Examination
SSY130 Applied Signal Processing

14:00-18:00, April 16, 2015

Instructions

- *Responsible teacher:* Tomas McKelvey, ph 031-7728061. Teacher will visit the site of examination at approximately 15:00 and will thereafter be available by telephone.
- Score from the written examination will together with course determine the final grade according to the Course PM.
- Solutions are published on the course home-page latest on Monday, April 20.
- Your preliminary grade is reported to you via email.
- Exam grading review will be held in room 7434 on level 7 at 12:30-13:00 on Thursday, April 30.

Allowed aids at exam:

- L. Råde and B. Westergren, Mathematics Handbook (any edition, including the old editions called Beta or copied sections from it), Formulaires et tables Mathématiques, Physique, Chimie, or similar.
- Any calculator
- One a4 size single sheet of paper with written notes on both sides.

Other important issues:

- The exam consists of 6 numbered problems.
- The ordering of the questions is arbitrary.
- All solutions should be well motivated and clearly presented in order to render a full score. Unclear presentation or adding, for the problem in questing, irrelevant information render a reduction of the score.
- Write solutions to each problem on a *separate* sheet of paper.
- The maximum score is 52 points.

1. A signal interpolator is commonly used to increase the sample rate of a signal but still preserving the information in it. Describe the two processing blocks found in a signal interpolator. (2pt)

2. A sampled signal $x(n)$ has a DTFT of the following shape

$$|X(\omega)| = \frac{|\omega|}{\pi}, \quad |\omega| < \pi$$

   where ω is relative with respect to the sample rate (i.e. rad/sample).

   (a) Sketch the magnitude of the DTFT of $x(n)$. (1pt)

   (b) A new signal $y(n)$ is created by up-sampling $x(n)$ a factor 4. Show exactly how $y(n)$ is defined in terms of $x(n)$. (2pt)

   (c) Sketch the magnitude of the DTFT of $y(n)$. Use a frequency scale relative to the new sample rate. (3pt)

3. A continuous time real valued signal containing one sinusoidal component is sampled without any antialiasing filter and 64 samples are collected.

   (a) Assume the frequency of the sinusoidal component is less than 12.5 kHz and that the signal is sampled with a sample rate of 25 kHz. The magnitude of the 64 point FFT of the signal is illustrated in the top graph in Figure 2. The first peak appear at index 28. Derive the index of the second peak and determine the frequency (in Hz) of the sinusoidal component. (3pt)

   (b) Assume we know that the frequency of the sinusoidal component is larger than 62.5 kHz and less than 70 kHz and the sample rate is still 25 kHz. The resulting FFT is again illustrated in the top graph in Figure 2. What is the frequency of the sinusoidal component? (4pt).

   (c) Assume we know that the frequency is less than 100 kHz and the signal is sampled twice. The first 64 samples (data set A) are collected with a sample rate of 25 kHz and the second batch of 64 samples (data set B) are collected with 30 kHz sample rate. Top graph in Figure 2 corresponds to the sample rate of 25 kHz and the bottom graph corresponds to the sample rate of 30 kHz. In the bottom graph the first peak is located at index 13. What is the frequency of the sinusoidal signal? (5pt)

4. A digital signal processing filter specification has the following requirements:

   - sample rate 2 kHz
   - 3 dB maximum passband ripple
   - low pass filter
   - pass-band edge frequency 100 Hz
   - a minimum of 40 dB stop-band attenuation
   - stop-band edge frequency at 400 Hz
   - gain of 1 at frequency 0 (DC-level)

   In Table 1 you find tabulated values of the amplitude of a digital low pass filter with monotonous amplitude curve.

   (a) Which of the specifications above does the filter in Table 1 meet? (4p)

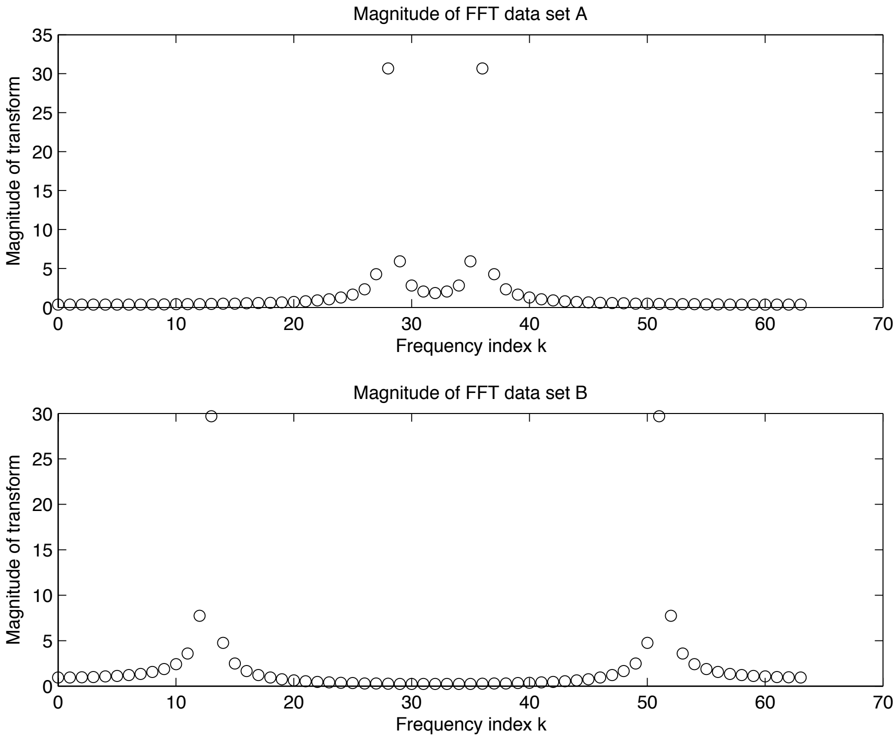

Figure 1: 64 point FFT for problem 1

(b) Design a first order digital Butterworth filter such that when connected in series with the given filter in Table 1 that all the specifications above are met. (6p)
Hint: A low-pass analog Butterworth prototype filter of order 1 has the Laplace transform:

$$H_{\mathrm{LP}}(s) = \frac{1}{s+1}$$

the Bilinear-transform is

$$s = 2\frac{(z-1)}{z+1}$$

and the frequency warping function is

$$\Omega = 2\tan(\omega/2)$$

where Ω is the analog filter frequency in rad/s and ω is the digital filter frequency in rad/sample (i.e. normalized w.r.t. sample frequency).

5. An engineer is monitoring a slowly varying physical value by sampling it. He notices a large variation between consecutive samples and tries to remove the variation by caluclating a running average

$$\bar{x}(n) = \frac{1}{N}\sum_{l=n}^{n-N+1} x(l).$$

where $x(n)$ is the sampled signal and $\bar{x}(n)$ is the running average. To test how the running averaging works he uses a sinusodal signal as a test input and records the magnitude of the running average. The magnitude as a function of the frequency of the input signal is shown in Figure 2. The unit on the frequency axis is normalized with respect to the sampling frequency.

f [Hz]	Amplitude
0	1
100	1
200	0.7
300	0.3
400	0.09
500	0.03
600	0.01
700	4×10^{-3}
800	1×10^{-3}
900	0.1×10^{-3}
1000	0

Table 1: Amplitude response of digital filter.

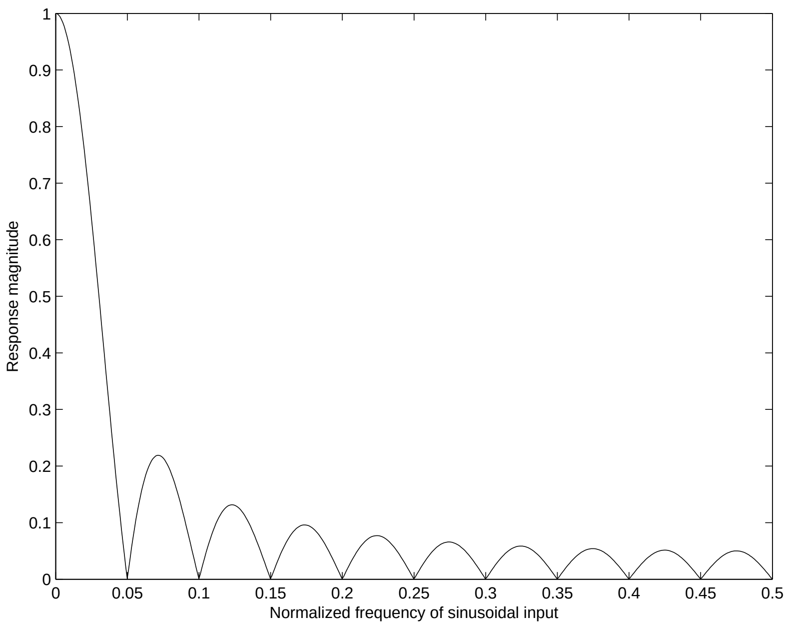

Figure 2: Magnitude of running average signal as a function of frequency.

(a) Derive an analytical expression for the graph in Figure 2. (5pt)

(b) Suggest a length N and sampling frequency such that a 50 Hz disurbance is completely cancelled by the averaging operation. Is the solution unique? (5pt)

6. In this problem an LMS-filtering setup, similar to the one used in the second project is used. The output of the LMS filter is $\hat{x} = y * h$, (here $*$ denotes convolution) and the filtering error which is minimized by the adaptation is is $e = x - \hat{x}$.

   Consider the following scenarios and, for each scenario, derive en expression of what h will converge to and what the error signal e become after convergence. The impulse responses g and f are FIR filters and the signals s, w, w_1, w_2 are independent of each other and each of them are zero mean random processes with a constant power spectrum (a white noise signal). Assume that the filter is tuned to converge to the optimal filter and the length of the LMS-filter h is at least as long as the length of $f * g$.

   (a) $y = w$ and $x = s + 2w * g$ (3pt)

   (b) $y = w_1$ and $x = g * (s + w_1 + w_2)$ (3pt)

   (c) $y = w$, $x = g * (s + f * w)$ (3pt)

   (d) $s_1 = w_2 + f * w_1$, $y = w_1$ and $x = g * (s_1 + w_1)$ (3pt)

END - Good Luck!