Solution suggestions to examination SSY130 Applied Signal Processing

14:00-18:00, August 21, 2008

1. *Solution:* Sample rate conversion is achieved by combining interpolation and decimation. The ratio between the rates is $48/44.1 = 160/147$ which means a 160 fold interpolation followed by an 147 fold decimation is the correct rates. The interpolator is composed by an 160 fold upsampler and a low pass filter which removes the images created in the upsampling process. This filter should be of low pass character with a band width of 22.05 kHz. The decimator step needs no anti aliasing filter as the upsampled data is limited to 22.05 kHz bandwidth. To save computations the low pass filter can be implemented using the polyphase structure which means that the filtering takes place at the 44.1 kHz rate. In order to limit the requirements on the filter specifications is advantageous to do the rate conversions by cascading several rate changing blocks, i.e. 10:7, then 8:7 then 2:3. □

2. *Solution:*

   (a) To produce one output sample M multiplications are required. Hence for a block of length N a total of NM multiplications are needed.

   (b) Filtering with FFT can be applied. First the length of the FFT is determined (N_{FFT}) as the number which is a power of 2 such that $N_{\text{FFT}} >= N + M - 2$. Zero pad the input and the filter to the FFT length and calculate the FFT of both signals. Take the product between them and then apply the inverse FFT to calculate the output of the filter. In each FFT we then need in the order of $\log_2 N_{\text{FFT}} N_{\text{FFT}}$ and N_{FFT} multiplications.

   □

3. *Solution:*

   (a) The optimal FIR Winer filter is the filter which minimizes the variance of the filtering error $e(n)$. Forming the variance we obtain

   $$\mathbf{E}[e(n)^2] = \mathbf{E}[d(n)^2] - 2\mathbf{h}^T\boldsymbol{\gamma}_{d\mathbf{x}} + \mathbf{h}^T\boldsymbol{\Gamma}_{\mathbf{x}}\mathbf{h}$$

   Differentiating with respect to $\mathbf{h}$ and set the gradient to zero we obtain solution to the minimum variance problem

   $$\mathbf{h}_0 = \boldsymbol{\Gamma}_{\mathbf{x}}^{-1}\boldsymbol{\gamma}_{d\mathbf{x}}$$

(b) From the signal relations and the noise properties we obtain

$$\begin{aligned}
\gamma_{xx}(0) &= \mathbf{E}[x(n)x(n)] = 2^2 + 1 = 5 \\
\gamma_{xx}(1) &= \mathbf{E}[x(n)x(n-1)] = 2 \\
\gamma_{xx}(k) &= \mathbf{E}[x(n)x(n-k)] = 0, \quad k > 1 \\
\gamma_{dx}(0) &= \mathbf{E}[d(n)x(n)] = 4 + 3 = 7 \\
\gamma_{dx}(1) &= \mathbf{E}[d(n)x(n-1)] = 6 + 1 = 7 \\
\gamma_{dx}(2) &= \mathbf{E}[d(n)x(n-2)] = 2 \\
\gamma_{dx}(k) &= \mathbf{E}[d(n)x(n-k)] = 0, \quad k > 2
\end{aligned}$$

which then yields the vector and matrix

$$\mathbf{\Gamma_x} = \begin{bmatrix} 5 & 2 & 0 \\ 2 & 5 & 2 \\ 0 & 2 & 5 \end{bmatrix}, \quad \gamma_{d\mathbf{x}} = \begin{bmatrix} 7 \\ 7 \\ 2 \end{bmatrix}$$

(c) Define $\tilde{\mathbf{h}}(n) \triangleq \bar{\mathbf{h}}(n) - \mathbf{h}_0$.
Then

$$\begin{aligned}
\|\tilde{\mathbf{h}}(n)\|^2 &= \tilde{\mathbf{h}}(n)^T\tilde{\mathbf{h}}(n) = \tilde{\mathbf{h}}(0)^T(\mathbf{I} - \mu\mathbf{\Gamma})^{2n}\tilde{\mathbf{h}}(0) = \tilde{\mathbf{h}}(0)^T(\mathbf{I} - \mu\mathbf{U\Lambda U}^T)^{2n}\tilde{\mathbf{h}}(0) \\
&= \tilde{\mathbf{h}}(0)^T\mathbf{U}(\mathbf{I} - \mu\mathbf{\Lambda})^{2n}\mathbf{U}^T\tilde{\mathbf{h}}(0) \\
&= \begin{bmatrix} \tilde{\mathbf{h}}(0)^T\mathbf{u}_0 & \cdots & \tilde{\mathbf{h}}(0)^T\mathbf{u}_{M-1} \end{bmatrix} (\mathbf{I} - \mathbf{\Lambda})^{2n} \begin{bmatrix} \mathbf{u}_0^T\tilde{\mathbf{h}}(0) \\ \vdots \\ \mathbf{u}_{M-1}^T\tilde{\mathbf{h}}(0) \end{bmatrix} \\
&= \sum_{k=0}^{M-1}(1 - \mu\lambda_k)^{2n}(\mathbf{u}_k^T\tilde{\mathbf{h}}(0))^2
\end{aligned}$$

(d) To guarantee convergence then each factor $(1 - \mu\lambda_k)$ must have a magnitude less than 1. Since $\mathbf{\Gamma_x}$ is a symmetric positive definite matrix then all $\lambda_k > 0$. Hence the lower bound on μ is 0. The upper bound is then determined by the maximum eigenvalue and we need $\mu\lambda_{\max} < 2$ which yields $0 < \mu < 2/\lambda_{\max}$

□

4. *Solution:*

(a) α weights the criterion function and thus should be a positive number. If α is larger than 1 then the criterion would be dominated by the data in the far past which is not desired in an adaptive algorithm.

(b) The parameter α control the influence of the past errors. If α is 1 then all historical data has equal weight. If α is decreased more and more emphasis is on the recent data.

(c) Adaptive algorithms are used to track a changing environment. If the change is fast only a short history of the data should be used by the algorithm. However, the algorithm makes an estimate of the parameters of a filter based on the measurements and measurements always contain noise which will result in estimation errors and these errors are averaged out if a long data set is used. The estimation error of course leads to a degradation of the filtering performance. Hence in each tracking application the trade-off is between tracking performance and estimation performance and is controlled by the value of α in the RLS algorithm

□

5. *Solution:*

(a) The frequency function is given by

$$
\begin{aligned}
H(\omega) &= \sum_{k=0}^{M-1} h(k)e^{-j\omega k} \\
&= e^{-j\omega\frac{M-1}{2}} \sum_{k=0}^{M-1} h(k)e^{-j\omega k}e^{j\omega\frac{M-1}{2}} \\
&= e^{-j\omega\frac{M-1}{2}} \sum_{s=\frac{M-1}{2}}^{\frac{M-1}{2}} h(k)e^{-j\omega(k-\frac{M-1}{2})} \\
&= e^{-j\omega\frac{M-1}{2}} \sum_{s=-\frac{M-1}{2}}^{\frac{M-1}{2}} h(s+\frac{M-1}{2})e^{-j\omega s} \\
&= e^{-j\omega\frac{M-1}{2}} \sum_{s=-\frac{M-1}{2}}^{\frac{M-1}{2}} h(s+\frac{M-1}{2})\left(\cos(-\omega s)+j\sin(-\omega s)\right) \\
&== e^{-j\omega\frac{M-1}{2}} \sum_{s=-\frac{M-1}{2}}^{\frac{M-1}{2}} h(s+\frac{M-1}{2})\cos(-\omega s)
\end{aligned}
$$

where the last equality follows from the symmetry of $h(k)$.

(b) As illustrated in the previous sub problem a linear phase filter will delay all frequency components with an equal amount of samples. This means that the pulse shape of a signal is preserved which in some applications is desirable.

(c) From the derivation in (a) is follows that the delay is $(M-1)/2$ samples.

□

END

Good Luck!