Examination
SSY130 Applied Signal Processing

14:00-18:00, August 25, 2015

Instructions

- *Responsible teacher:* Tomas McKelvey, ph 8061. Teacher will visit the site of examination at approximately 15:00.

- Score from the written examination will together with course score determine the final grade according to the Course PM.

- Your preliminary grade is reported to you via email.

- Exam grading review will be arranged upon request by email to the examiner. Requests should be made within 2 weeks after preliminary grade is announced.

Allowed aids at exam:

- L. Råde and B. Westergren, Mathematics Handbook (any edition, including the old editions called Beta or copied sections from it), Formulaires et tables Mathématiques, Physique, Chimie, or similar.

- Any calculator

- One a4 size single sheet of paper with written notes on both sides.

Other important issues:

- The exam consists of 5 numbered problems.

- The ordering of the questions is arbitrary.

- All solutions should be well motivated and clearly presented in order to render a full score. Unclear presentation or adding, for the problem in questing, irrelevant information render a reduction of the score.

- Write solutions to each problem on a *separate* sheet of paper.
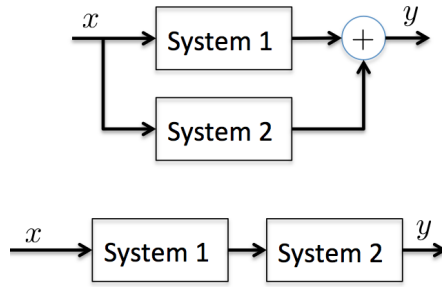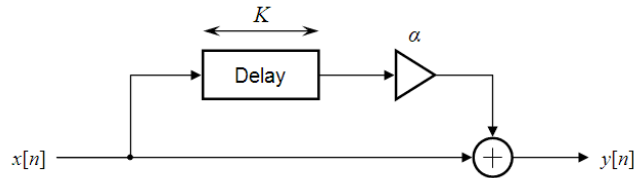
- The maximum score is 52 points.

Figure 1: Block diagram of parallell and serial filter connections.

Problems

1. The comb filter is a filter with the general structure as illustrated below

   which means that the filtered output is the present input added with the input delayed K samples, where K is a positive integer, and scaled with α.

   (a) Explain why the filter is particularly inexpensive to implement if $\alpha = \pm 1$ (2pt)

   (b) Derive the frequency function for the comb filter   (3pt)

   (c) Which choice $\alpha = +1$ or $\alpha = -1$ would yield a filter where the lowest frequencies will pass through the filter?   (2pt)

   (d) Assume $\alpha = -1$. At which frequencies will the frequency function have magnitude zero? *Hint:* Recall the complex solutions to the algebraic equation $x^K = 1$ (3pt)

2. (a) Design a causal, linear-phase, low pass FIR filter of length $M = 5$ with a cut-off frequency at 2 kHz using the window method. Use a sine squared window function $(w(n) = \sin^2(\frac{(n+1)\pi}{M+1}), \quad n = 0, 1 \ldots, M-1)$ and make sure the DC-level of the frequency response have a gain of 1 (unity). The algorithm runs on a unit with the sample rate of 10 kHz.   (5pt)

   (b) A possible approach to FIR filter design would be to simply use the inverse DFT of the amplitude specification. This would lead to a FIR filter which exactly satisfies the amplitude specification at the frequencies corresponding to the DFT frequency grid. Explain why this in general is not a good design method.   (5pt)

3. Consider the block diagrams in Figure 1. The blocks marked System 1 and Systm 2 are FIR filters. System 1 and System 2 have the impulse responses

$$h_1(k) = \begin{cases} 1, & k = 0 \\ 2, & k = 1 \\ 3, & k = 2 \\ 4, & k = 3 \\ 0, & \text{otherwise} \end{cases} \qquad h_2(k) = \begin{cases} 1, & k = 0 \\ 2, & k = 1 \\ 4, & k = 2 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Derive the impulse response of the equivalent filter for the upper blockdiagram in Figure 1, i.e. the filter which satisfy $y(n) = h(n) * x(n)$ (3pt)

(b) Derive the impulse response of the equivalent filter for the lower filter in Figure 1, i.e. the filter which satisfy $y(n) = h(n) * x(n)$ (3pt)

(c) Derive the impulse response of the equivalent filter for the lower filter in Figure 1, i.e. the filter which satisfy $y(n) = h(n) * x(n)$ when the blocks System 1 and System 2 have changed places. (4pt)

4. Assume we measure the signal
$$y(n) = s(n) + w(n)$$
where $s(n)$ is the signal we want to recover with autocorrelation function
$$\phi_{ss}(n) = (-1)^{|n|}, \quad n = 0, \pm 1, \ldots, \pm\infty$$
and $w(n)$ is a zero mean noise signal which is uncorrelated with $s(n)$ and has autocorrelation function
$$\phi_{ww}(n) = \sigma_w^2 e^{-|n|}, \quad n = 0, \pm 1, \ldots, \pm\infty.$$
Define $\hat{s}(n)$ as the output we obtain from filtering $y(n)$ using a causal FIR filter of length 2
$$\hat{s}(n) = h_0 y(n) + h_1 y(n-1)$$

(a) Derive the filter coefficients which minimizes
$$\text{MSE} = \mathbf{E}(\hat{s}(n) - s(n))^2.$$

(6pt)

(b) Derive the value of the resulting optimal MSE and compare it with the MSE of the trivial filter $\hat{s}(n) = y(n)$. (6pt)

5. FFT can be used to perform linear filtering, (or equivalently convolution). Consider the following two MATLAB scripts where x and h are vectors with the input signal and impulse response of the finite impulse response filter (FIR) respectively.

Case 1

```
N = length(x);
M = length(h);
P = N+M-1;
y = ifft(fft(x,P).*fft(h,P));
```

Case 2

```
N = length(x);
M = length(h);
if N>=M
   y = ifft(fft(x,N).*fft(h,N));
else
   error('This code does not handle the case when M>N')
end
```

(a) Describe what underlying assumption regarding the properties of the input x which motivates the codes in the two cases (5pt)

(b) The code in the second case (Case 2) only handles the situation when the length of the signal vector is longer or equal to the length of the impulse response of the filter. Add MATLAB code to also correctly handle the case when the impulse response of the filter is longer than the input signal vector. (5pt)

END - Good Luck!