Examination
SSY130 Applied Signal Processing

14:00-18:00, April 7, 2016

Instructions

- *Responsible teacher:* Ayca Ozcelikkale, ph 1724. Teacher will visit the site of examination at approximately 14:45 and 16:30.
- Score from the written examination will together with course score determine the final grade according to the Course PM.
- Solutions are published on the course home-page latest on April 11
- Your preliminary grade is reported to you via email.
- Exam grading review will be held between 12:00 and 13:00 on April 26 in room 7434.

Allowed aids at exam:

- L. Råde and B. Westergren, Mathematics Handbook (any edition, including the old editions called Beta or copied sections from it), Formulaires et tables Mathématiques, Physique, Chimie, or similar.
- Any calculator
- One a4 size single sheet of paper with written notes on both sides.

Other important issues:

- The exam consists of 5 numbered problems.
- The ordering of the questions is arbitrary.
- All solutions should be well motivated and clearly presented in order to render a full score. Unclear presentation or adding, for the problem in questing, irrelevant information render a reduction of the score.
- Write solutions to each problem on a *separate* sheet of paper.
- The maximum score is 52 points.

Problems

1. The Discrete Fourier Transform (DFT) is a commonly used tool in signal processing algorithms. For a signal $x(n)$ of length N, the DFT is defined as

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, \ldots, N-1. \tag{1}$$

The Radix-2 Fast Fourier Transform (FFT) is a computationally efficient procedure to calculate the DFT. Derive the Radix-2 FFT algorithm for the case when $N = 2^p$ for a positive integer p and compare the complexity between the Radix-2 FFT algorithm and a direct calculation of the DFT according to equation (1) with respect to the number of multiplications needed. (12pt)

2. It is desired to sample and store a continuous real-valued signal $s(t)$ to enable an offline analysis of the properties of the signal. It is known that $s(t)$ is bandlimited and has a constant signal power between 0 kHz and 4 kHz. Due to noise in the environment the measurements of $s(t)$ is contaminated with a real-valued noise signal $e(t)$ that has a constant power spectral density between 0 kHz and 10 kHz and zero above. Within the 0 to 4 kHz frequency band the signal to noise ratio (SNR) is -50 dB.

   (a) Describe a signal processing solution which samples the signal $x(t) = s(t) + e(t)$ using as low sample frequency as possible while preserving the information in $s(t)$. Explain how the signal to noise ratio of the sampled signal is influenced by the specifications of the anti-aliasing filter. (5pt)

   (b) Describe a signal processing solution which samples the signal $x(t) = s(t) + e(t)$ using 4 times oversampling followed by a 4 times decimation stage while preserving the information in $s(t)$. Explain how the signal to noise ratio of the sampled signal is influenced by the choice (specifications) of both the digital and analog anti-aliasing filters used. (5pt)

   (c) Compare the two solutions by discussing the advantages and disadvantages with both approaches when they have comparable performance in terms of SNR in the sampled signal. (5pt)

3. Explain the difference between circular convolution and linear convolution and give examples where each method is applicable. (5pt)

4. Consider the adaptive filtering problem

$$\hat{x}(n) = \mathbf{h}(n)^T \mathbf{y}(n)$$
$$e(n) = x(n) - \hat{x}(n)$$

The recursive least-squares algorithm (RLS) can be formulated to minimize (with respect to $\mathbf{h}(n)$) the following functional

$$L = \sum_{k=0}^{n} \alpha^{n-k} e(k)^2$$

   (a) What is the name of the parameter α? (2pt)

   (b) In what numerical range should α be selected to make the algorithm meaningful? (2pt)

   (c) Two RLS algorithms are run on the same input data from a time-invariant scenario and with the same length of $\mathbf{h}(n)$. Both algorithms are started at sample $n = 0$. The only difference is the parameter α. Figure 1 shows the histogram for $e(n)$ calculated from samples $9000 \leq n < 10000$. Which algorithm has the largest α? (3pt)

   (d) Give an example of a case where $\alpha = 1$ is the right choice. (3pt)

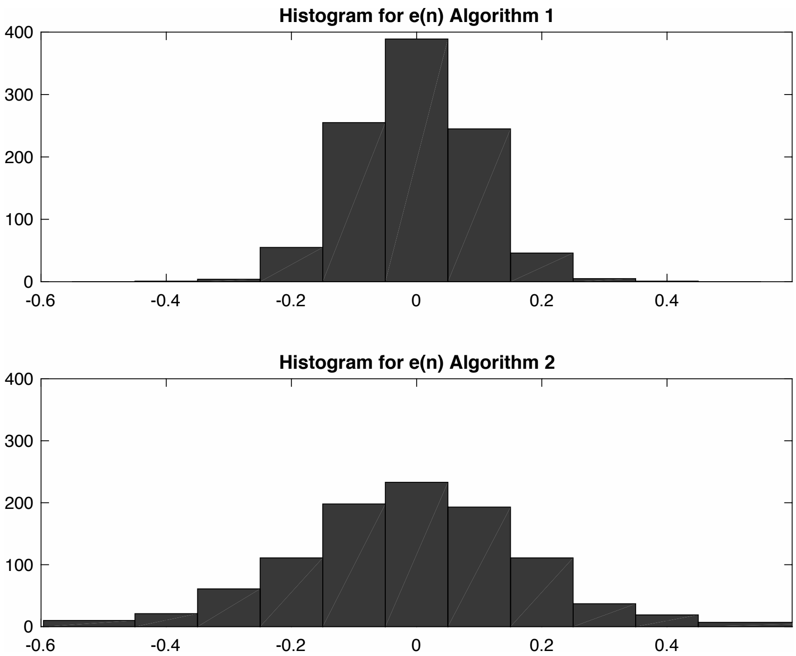

Figure 1: Histogram for the error signal for two different RLS filters

5. The goal of this problem is to recover a signal from noise using a FIR Wiener filter. Assume the desired signal $x(n)$ is described as

$$x(n) - 0.8x(n-1) = e(n)$$

where $e(n)$ is zero-mean white noise with variance $\sigma_e^2 = 1$. The measured signal $y(n)$ is given by

$$y(n) = x(n) - 0.5x(n-1) + v(n)$$

where $v(n)$ is zero-mean white noise with variance $\sigma_v^2 = 0.1$. The signals $v(n)$ and $e(n)$ are independent. Determine a filter

$$\hat{x}(n) = w_0 y(n) + w_1 y(n-1)$$

such that $E\left[(x(n) - \hat{x}(n))^2\right]$ is minimized! (10p)

END - Good Luck!