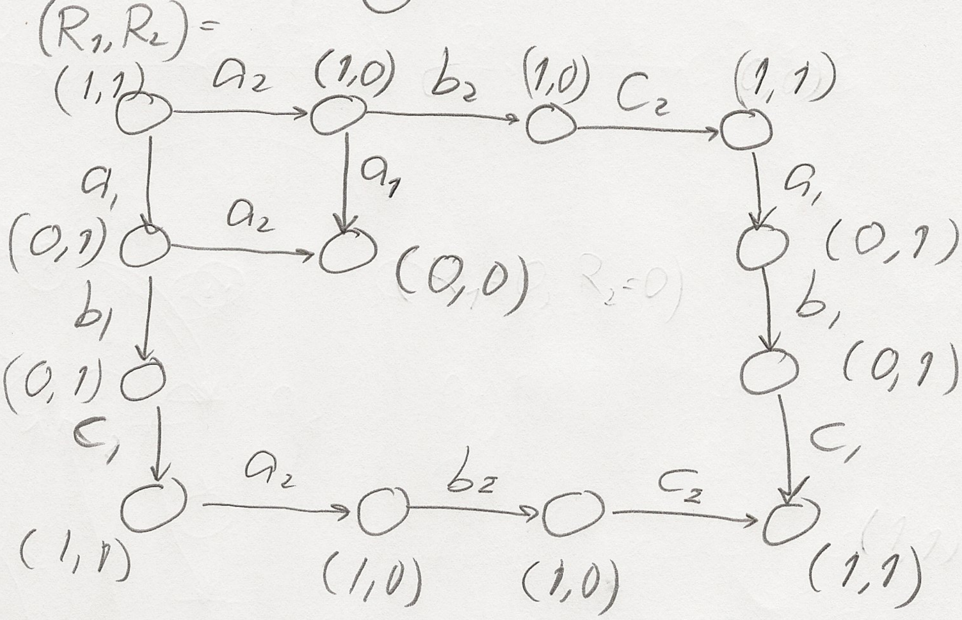
$$= \frac{\rho}{1-\rho}$$

6.

a)



b) $E_1 || E_2$



c) Added control guard

at a_1: $R_2 = 1$

at a_2: $R_1 = 1$