Examination
SSY130 Applied Signal Processing

14:00-18:00, January 13, 2016

Instructions

- *Responsible teacher:* Tomas McKelvey, ph 8061. Teacher will visit the site of examination at approximately 14:45 and 16:30.
- Score from the written examination will together with course score determine the final grade according to the Course PM.
- Solutions are published on the course home-page latest on January 14
- Your preliminary grade is reported to you via email.
- Exam grading review will be held between 12 and 13 on January 27 in room 7430 Landahlsrummet.

Allowed aids at exam:

- L. Råde and B. Westergren, Mathematics Handbook (any edition, including the old editions called Beta or copied sections from it), Formulaires et tables Mathématiques, Physique, Chimie, or similar.
- Any calculator
- One a4 size single sheet of paper with written notes on both sides.

Other important issues:

- The exam consists of 5 numbered problems.
- The ordering of the questions is arbitrary.
- All solutions should be well motivated and clearly presented in order to render a full score. Unclear presentation or adding, for the problem in questing, irrelevant information render a reduction of the score.
- Write solutions to each problem on a *separate* sheet of paper.
- The maximum score is 52 points.

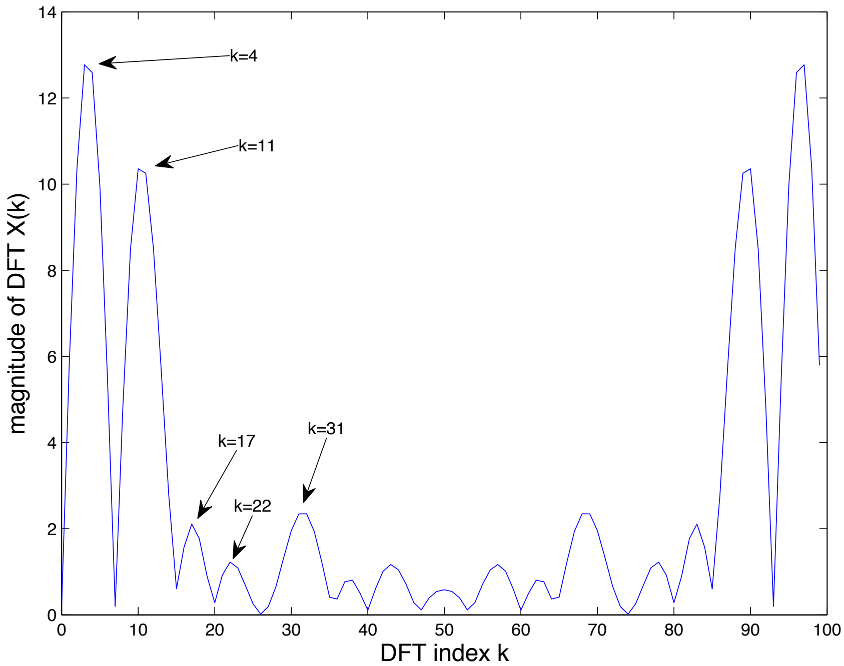

Figure 1: Magnitude of DFT.

Problems

1. A signal is known to contain 3 real valued sinusoids and is sampled at the rate of 12 kHz resulting in a total of 20 signal samples. Two of the sinusoids have equal amplitude and are large while the third sinusoid has an amplitude which is 5 times smaller. Figure 1 shows the magnitude of the DFT of the zero padded sampled signal with a total length of 100 samples. The locations of the 5 largest peaks are indicated in the graph. The location is given as the value of the DFT index k.

   (a) Assume all sinusoids have a frequency less than half the sampling frequency. Give an estimate of the frequencies (in Hz) of the 3 sinusoids. (4pt)

   (b) Explain why the magnitude of the DFT shown in Figure 1 have more than 3 peaks. (2pt)

   Solution:

   (a) Since all signals are real valued and have frequencies below the Nyquist frequency the peak locations below relative frequency 0.5 will give a good estimate of the freqency. Since two signals are larger than the third we expect to see two dominant peaks and one peak which is 5 times lower then the dominant ones. Peaks at locations $k = 4$, 11 and 31 är then suitable candidates. The frequencies are given as the index divided with the number of total signal samples including the zero-padding multiplied with the sampling frequency (here 100). Estimated frequencies are then $4/100 * 12$ kHz=0.48 kHz,$11/100 * 12$ kHz=1.3231 and 31/100*12 kHz=3.72 kHz.

   (b) Since the signal is real valued the DFT will also have three peaks corresponding to the negative frequencies. In the graph these are located above the Nyquist frequencies (since the DFT is periodic). The other peaks are the results from the sidelobes of the rectangular window. Clearly if the level of the third sinusoid would be even lower it would be difficult to make a correct estimation from the graph.

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2. Consider the following linear *time varying* system given by

$$y(n) = -a(n)y(n-1) + bx(n)$$

where $y(n)$ is the noise free output of the system and $x(n)$ is the input signal. The output is measured by a sensor which yields the measured signal

$$y_m(n) = y(n) + e(n)$$

where $e(n)$ is the measurement noise which has zero mean and is independent of $x(n)$. Given that the input signal $x(n)$ is known we want to track the time varying parameter $a(n)$ based on the measured signal $y_m(n)$.

   (a) Assume first that $a(n) = a_1$ for all n, i.e. there is no time variation and that $e(n) = 0$. Show that, (for specific values of α_0 and α_1) a FIR filter with structure

   $$\hat{x}(n) = \alpha_0 y_m(n) + \alpha_1 y_m(n-1)$$

   yields $\hat{x}(n) = x(n)$. (5pt)

   (b) Suggest an adaptive filter solution which will provide a method to track the time variability of the IIR filter coefficient $a(n)$. (7pt)

   (c) Assume two scenarios:

      i. $a(n) = 0.85 - 0.1\cos(0.01\pi n)$
      ii. $a(n) = 0.85 - 0.1\cos(0.001\pi n)$

   In which one of the two scenarios will the adaptive filter have the possibility to provide a more accurate tracking estimate. (3pt)

3. Assume the MATLAB vectors `x` and `h` are real valued and the length of the vector `x` is an even number. Consider the following MATLAB code:

```
Nx = length(x);
Nh = length(h);
xx = x(1:Nx/2)+i*x(Nx/2+1:Nx);
Nyy = Nx/2+Nh-1;
yy = ifft(fft(xx,Nyy).*fft(h,Nyy),Nyy);
y = [real(yy); zeros(Nx/2,1)] + [zeros(Nx/2,1); imag(yy)];
```

Explain what the code calculates. Recall that `i` $= \sqrt{-1}$ (10pt)

4. Consider a deterministic Wiener filter problem where the filtered output is given by

$$\hat{x}(n) = \sum_{k=0}^{2} h(k)y(n-k)$$

and we seek the filter coefficients $h(k)$, $k = 0, 1, 2$ which minimize

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} (x(n) - \hat{x}(n))^2.$$

The input $y(n)$ is given by

$$y(n) = \cos(\frac{\pi}{2}n).$$

Let $r_{yx}(k)$ denote the sample based cross-correlation limit according to

$$r_{yx}(k) \triangleq \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} y(n)x(n+k).$$

The relation between $y(n)$ and $x(n)$ is captured by the sample based cross-correlations with numerical values as $r_{yx}(0) = -1$, $r_{yx}(1) = 2$ and $r_{yx}(2) = 1$.

(a) Show that the sample based auto-correlation matrix R_{yy} is given by

$$R_{yy} \triangleq \begin{bmatrix} r_{yy}(0) & r_{yy}(1) & r_{yy}(2) \\ r_{yy}(1) & r_{yy}(0) & r_{yy}(1) \\ r_{yy}(2) & r_{yy}(1) & r_{yy}(0) \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & -1/2 \\ 0 & 1/2 & 0 \\ -1/2 & 0 & 1/2 \end{bmatrix}$$

(5pt)

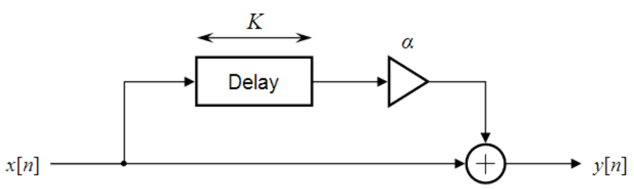
(b) Consider the three filters given below. Which ones of them are optimal?

	A	B	C
$h(0)$	-2	0	-3
$h(1)$	4	4	4
$h(2)$	0	-2	-1

(5pt)

(c) Derive an explicit expression for all optimal solutions. (5pt)

5. The comb filter is a filter with the general structure as illustrated below



which means that the filtered output is the present input added with the input delayed K samples, where K is a positive integer, and scaled with α.

(a) Derive the frequency function of the comb filter (3pt)

(b) Assume $\alpha = -1$. At which frequencies will the frequency function have magnitude zero? *Hint:* Recall the complex solutions to the algebraic equation $x^K = 1$ (3pt)

END - Good Luck!