# *Discrete Event Systems*

## *Course code: SSY165*

## *Examination 2011-10-20*

Time: 14:00-18:00, Location: V-building

Teacher: Bengt Lennartson, phone 3722

The examination includes 25 points, where grade three requires 10 points, grade four 15 points and grade five 20 points.

*The result of this examination* will be announced latest on Thursday *November 3* on the notice board of the division, at the entrance in the south east corner on floor 5 of the E-building. *Inspection* of the grading is done on Thursday *November 3* and Friday *November 4* at 12:30-13:00.

*Allowed aids at the examination:*

- Standard mathematical tables such as Beta, see also formulas in the end of this examination.
- Pocket calculator.

Good luck!

Department of Signals and Systems
Division of Automatic Control, Automation and Mechatronics
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**1**

Show that the following set expression is valid for all elements in the universal set

$$((A \subseteq B) \cap (A \cup \sim B)) \cup ((A \cup B) \cap (A \cup B \cup \sim A))$$

(3 p)

**2**

Two discrete event subsystems $A_1$ and $A_2$ are modeled by the following marked formal languages

$$\begin{aligned} L_m(A_1) &= (a(a+b))^* \\ L_m(A_2) &= (ac)^* \end{aligned}$$

a) Generate an automaton for the synchronized system $A_1||A_2$ when $b \notin \Sigma_{A_2}$ and $c \notin \Sigma_{A_1}$.

(2 p)

b) Generate a formal language for the synchronized system $A_1||A_2$

(1 p)

c) Generate a Petri net for the synchronized system $A_1||A_2$ and the related incidence matrix $A^+$, defining the weights of the arcs between the input transitions and their output places, and $A^-$, defining the weights of the arcs between the input places and their output transitions, as well as the initial marking vector $m_0$.

(2 p)

**3**

Generate a controllable and nonblocking supervisor for a plant $P = P_1||P_2||P_3$, where the automaton for the subplant $P_i$ has the transition relations

$$q_{i1} \xrightarrow{e_{i1}} q_{i2},\ q_{i2} \xrightarrow{e_{i2}} q_{i3},\ q_{i3} \xrightarrow{e_{i3}} q_{i1}$$

The state $q_{i1}$ is the initial state but also the marked state, the event $e_{i2}$ is uncontrollable while the events $e_{i1}$ and $e_{i3}$ are controllable, and the synchronized state $\langle q_{13}, q_{23}, q_{33} \rangle$ is forbidden.

(5 p)

**4**

Consider the plant $P$ in Task 3. A discrete state-space model with a state vector $x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$ and an event vector $e = \begin{bmatrix} e_1 & e_2 & e_3 \end{bmatrix}^T$ can be generated by representing the states in each plant $P_i$ by an integer such that $x_i = \ell$ corresponds to the state $q_{i\ell}$. The event signal $e_i = \ell$ when the event $e_{i\ell}$ occurs, and otherwise $e_i = 0$.

a) Generate a state vector model $x(t_k^+) = f(x(t_k), e(t_k))$ for the plant $P$, where the following type of syntax for mixed integer logical expressions is recommended. The expression $x * (e = \ell)$ is equal $x$ when $e = \ell$, and zero when $e \neq \ell$.
(2 p)

b) Add a guard condition to the state vector model such that the transition $q_{i1} \xrightarrow{e_{i1}} q_{i2}$ can only be executed when the other two subplants are in their initial states $q_{j1}$.
(1 p)

**5**

Consider two systems given by the languages

$$\begin{aligned} L(P_1) &= \overline{a(f+b)c^*} \\ L(P_2) &= \overline{a(f+c)c^*} \end{aligned}$$

Assume that the event $f$ is a failure event which is not observable, while the rest of the events are observable. Determine observers for the two systems that only include the observable events. The goal is to decide if the failure event $f$ has happened. Show that this is possible for $P_1$ but not for $P_2$, which means that $P_1$ is diagnosable but not $P_2$.
(3 p)

**6**

The temperature of a system is modeled by a first order differential equation

$$\dot{x}(t) = -ax(t) + bu(t)$$

where the control signal $u(t)$ is switched to one when $x \leq 19^\circ$ and switched back to zero when $x \geq 21^\circ$. Generate a hybrid automaton for this mixed continuous and discrete system, which is called a thermostat.
(2 p)

**7**

Consider a queueing system where the arrival rate of jobs is $\lambda$ and the service rate of jobs is $\mu$. The discrete states in the system are the number of jobs in the system, where state $q_j$ corresponds to $j$ jobs. Assume that the maximum number of jobs in the system including buffer and server is two (three states in the continuous-time Markov process).

a) Determine the stationary state probability for the three states, i.e. the probability that there are zero, one or two jobs in the system. Express the probabilities as function of the utilization factor $\rho = \lambda/\mu$.

(2 p)

b) Determine the average number of jobs in the system $\bar{N}$ as a function of $\rho$, and give specific values for $\rho = 0.5$ and $0.9$.

(2 p)

# Solution Exam Discrete Event Systems
## 2011-10-20

BL 11/029

1. Show that $((p_A(x) \rightarrow p_B(x)) \wedge (p_A(x) \vee \neg p_B(x))) \vee ((p_A(x) \vee p_B(x)) \wedge (p_A(x) \vee p_B(x) \vee \neg p_A(x)))$ is a tautology for all $x$

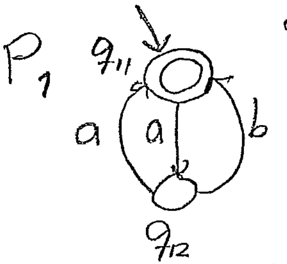
$$((\neg p_A \vee p_B) \wedge (p_A \vee \neg p_B)) \vee ((p_A \vee p_B) \wedge \overbrace{(\underbrace{p_A \vee \neg p_A}_{\mathbb{T}} \vee p_B)}^{\mathbb{T}}) \Leftrightarrow$$

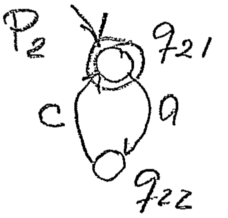
$$\underbrace{(\neg p_A \wedge p_A)}_{\mathbb{F}} \vee (\neg p_A \wedge \neg p_B) \vee (p_B \wedge p_A) \vee \underbrace{(p_B \wedge \neg p_B)}_{\mathbb{F}} \vee (p_A \vee p_B) \Leftarrow$$

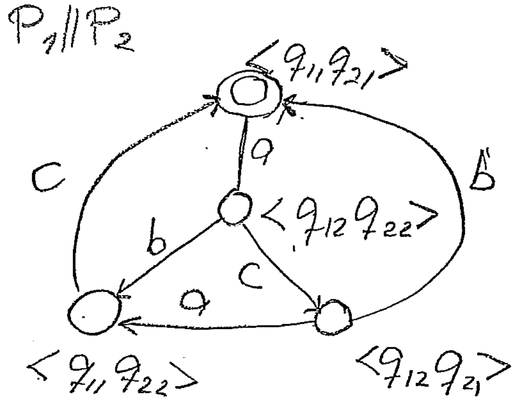
$$(\neg p_A \wedge \neg p_B) \vee (p_A \wedge p_B) \vee (\neg p_A \wedge p_B) \vee (p_A \wedge \neg p_B) \Leftrightarrow \mathbb{T}$$

2.

a)





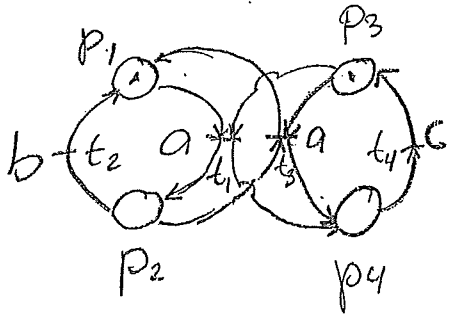


$L_m(P_1) = (a(b+a))^*$

$L_m(P_2 = (ac)^*$

b) $L_m(P_1 \| P_2) = (abc + acac + acb)^* = (a(ca+b)c + acb)^*$

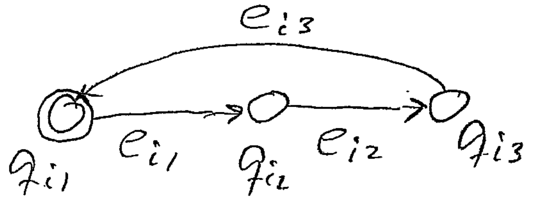
c)



$$A^+ = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \quad A^- = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

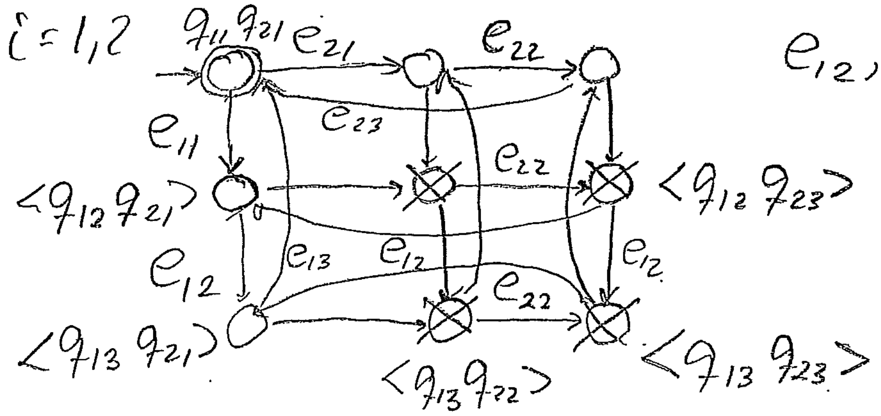
$$m_0 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

3.



$e_{i2} \in \Sigma_u$ $i = 1, \dots, n$

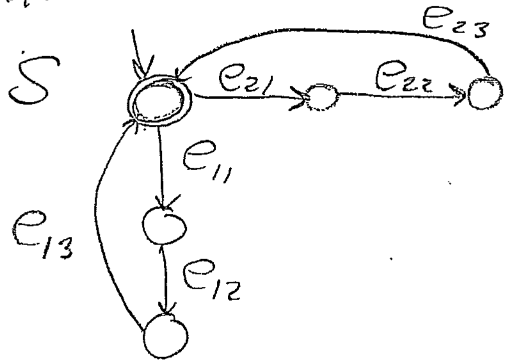
n=2



$e_{12}, e_{22} \in \Sigma_u$

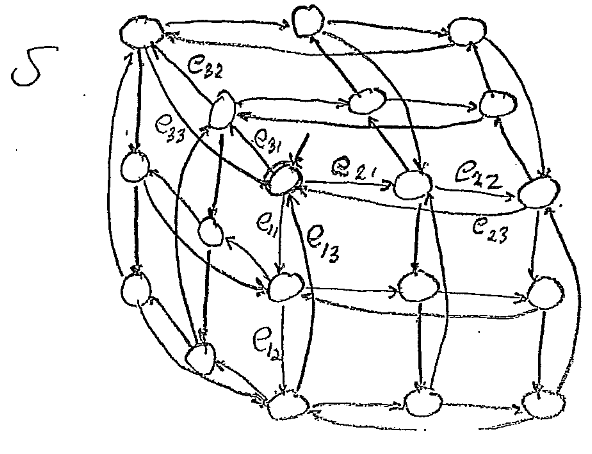
$\langle q_{13} q_{23} \rangle$ forbidden by the specification

$\langle q_{13} q_{22} \rangle$, $\langle q_{12} q_{22} \rangle$ and $\langle q_{12} q_{23} \rangle$ are then uncontrollable states since $e_{12}$ and $e_{22}$ are uncontrollable events



obviously every sub-plant i may continue to the critical state $q_{i3}$ when the event $e_{i1}$ has been executed.

For n=3 (3 subplants) it is ok to have 2 subplants in the state $q_{i3}$ but not all three. It gives the following supervisor



4. a) $x^+ = x(t_k^+)$ $x = x(t_k)$ $x(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

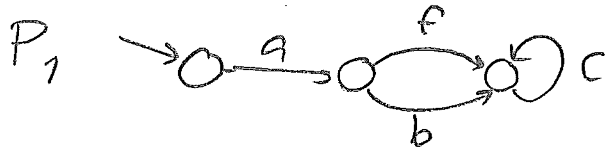
$$x_i^+ = x_i + (x_i=1)*(e_i=1) + (x_i=2)*(e_i=2) - 2*(x_i=3)*(e_i=3)$$

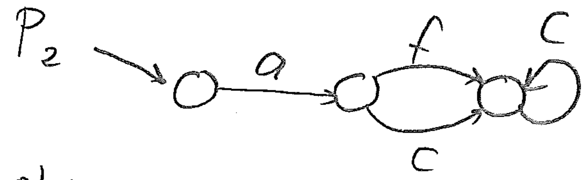
$i = 1, 2, 3$

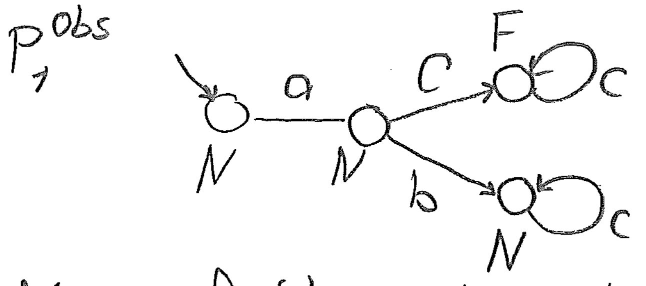
b)

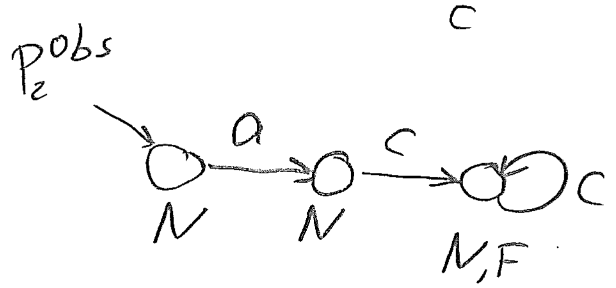
$$x_i^+ = x_i + (x_1=1)*(x_2=1)*(x_3=1)*(e_i=1) + (x_i=2)*(e_i=2) - 2*(x_i=3)*(e_i=3)$$

5.





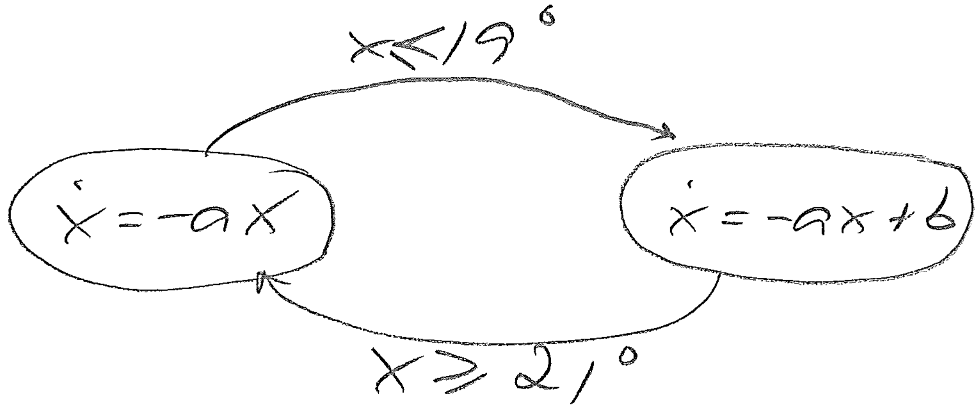




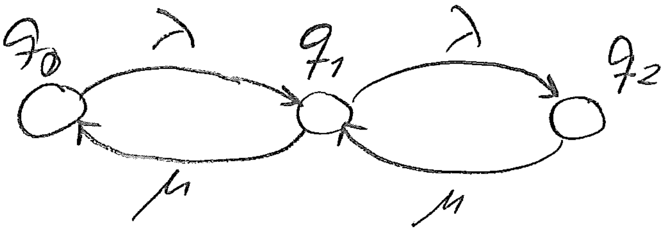
N = no failure has happened
F = failure — " —

For $P_1^{obs}$ the execution of the nonobservable failure event f can be decided, since the string ac ⇒ failure while ab ⇒ no failure. This cannot be distinguished for $P_2^{obs}$ since ac ⇒ a failure cannot be decided

6.



7.



a)

$$\lambda p_0 = \mu p_1 \quad \Rightarrow \quad p_1 = \frac{\lambda}{\mu} p_0 = \rho p_0$$

$$\lambda p_0 + \mu p_2 = (\lambda + \mu) p_1 \quad \Rightarrow \quad p_2 = \frac{\lambda}{\mu} p_1 = \rho p_1 = \rho^2 p_0$$

$$p_0 + p_1 + p_2 = (1 + \rho + \rho^2) p_0 = 1$$

$$p_0 = \frac{1}{1+\rho+\rho^2} \qquad p_i = \frac{\rho^i}{1+\rho+\rho^2}$$

b)

$$\bar{N} = 0 \cdot p_0 + 1 \cdot p_1 + 2 \cdot p_2 = \frac{\rho + 2\rho^2}{1+\rho+\rho^2}$$

$$= \frac{\sum_{i=0}^{2} i \rho^i}{\sum_{i=0}^{2} \rho^i} = \begin{cases} \frac{0.5+2\cdot 0.25}{1.75} = \frac{4}{7} & \rho = 0.5 \\ \frac{0.9+2\cdot 0.81}{1+0.9+0.81} = \frac{2{,}52}{2{,}71} & \rho = 0.9 \end{cases}$$