Introduction to Discrete Event Systems

Course code: SSY165, ESS200

Examination 2008-10-23

Time: 8:30-12:30, Lokal: V-building

Teacher: Tord Alenljung, phone 1799

The examination includes 25 points, where grade three requires 10 points, grade four 15 points and grade five 20 points.

The result of this examination will be announced latest on Thursday *November 6* on the notice board of the division, at the entrance in the south east corner on floor 5 of the E-building. *Inspection* of the grading is done on Thursday *November 6* and Friday *November 7* at 12:30-13:00.

Allowed aids at the examination:

- Standard mathematical tables such as Beta, see also formulas in the end of this examination.
- Pocket calculator.

Good luck!

Department of Signals and Systems
Division of Automatic Control, Automation and Mechatronics
Chalmers University of Technology

1

Show that the following logical expression is a tautology.

$$(p \rightarrow q) \wedge (\neg q \vee p) \vee ((p \vee q) \wedge (p \vee q \vee \neg p))$$

(2 p)

2

Show the following set equivalence by equivalence relations based on predicate expressions

$$A \cap B \subseteq B \cap C \Leftrightarrow A \cap B \subseteq C$$

(2 p)

3

Three robots P_1, P_2 and P_3 share a common zone. When a robot P_i enters the common zone the event a_i occurs. When it leaves the zone the event b_i occurs.

a) Formulate an automaton model for robot P_i including the events a_i and b_i and give the formal language $L(P_i)$ for robot P_i.

(1 p)

b) Generate a Petri net including the three robots and a mutual exclusion place which guarantees that only one robot at a time is in the common zone.

(1 p)

c) The Petri net model represents a closed loop system. Identify based on this net a supervisor S, which together with the robot models P_1, P_2 and P_3 generates the closed loop system $P_1||P_2||P_3||S$ corresponding to the Petri net model above.

(1 p)

d) Generate the reachability graph for the Petri net in b), and verify that the states where more than one robot is in the common zone is not reachable.

(2 p)

4

Consider a system with binary signals y, v and z. The event $y\uparrow$ occurs when y raises from zero to one and the event $y\downarrow$ occurs when it goes back to zero. The same notation is valid for v and z. The initial values for the signals are assumed to be zero.

Two subsystems A and B model signal behaviors given by the following two marked languages

$$L_m(A) = ((y\uparrow y\downarrow + v\uparrow v\downarrow)e)^*$$

$$L_m(B) = ((y\uparrow y\downarrow + z\uparrow z\downarrow)e)^*$$

and their corresponding non-marked languages $L(A) = \overline{L_m(A)}$ and $L(B) = \overline{L_m(B)}$. Observe the common event e which is included to model a synchronized reset action back to the initial state. This reset action does not depend on any external signal and is only included here to model the synchronization between the two subsystems.

a) Generate corresponding automata A and B and a Petri net for the composed system $A||B$.

(2 p)

b) Formulate boolean state equations representing the composed system $A||B$.

(2 p)

c) Assume now that the signal value $z = 1$ is a forbidden state in subsystem B. Identify all states in the composed system $A||B$ that are forbidden.

(1 p)

d) Also identify the resulting blocking states in the composed system and the resulting automaton for the nonblocking composed system when the forbidden signal value $z = 1$ is taken into account.

(1 p)

5

Consider a plant P with a formal language $L(P) = \overline{a(b + d) + cd}$ and two supervisors S_1 and S_2 with formal languages $L(S_2) = \overline{ab + cd}$ and $L(S_2) = \overline{cd}$. Assume that the event d is uncontrollable that is $\Sigma_u = \{d\}$.

a) Show that the supervisor S_1 is uncontrollable, while S_2 is controllable, by applying the language controllability definition, which says that a supervisor S is controllable with respect to a plant P if

$$L(S)\Sigma_u \cap L(P) \subseteq L(S)$$

(3 p)

b) Give the corresponding automata for the languages $L(P)$, $L(S_1)$ and $L(S_2)$ and explain based on these automata why S_1 is not controllable.

(2 p)

6

Two people named A and B are playing a simple game. A number of sticks are lain out on the ground and the players take alternately one or two sticks. Note that at least one stick must be picked. The player that ends up with the last stick has lost the game. Player A is always the one that starts picking sticks.

a) Model this game by an automaton, with an initial number of five sticks. Hint: identify the events and the states.

(2 p)

b) Introduce a marked state specifying that player A is to win and player B is to loose. Remember that the player left with only the final stick to pick, is the loser.

(1 p)

c) Generate a supervisor that guarantees that player A wins the game. Note that the set of uncontrollable events must first be decided.

(2 p)

Table 1.1 Equivalence relations.

E_1	$\neg\neg p \Leftrightarrow p$		
E_2	$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$	E_3	$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$
E_4	$p \vee q \Leftrightarrow q \vee p$	E_5	$p \wedge q \Leftrightarrow q \wedge p$
E_6	$p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$	E_7	$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$
E_8	$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$	E_9	$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$
E_{10}	$p \vee p \Leftrightarrow p$	E_{11}	$p \wedge p \Leftrightarrow p$
E_{12}	$p \vee \mathbf{F} \Leftrightarrow p$	E_{13}	$p \wedge \mathbf{T} \Leftrightarrow p$
E_{14}	$p \vee \mathbf{T} \Leftrightarrow \mathbf{T}$	E_{15}	$p \wedge \mathbf{F} \Leftrightarrow \mathbf{F}$
E_{16}	$p \vee \neg p \Leftrightarrow \mathbf{T}$	E_{17}	$p \wedge \neg p \Leftrightarrow \mathbf{F}$
E_{18}	$p \vee (p \wedge q) \Leftrightarrow p$	E_{19}	$p \wedge (p \vee q) \Leftrightarrow p$
E_{20}	$p \rightarrow q \Leftrightarrow \neg p \vee q$	E_{21}	$\neg(p \rightarrow q) \Leftrightarrow p \wedge \neg q$
E_{22}	$p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$	E_{23}	$p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$
E_{24}	$\neg(p \leftrightarrow q) \Leftrightarrow p \leftrightarrow \neg q$	E_{25}	$p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$
E_{26}	$p \leftrightarrow q \Leftrightarrow (p \wedge q) \vee (\neg p \wedge \neg q)$		

Table 1.2 Implication relations.

I_1	$p \wedge q \Rightarrow p$	I_2	$p \wedge q \Rightarrow q$
I_3	$p \Rightarrow p \vee q$	I_4	$q \Rightarrow p \vee q$
I_5	$\neg p \Rightarrow p \rightarrow q$	I_6	$q \Rightarrow p \rightarrow q$
I_7	$\neg(p \rightarrow q) \Rightarrow p$	I_8	$\neg(p \rightarrow q) \Rightarrow \neg q$
I_9	$\neg p \wedge (p \vee q) \Rightarrow q$	I_{10}	$p \wedge (p \rightarrow q) \Rightarrow q$
I_{11}	$\neg q \wedge (p \rightarrow q) \Rightarrow \neg p$	I_{12}	$(p \rightarrow q) \wedge (q \rightarrow r) \Rightarrow p \rightarrow r$
I_{13}	$(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r) \Rightarrow r$		

$$A||B = \left\langle Q^A \times Q^B, \Sigma^A \cup \Sigma^B, \delta, \langle q_i^A, q_i^B \rangle, Q_m^A \times Q_m^B, (Q_x^A \times Q^B) \cup (Q^A \times Q_x^B) \right\rangle$$

$$\delta(\langle q^A, q^B \rangle, \sigma) = \begin{cases} \delta^A(q^A, \sigma) \times \delta^B(q^B, \sigma) & \sigma \in \Sigma^A \cap \Sigma^B \\ \delta^A(q^A, \sigma) \times \{q^B\} & \sigma \in \Sigma^A \setminus \Sigma^B \\ \{q^A\} \times \delta^B(q^B, \sigma) & \sigma \in \Sigma^B \setminus \Sigma^A \end{cases}$$

Solution to Introduction to Discrete Event Systems Examination 081023

BL 081022

1. $(p\to q)\wedge(\neg q\vee p)\vee((p\vee q)\wedge(p\vee q\vee\neg p))\Leftrightarrow$

$((\neg p\vee q)\wedge\neg q)\vee((\neg p\vee q)\wedge p)\vee((p\vee q)\wedge(\underbrace{p\vee\neg p}_{T}\vee q))\Leftrightarrow$

$(\neg p\wedge\neg q)\vee(\underbrace{q\wedge\neg q}_{F})\vee(\underbrace{\neg p\wedge p}_{F})\vee(q\wedge p)\vee((p\vee q)\wedge T)\Leftrightarrow$

$\underbrace{\neg(p\vee q)\vee(p\vee q)}_{T}\vee(p\wedge q)\vee F\Leftrightarrow T\vee(p\wedge q)\Leftrightarrow T$

2. $A\cap B\subseteq B\cap C\Leftrightarrow A\cap B\subseteq C$?

$p_A(x)\wedge p_B(x)\to p_B(x)\wedge p_C(x)\Leftrightarrow\neg(p_A(x)\wedge p_B(x))\vee(p_B(x)\wedge p_C(x))$

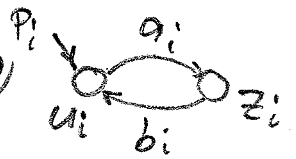
$\Leftrightarrow((\neg p_A(x)\vee\neg p_B(x))\vee p_B(x))\wedge\neg(p_A(x)\wedge p_B(x))\vee p_C(x)$

$\Leftrightarrow(\neg p_A(x)\vee\underbrace{\neg p_B(x)\vee p_B(x)}_{T})\wedge(p_A(x)\wedge p_B(x))\to p_C(x)$

$\Leftrightarrow\underbrace{(\neg p_A(x)\vee T)}_{T}\wedge(p_A(x)\wedge p_B(x))\to p_C(x)$

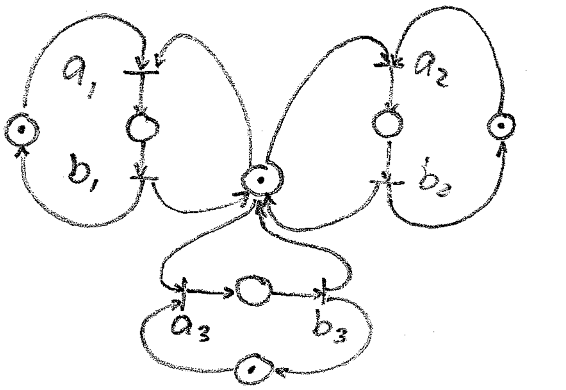
$\Leftrightarrow(p_A(x)\wedge p_B(x))\to p_C(x)$ ∴ $A\cap B\subseteq B\cap C\Leftrightarrow A\cap B\subseteq C$

3. a)

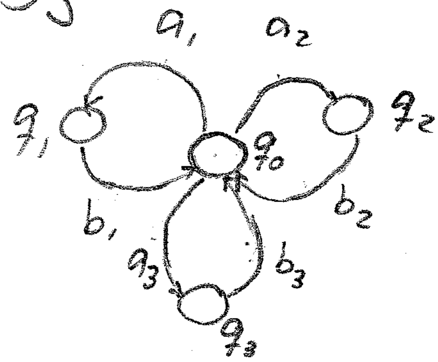


$L(P_i)=\overline{(a_ib_i)^*}$

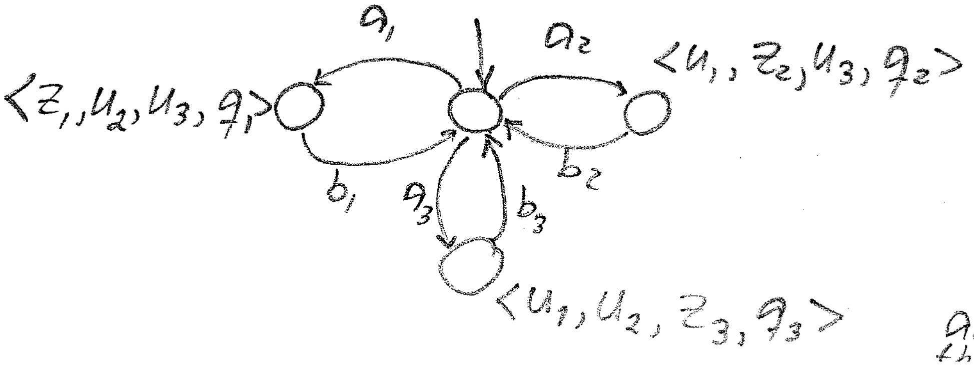
b)



c) S



d)



∴ The states

$\langle z_1,z_2,z_3,\cdot\rangle$

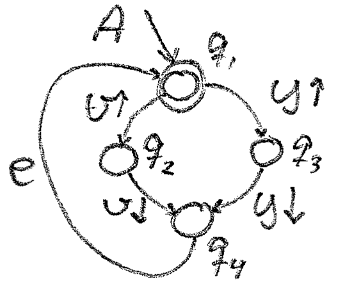
$\langle z_1,z_2,u_3,\cdot\rangle$

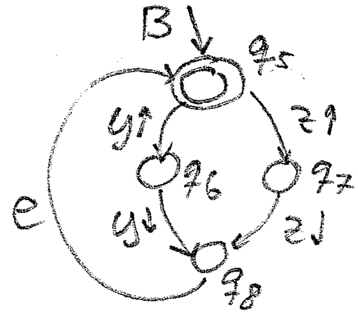
$\langle z_1,u_2,z_3,\cdot\rangle$

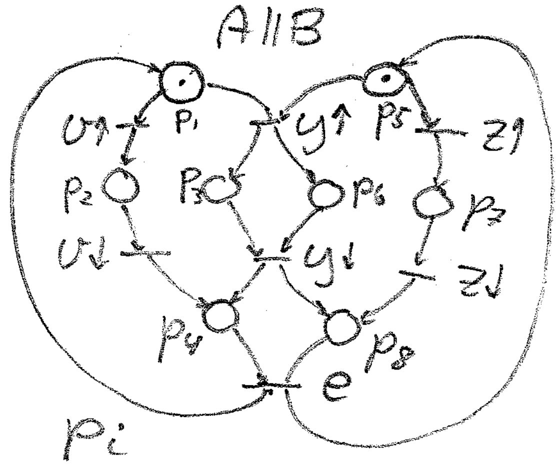
$\langle u_1,z_2,z_3,\cdot\rangle$

are not reachable in the closed loop system

4. a)







b) x_i = number of tokens i p_i

$x_1^+ = (x_4 \wedge x_8) \vee init \vee (x_1 \wedge \neg v \wedge \neg y)$

$x_2^+ = (x_1 \wedge v) \vee (x_2 \wedge v)$

$x_3^+ = (x_1 \wedge y) \vee (x_3 \wedge y)$

$x_4^+ = (x_2 \wedge \neg v) \vee (x_3 \wedge x_6 \wedge \neg y) \vee (x_4 \wedge \neg x_8)$

$x_5^+ = (x_4 \wedge x_8) \vee init \vee (x_5 \wedge \neg y \wedge \neg z)$

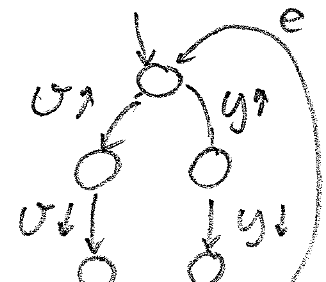
$x_6^+ = (x_5 \wedge y) \vee (x_6 \wedge y)$

$x_7^+ = (x_5 \wedge z) \vee (x_7 \wedge z)$

$x_8^+ = (x_3 \wedge x_6 \wedge \neg y) \vee (x_7 \wedge \neg z) \vee (x_8 \wedge \neg x_4)$

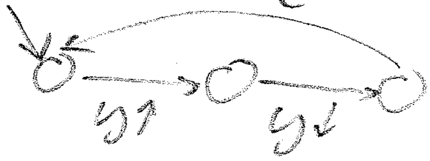
c) The local forbidden state in B is q_7
In A the states q_1, q_2 and q_4 are then reachable but not q_3
∴ The forbidden states in A||B are then $\langle q_1, q_7 \rangle$, $\langle q_2, q_7 \rangle$, $\langle q_4, q_7 \rangle$ corresponding to the marking vectors in the PN $m = [1000\,0010]^T$ $m = [0100\,0010]^T$
$m = [0001\,0010]^T$

d)



Blocking states $\langle q_2, q_5 \rangle$, $\langle q_4, q_5 \rangle$

Nonblocking system



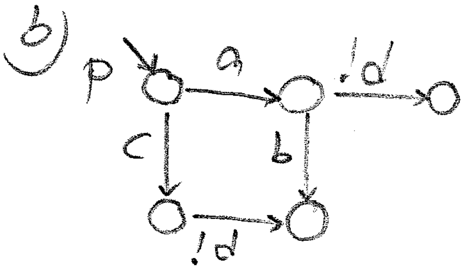
5. $L(P) = \{\varepsilon, a, ab, ad, c, cd\}$ $\Sigma_u = \{d\}$

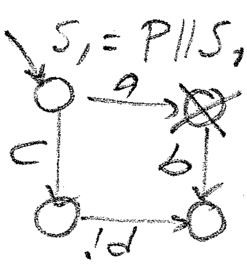
$L(S_1) = \{\varepsilon, a, ab, c, cd\}$ $L(S_2) = \{\varepsilon, c, cd\}$

a) $L(S_1)\Sigma_u \cap L(P) = \{\varepsilon, a, ab, c, cd\}\{d\} \cap \{\varepsilon, a, ab, ad, c, cd\} = \{d, ad, abd, cd, cdd\} \cap \{\varepsilon, a, ab, ad, c, cd\}$
$= \{ad, cd\} \not\subseteq \{\varepsilon, a, ab, c, cd\} = L(S_1)$

$L(S_2)\Sigma_u \cap L(P) = \{\varepsilon, c, cd\}\{d\} \cap L(P) =$
$= \{d, cd, cdd\} \cap L(P) = \{cd\} \subseteq L(S_2)$

b)





uncontrollable state since the plant can execute the uncontrollable event d which S_1 cannot

6.

a_i = take i stick(s)
b_i = take i stick(s) (uncontrollable