Examination
SSY130 Applied Signal Processing

14:00-18:00, August 21, 2008

Instructions

- *Responsible teacher:* Ingemar Andersson, ph 1784. Teacher will visit the site of examination at 15 and 17.
- Score from the written examination will together with course score determine the final grade according to the Course PM.
- Solutions are published on the course home-page latest 12 noon August 22.
- Preliminary grade is reported to you via email latest 21 days after the exam.
- Exam review will be held in the "Blue Room" on level 6 at 12:30-13:00 on September 11.

Allowed aids at exam:

- L. Råde and B. Westergren, Mathematics Handbook (any edition, including the old editions called Beta).
- Any calculator
- One a4 size single page with written notes

Other issues:

- All solutions should be well motivated in order to render a full score.
- The maximum score is 52 points.

Problems

1. In an audio studio the producer wants to mix a sound track from a CD disc sampled at 44.1 kHz with a sound track sampled at 48 kHz. The mix should then be stored on a Digital Audio Tape (DAT) with a rate of 48 kHz. Suggest a signal processing architecture which solves the mixing task and briefly discuss the purpose of the used components and how the parameters of them should be selected to balance the complexity of the processing and the achieved quality. (5pt)

2. A block of data of length N is to be processed by an FIR filter of length M.
   (a) Directly solving the convolution sum requires multiplications and summations. How many multiplications are needed to filter the data bock? (1pt)
   (b) Suggest an alternative method which solves the filtering problem with fewer multiplications. How many multiplications are required for the alternative method. (4pt)

3. Consider an LMS filtering problem where the filtering equations are the in the standard form:

$$\hat{d}(n) = \mathbf{h}(n)^T \mathbf{x}(n)$$
$$e(n) = d(n) - \hat{d}(n)$$
$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mu \mathbf{x}(n) e(n)$$

where

$$\mathbf{x}(n) = \begin{bmatrix} x(n) & x(n-1) & x(n-2) & \cdots & x(n-M+1) \end{bmatrix}^T$$
$$\mathbf{h}(n) = \begin{bmatrix} h_0(n) & h_1(n) & h_2(n) & \cdots & h_{M-1}(n) \end{bmatrix}^T$$

and we assume $x(n)$ and $d(n)$ to be stationary ergodic stochastic processes. For analysis of the LMS algorithm the following matrix and vector play key roles

$$\mathbf{\Gamma_x} = \mathbf{E}\left[\mathbf{x}(n)\mathbf{x}(n)^T\right], \quad \boldsymbol{\gamma}_{d\mathbf{x}} = \mathbf{E}\left[d(n)\mathbf{x}(n)\right]$$

   (a) If non-divergent, the LMS algorithm will make $\mathbf{h}(n)$ approach the coefficients of the optimal FIR Wiener Filter as n increases. Derive the explicit expression for coefficients of the optimal filter. (5pt)
   (b) In a specific application the signals $x(n)$ and $d(n)$ are generated as

$$x(n) = 2w(n) + w(n-1)$$
$$d(n) = x(n) + x(n-1) + 0.2v(n)$$

   where $w(n)$ and $v(n)$ zero mean white noise stochastic processes with unit variance and independent of each other. Derive numerical expressions for $\mathbf{\Gamma_x}$ and $\boldsymbol{\gamma}_{d\mathbf{x}}$ (5pt)
   (c) With some assumptions a convergence analysis of the LMS algorithm leads to the equation

$$\bar{\mathbf{h}}(n) - \mathbf{h}_0 = (\mathbf{I} - \mu\mathbf{\Gamma_x})^n(\bar{\mathbf{h}}(0) - \mathbf{h}_0)$$

   where $\mathbf{I}$ is the identity matrix, $\bar{\mathbf{h}}(n)$ is the mean value filter coefficients at sample time n and $\mathbf{h}_0$ is the optimal Wiener Filter. Hence, $\bar{\mathbf{h}}(n) - \mathbf{h}_0$ is the estimation error which should be driven towards zero. Show that

$$\|\bar{\mathbf{h}}(n) - \mathbf{h}_0\|^2 = \sum_{k=0}^{M}(1-\mu\lambda_k)^{2n}(\mathbf{u}_k^T(\bar{\mathbf{h}}(0) - \mathbf{h}_0))^2$$

   where $\mathbf{\Gamma_x} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^T$ is the eigenvalue decomposition and

$$\mathbf{U} = \begin{bmatrix} \mathbf{u}_o & \mathbf{u}_1 & \cdots & \mathbf{u}_{M-1} \end{bmatrix}$$

   is an orthonormal matrix ($\mathbf{U}^T\mathbf{U} = \mathbf{I}$) and $\mathbf{\Lambda}$ is a diagonal matrix with the eigenvalues λ_k, $k = 0, \ldots, M-1$ on the diagonal. (7pt)

(d) Use the result above and derive a bound on the step length μ which guarantees convergence (in the mean value analysis). (5pt)

4. Consider the adaptive filtering problem

$$\hat{d}(n) = \mathbf{h}(n)^T \mathbf{x}(n)$$
$$e(n) = d(n) - \hat{d}(n)$$

The recursive least-squares algorithm (RLS) can be formulated to minimize (with respect to $\mathbf{h}(n)$) the following functional

$$L = \sum_{k=0}^{n} \alpha^{n-k} e(k)^2$$

(a) In what numerical range should α be selected to make the algorithm meaningful? (2pt)

(b) Explain the effect of the parameter α. (3pt)

(c) Given one filtering application compare the effect of two algorithms where the only difference is the choice of α and $\alpha_1 > \alpha_2$. (2pt)

(d) In every adaptive filter application there is a fundamental trade-off between two properties. The selection of α balances this trade off. Explain the nature of the trade-off. (3pt)

5. A FIR filter with coefficients satisfying $h(k) = h(M-1-k)$, $k = 0, \ldots, M-1$ is called *symmetric*. A *linear phase* filter has the frequency function

$$H(\omega) = H_r(\omega) e^{-j\tau\omega}$$

where $H_r(\omega)$ is a real valued function and τ is a real scalar.

(a) Assume M is odd and show that a symmetric impulse response imply a linear phase frequency function. (5pt)

(b) Explain the benefit of linear phase filters (3pt)

(c) What is the delay of a causal symmetric FIR filter with M coefficients and assume M is odd? (2pt)

END

Good Luck!