Introduction to Discrete Event Systems

Course code: SSY165, ESS200

Examination 2009-10-23

Time: 14:00-18:00, Location: HA, HB, HC

Teacher: Bengt Lennartson, phone 3722

The examination includes 25 points, where grade three requires 10 points, grade four 15 points and grade five 20 points.

The result of this examination will be announced latest on Thursday *November 5* on the notice board of the division, at the entrance in the south east corner on floor 5 of the E-building. *Inspection* of the grading is done on Thursday *November 5* and Friday *November 6* at 12:30-13:00.

Allowed aids at the examination:

- Standard mathematical tables such as Beta, see also formulas in the end of this examination.
- Pocket calculator.

Good luck!

Department of Signals and Systems
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1

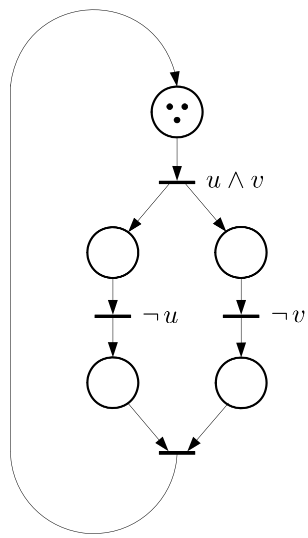
Show the following set implication by relations based on predicate expressions

$$A \subseteq B \Rightarrow (A \setminus D) \cap C \subseteq B \cap C \cap \sim D$$

(3 p)

2

Consider the following Petri net where the enabling of the transitions are dependent on the logical variables u and v.



Generate a state space model

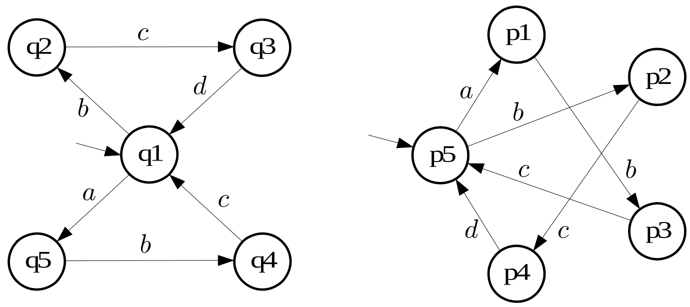
$$x^+ = f(x, u, v)$$

where each state variable x_i represents the number of tokens in corresponding place. The non-linear function f may include inequality and propositional logical operators.

(3 p)

3

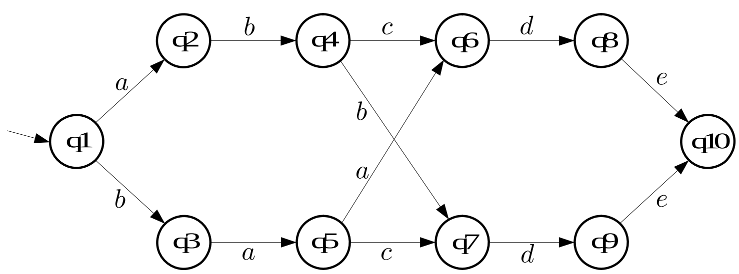
Show that the following two automata are structurally equivalent. This means that for all strings s in the langauge generated by the two automata, there is a unique one-to-one mapping between corresponding states in the automata reached by the same string s.



(3 p)

4

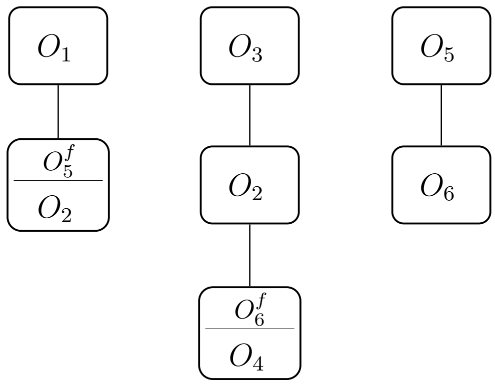
Generate a controllable and nonblocking supervisor, by the fix point algorithm presented in the lecture notes, for the plant P given below. Assume that the event c is uncontrollable, while the other events are controllable. The specification $Sp = P$ with the additional demand that q_8 is a forbidden state and q_{10} is the only marked state. Show the resulting automaton after each Backward_Reachability computation.



(4 p)

5

A number of non repeated operations need to be coordinated. Generally, an operation O_k starts when the event s_k occurs and it is completed when the event c_k is fired. Six operations $O_1, \ldots Q_6$ are going to be executed. In principle all operations can be executed concurrently, except for a couple of restrictions between the operations that are shown in the figure below. The operation sequences are executed from top to bottom, but additional preconditions are included above some of the operation names (O_k^f means final state for operation O_k). This implies that for instance O_2 in the left sequence must wait until both O_1 and O_5 have been completed. Furthermore, the middle sequence specifies that O_2 also must wait until O_3 has been completed.



a) Formulate a Petri net that specify a coordination between the different operations such that the given operation restrictions are satisfied.

(2 p)

b) Formulate an alternative modular model, composed of a number of automata synchronized by common events, that specify the same behavior as in a).

(2 p)

c) Show that the final state, where all operations have been completed, is reachable, by giving one string of events that is included in the language generated by the Petri net in a) and the automata models in b). A correct solution of a) and b) means of course that they generate the same language.

(1 p)

6

To be able to show that the closed loop system $P||S$ is controllable if and only of the supervisor S is controllable, the following result is useful

$$L_A\Sigma \cap L_B\Sigma = (L_A \cap L_B)\Sigma$$

where L_A and L_B are languages, and Σ is a set of events (strings of length one).

a) Show that the language intersection/concatenation expressions above are equal. According to Beta (Math Handbook) $L_1 L_2 = \{s_1 s_2 | s_1 \in L_1 \wedge s_2 \in L_2\}$
(2 p)

b) Assume that the alphabets $\Sigma_P = \Sigma_S$, and show that the closed loop system $P||S$ is controllable with respect to the plant, if and only of the supervisor S is controllable also with respect to the plant, i.e.

$$L(P||S)\Sigma_u \cap L(P) \subseteq L(P||S) \;\; \Leftrightarrow \;\; L(S)\Sigma_u \cap L(P) \subseteq L(S)$$

The following additional results are then required:

1. $L(P||S) = L(P) \cap L(S)$ when $\Sigma_P = \Sigma_S$
2. $A \cap B \subseteq C \cap B \;\; \Leftrightarrow \;\; A \cap B \subseteq C$
3. $(L(P)\Sigma_u \cap L(P)) \cap (L(S)\Sigma_u \cap L(P)) \subseteq L(S) \;\; \Leftrightarrow \;\; L(S)\Sigma_u \cap L(P) \subseteq L(S)$

(2 p)

7

A machine has one operating state q_1 and one broken (idle) state q_2. The conditional transition probabilities are given in the transition probability matrix

$$\mathcal{P} = \begin{bmatrix} 0.9 & 0.1 \\ 0.4 & 0.6 \end{bmatrix}$$

a) Draw a state transition diagram for this discrete-time Markov chain.
(1 p)

b) Calculate the state probability after one time instant when the initial state is q_1 (the machine is working), i.e. calculate $p(t_1)$ when $p(t_0) = \begin{bmatrix} 1 & 0 \end{bmatrix}$.
(1 p)

c) Calculate the stationary state probability $p = \begin{bmatrix} p_1 & p_2 \end{bmatrix}$ (note that the sum of the two probabilities is equal one).
(1 p)

Table 1.1 Equivalence relations.

E_1	$\neg\neg p \Leftrightarrow p$		
E_2	$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$	E_3	$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$
E_4	$p \vee q \Leftrightarrow q \vee p$	E_5	$p \wedge q \Leftrightarrow q \wedge p$
E_6	$p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$	E_7	$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$
E_8	$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$	E_9	$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$
E_{10}	$p \vee p \Leftrightarrow p$	E_{11}	$p \wedge p \Leftrightarrow p$
E_{12}	$p \vee \mathbf{F} \Leftrightarrow p$	E_{13}	$p \wedge \mathbf{T} \Leftrightarrow p$
E_{14}	$p \vee \mathbf{T} \Leftrightarrow \mathbf{T}$	E_{15}	$p \wedge \mathbf{F} \Leftrightarrow \mathbf{F}$
E_{16}	$p \vee \neg p \Leftrightarrow \mathbf{T}$	E_{17}	$p \wedge \neg p \Leftrightarrow \mathbf{F}$
E_{18}	$p \vee (p \wedge q) \Leftrightarrow p$	E_{19}	$p \wedge (p \vee q) \Leftrightarrow p$
E_{20}	$p \rightarrow q \Leftrightarrow \neg p \vee q$	E_{21}	$\neg(p \rightarrow q) \Leftrightarrow p \wedge \neg q$

Table 1.2 Implication relations.

I_1	$p \wedge q \Rightarrow p$	I_2	$p \wedge q \Rightarrow q$
I_3	$p \Rightarrow p \vee q$	I_4	$q \Rightarrow p \vee q$
I_5	$\neg p \Rightarrow p \rightarrow q$	I_6	$q \Rightarrow p \rightarrow q$
I_7	$\neg(p \rightarrow q) \Rightarrow p$	I_8	$\neg(p \rightarrow q) \Rightarrow \neg q$

$$A||B = \left\langle Q^A \times Q^B, \Sigma^A \cup \Sigma^B, \delta, \langle q_i^A, q_i^B \rangle, Q_m^A \times Q_m^B, (Q_x^A \times Q^B) \cup (Q^A \times Q_x^B) \right\rangle$$

$$\delta(\langle q^A, q^B \rangle, \sigma) = \begin{cases} \delta^A(q^A, \sigma) \times \delta^B(q^B, \sigma) & \sigma \in \Sigma^A \cap \Sigma^B \\ \delta^A(q^A, \sigma) \times \{q^B\} & \sigma \in \Sigma^A \setminus \Sigma^B \\ \{q^A\} \times \delta^B(q^B, \sigma) & \sigma \in \Sigma^B \setminus \Sigma^A \end{cases}$$

Solution to examination in IDES 09/023

1. $A \subseteq B \Rightarrow (A \setminus D) \cap C \subseteq B \cap C \cap \sim D$?

$p_A(x) \wedge \neg p_D(x) \wedge p_C(x) \to p_B(x) \wedge p_C(x) \wedge \neg p_D(x) \Leftrightarrow$

$\neg(p_A(x) \wedge \neg p_D(x) \wedge p_C(x)) \vee (p_B(x) \wedge p_C(x) \wedge \neg p_D(x)) \Leftrightarrow$

$\neg p_A(x) \vee \neg(\underbrace{p_C(x) \wedge \neg p_D(x)}_{p_X(x)}) \vee (p_B(x) \wedge \underbrace{p_C(x) \wedge \neg p_D(x)}_{p_X(x)}) \Leftrightarrow$

$\neg p_A(x) \vee (p_B(x) \wedge p_X(x)) \vee \neg p_X(x) \Leftrightarrow$

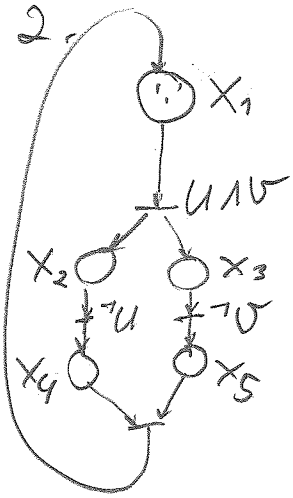
$(\neg p_A(x) \vee p_B(x)) \wedge (\neg p_A(x) \vee p_X(x)) \vee \neg p_X(x) \Leftrightarrow$

$\underbrace{(p_A(x) \to p_B(x))}_{\mathbb{T} \text{ since } A \subseteq B} \wedge (\neg p_A(x) \vee p_X(x)) \vee \neg p_X(x) \Leftrightarrow$ $\neg p_A(x) \vee \underbrace{p_X(x) \vee \neg p_X(x)}_{\mathbb{T}}$

$\Leftrightarrow \neg p_A(x) \vee \mathbb{T} \Leftrightarrow \mathbb{T}$

$\therefore (A \setminus D) \cap C \subseteq B \cap C \cap \sim D$ when $A \subseteq B$

2.



$x_1^+ = x_1 + (x_4>0) \wedge (x_5>0) - (x_1>0) \wedge (u \wedge v)$

$x_2^+ = x_2 + (x_1>0) \wedge (u \wedge v) - (x_2>0) \wedge \neg u$

$x_3^+ = x_3 + (x_1>0) \wedge (u \wedge v) - (x_3>0) \wedge \neg v$

$x_4^+ = x_4 + (x_2>0) \wedge \neg u - (x_4>0) \wedge (x_5>0)$

$x_5^+ = x_5 + (x_3>0) \wedge \neg v - (x_4>0) \wedge (x_5>0)$

3.

$p_5 = f(\varepsilon, q_1)$ $\quad p_2 = f(b, q_2)$ $\quad p_4 = f(bc, q_3)$

$p_5 = f(bcd, q_1)$ ....

$p_1 = f(a, q_5)$ $\quad p_3 = f(ab, q_4)$ $\quad p_5 = f(abc, q_1)$

$p_5 = f((abc)^* + (bcd)^*, q_1)$

$p_2 = f(((abc)^* + (bcd)^*)\,b, q_2)$

$p_4 = f(((abc)^* + (bcd)^*)\,bc, q_3)$

$p_1 = f(((abc)^* + (bcd)^*)\,a, q_5)$

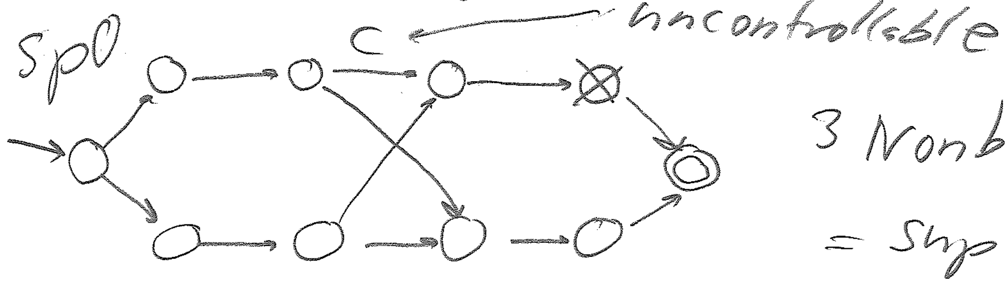
$p_3 = f(((abc)^* + (bcd)^*)\,ab, q_4)$

$\therefore$ A unique state mapping for each strings

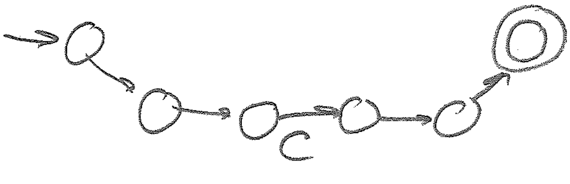
$\Sigma_A = \{a, b, c, d\} = \Sigma_B$, Including the state mapping above, the transition function $\delta_A = \delta_B$ and the initial states are the same.

$\Rightarrow$ The two automata are structurally equal.

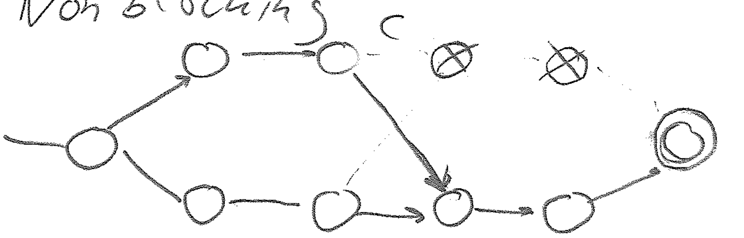
4. Sp0



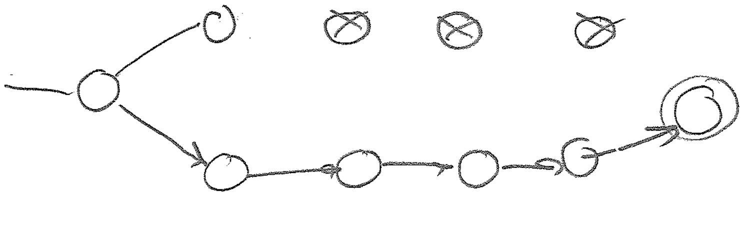
3 Nonblocking = Supervisor S



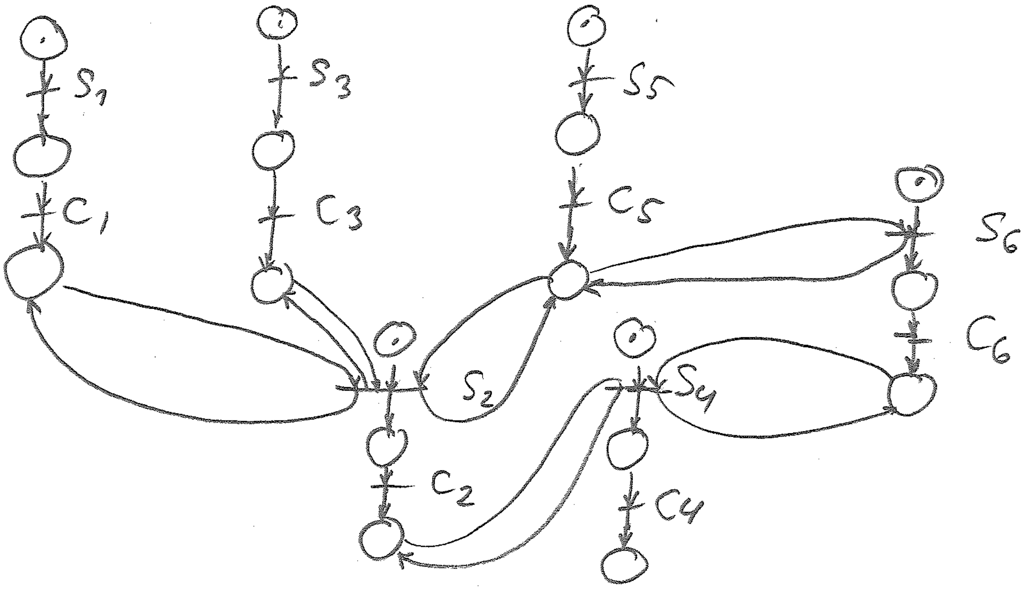
1 Nonblocking



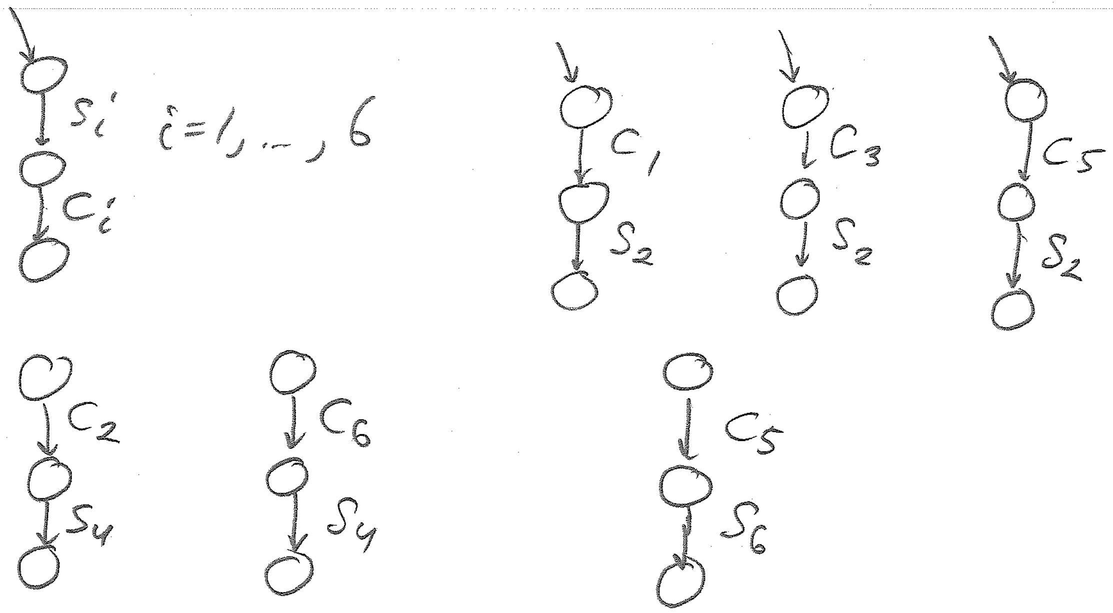
2 Controllable



5. a)



b)



c) $s_1\ c_1\ s_3\ c_3\ s_5\ c_5\ s_2\ c_2\ s_6\ c_6\ s_4\ c_4$

6. a) $L_A\Sigma \cap L_B\Sigma = \{se \mid s\in L_A \wedge e\in\Sigma\} \cap$

$\cap \{s'e \mid s'\in L_B \wedge e\in\Sigma\} = \{se \mid s\in L_A \wedge s\in L_B \wedge e\in\Sigma\}$

$= (L_A \cap L_B)\Sigma$

b) $L(P\|S)\Sigma_u \cap L(P) \subseteq L(P\|S) \Leftrightarrow$

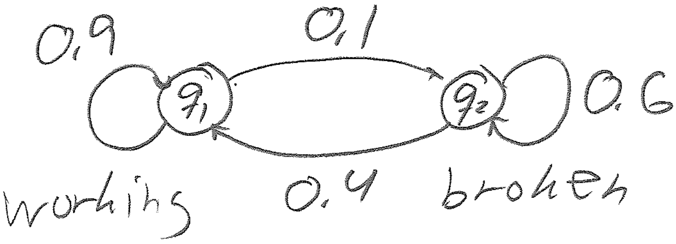
$(L(P)\cap L(S))\Sigma_u \cap L(P) \subseteq L(P)\cap L(S) \Leftrightarrow$

$L(P)\Sigma_u \cap L(S)\Sigma_u \cap L(P) \cap L(P) \subseteq L(S) \Leftrightarrow$

$(L(P)\Sigma_u \cap L(P)) \cap (L(S)\Sigma_u \cap L(P)) \subseteq L(S) \Leftrightarrow$

$L(S)\Sigma_u \cap L(P) \subseteq L(S)$

7. a)



b) $p(t_0)P = [1\ 0]\begin{bmatrix}0.9 & 0.1\\0.4 & 0.6\end{bmatrix} = [0.9\ \ 0.1] = p(t_1)$

c) $p = [p_1\ \ (1-p_1)] = [p_1\ \ (1-p_1)]\underbrace{\begin{bmatrix}0.9 & 0.1\\0.4 & 0.6\end{bmatrix}}_{P}$

$= [0.9p_1 + 0.4(1-p_1)\quad 0.1p_1 + 0.6(1-p_1)]$

$p_1 - 0.9p_1 + 0.4p_1 = 0.4 \Rightarrow p_1 = \frac{0.4}{0.5} = 0.8$

$\therefore\ p = [0.8\ \ 0.2]$ $\qquad p_2 = 1 - p_1 = 0.2$