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Exam in the course Antenna Engineering 2015-06-05

ANTENNA ENGINEERING (SSY100)
(E4) 2014/15 (Period IV)

Friday 5 June 1400-1800 hours.

Teachers: Prof. Per-Simon Kildal, Associate Prof. Jian Yang, Assistant Prof. Rob Maaskant, Assistant Prof. Ashraf Uz Zaman, Assistant Prof. Andres Alayon Glazunov.

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The exam consists of 2 parts. Part A is printed on colored paper and must be solved without using the textbook. When you have delivered the colored text and the solutions of Part A (latest 17:00), the textbook can be used for Part B of the exam.

You are allowed to use the following:
For Part A: **Pocket calculator of your own choice**
For Part B only: **Mathematical tables including Beta**
Pocket calculator of your own choice
Kildal's compendium "Foundations of Antennas: A Unified Approach for LOS and Multipath"
(The textbook can contain own notes and marks on its original printed pages. No other notes are allowed.)

Tentamen består av 2 delar. Del A har tryckts på färgade papper och skall lösas utan att använda läroboken. När du har inlämnat dom färgade arken med uppgifterna för del A och dina svar på dessa uppgifter (senast 17:00), kan du ta fram läroboken för att lösa del B.

Tillåtna hjälpmedel:
För del A: Valfri räknedosa
För del B: Matematiska Tabeller inkluderad Beta
Valfri räknedosa
Kildals lärobok "Foundations of Antennas: A Unified
Approach for LOS and Multipath"

(Boken kan innehålla egna noteringar skrivna på de inbundna sidorna. Extra ark med noteringar tillåts inte.)

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PART A (must be delivered before textbook can be used)

1.0 Foundations of Antenna Engineering (25p)

1.1. What is the phase centre of an antenna? What is the difference between a phase reference point and the phase centre? (2p)

A: The phase centre is the particular phase reference point which minimizes the phase variation of the co-polar far-field function of an antenna over a given solid angle of interest. The phase center is determined by the antenna geometry but the phase reference point can be any point, free from the antenna geometry.

1.2. Consider an antenna with a known far field function $\boldsymbol{G}(\theta, \varphi)$ and phase center location $\boldsymbol{r}_0$ in a given coordinate system, shown in Fig. 1.2a. Write the far field function of an array built up by two such antennas pointing in the same direction but with different locations. The locations are such that one has its phase centre located at $\boldsymbol{r}_1$ and the other at $\boldsymbol{r}_2$, shown in Fig1.2b. Note that the phase centre is calculated for a certain angular region $0 \le \theta \le \theta_{max}$ in a given φ-plane ($\varphi = \varphi_0$). (2p)

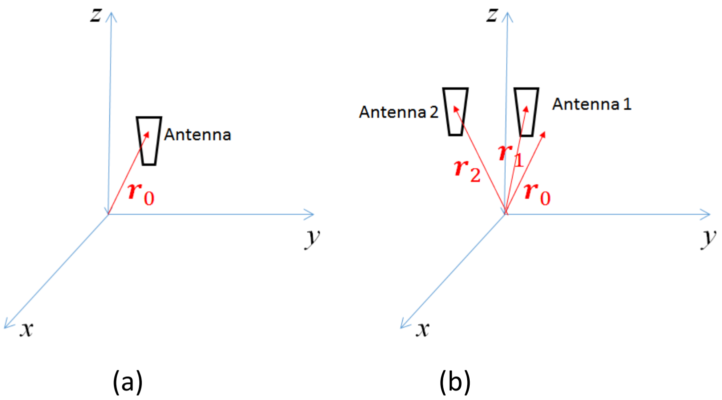

Fig. 1.2 The configuration of (a) an antenna with its phase center located at $\boldsymbol{r}_0$, and (b) the same two antennas with their phase centres located at $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$, respectively.

A:

$$\boldsymbol{G}_{array}(\theta, \varphi) = \boldsymbol{G}(\theta, \varphi) e^{jk(\boldsymbol{r}_1 - \boldsymbol{r}_0)\cdot\hat{\boldsymbol{r}}} + \boldsymbol{G}(\theta, \varphi) e^{jk(\boldsymbol{r}_2 - \boldsymbol{r}_0)\cdot\hat{\boldsymbol{r}}}$$

1.3. Which requirement do we need to put on $\boldsymbol{r}_2 - \boldsymbol{r}_1$ in order to assure that there is no grating lobes within $0 \le \theta \le \theta_{max}$. (1p)

A:
One of them is OK

i) $|\boldsymbol{r}_2 - \boldsymbol{r}_1| \le \frac{\lambda}{1+cos\theta_{max}}$

ii) $|\boldsymbol{r}_2 - \boldsymbol{r}_1| \le \lambda$

iii) $|\boldsymbol{r}_2 - \boldsymbol{r}_1| \le \lambda/2$

1.4. Please find the phase centre of the array antenna in 1.2 within the same angular region $0 \leq \theta \leq \theta_{max}$ in a given φ-plane ($\varphi = \varphi_0$). (2p)
A: With a new phase reference point $\boldsymbol{r}_{0new}$, the far field function is

$$\boldsymbol{G}_{array,new}(\theta,\varphi) = \boldsymbol{G}_0(\theta,\varphi)e^{jk(\boldsymbol{r}_1-\boldsymbol{r}_0-\boldsymbol{r}_{0\text{new}})\cdot\hat{\boldsymbol{r}}} + \boldsymbol{G}_0(\theta,\varphi)e^{jk(\boldsymbol{r}_2-\boldsymbol{r}_0-\boldsymbol{r}_{0\text{new}})\cdot\hat{\boldsymbol{r}}}$$

If we choose $\boldsymbol{r}_{0new} = \frac{\boldsymbol{r}_1-\boldsymbol{r}_2}{2} - \boldsymbol{r}_0$, we have

$$\boldsymbol{G}_{array,new}(\theta,\varphi) = \boldsymbol{G}_0(\theta,\varphi)2\cos\left[(k\frac{\boldsymbol{r}_1-\boldsymbol{r}_2}{2})\cdot\hat{\boldsymbol{r}}\right]$$

No phase error introduced. So the phase center of the array antenna is at

$$\boldsymbol{r}_{PC,array} = \boldsymbol{r}_{0new} = \frac{\boldsymbol{r}_1-\boldsymbol{r}_2}{2} - \boldsymbol{r}_0$$

1.5. Please answer for each of these antennas if they are a BOR_0, BOR_1, or non-BOR antenna. (2.5p)
a) A dipole along the z-axis;
A: BOR_0
b) An incremental dipole along the x-axis;
A: BOR_1
c) A resonant-loop helical antenna with its spiral shaped wire around z-axis (resonant loop means that one loop has a length equal to one wavelength);
A: non-BOR antenna
d) A conical horn antenna excited by a TE11 waveguide mode and located with its symmetrical axis along the *z*-axis;
A: BOR_1
e) A big circular planar array in *xy*-plane with small loop antennas as elements with element spacing of half wavelength and a uniform excitations of all elements.
A: BOR_0 because the elements (small loop antenna) are BOR_0 and the aperture is a planar circle with uniform distribution.

1.6. We can characterize an antenna system by using the figure of merit G/T_{syst}. Explain what G and T_{syst} stand for. How can you improve G/T_{syst} when you are designing an antenna system with a given diameter? You should point out at least three factors you would like to improve. (3p)
A:
G- antenna gain; Tsys – noise temperature of the system.
Improve i) antenna aperture efficiency to increase *G;* ii) reduce the omic loss (or by other words increase the radiation efficiency) to increase G and reduce T_{sys}; iii) increase spillover efficiency to reduce T_{sys}; iv) reduce the physical temperature of the antenna so the noise temperature due to the omic loss decreases so T_{sys} is reduced; v) reduce the noise temperature of the receivers used in the system; vi) match between the antenna and the receiver.

1.7. Linear and planar antennas can generally be analyzed in terms of two factors: the first being one of the three incremental elementary sources, and the second the Fourier transform of their excitation distribution. In the analysis of each of the following three antennas, state which incremental source should be used and write the expression for the far field function due to the incremental source. (3p)

a) A dipole along x-axis with the center of the dipole at the origin of the coordinate system;

A: x-polarized incremental electric current;

$$\boldsymbol{G}_{id}(\theta,\varphi) = C_k[\hat{\boldsymbol{x}} - (\hat{\boldsymbol{x}}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}}] = C_k(cos\theta cos\varphi\hat{\boldsymbol{\theta}} - sin\varphi\hat{\boldsymbol{\varphi}})$$

b) A half wavelength long slot in an infinite large ground plane, where the slot is oriented along y-axis with its center at the origin of the coordinate system and the ground plane is lying in xy-plane.

A: x-polarized incremental magnetic current;

$$\boldsymbol{G}_{im}(\theta,\varphi) = C_k[\hat{\boldsymbol{y}} \times \hat{\boldsymbol{r}}] = C_k(cos\varphi\hat{\boldsymbol{\theta}} - cos\theta sin\varphi\hat{\boldsymbol{\varphi}})$$

c) A large paraboloidal reflector aperture with y-polarization. The aperture is at the xy-plane with its center at the origin of the coordinate system.

A: y-polarized Huygen's source;

$$\boldsymbol{G}_{H}(\theta,\varphi) = 2C_k\cos^2(\theta/2)(sin\varphi\hat{\boldsymbol{\theta}} + cos\varphi\hat{\boldsymbol{\varphi}})$$

1.8. A large antenna has a circular planar aperture with the radius $r = 10\lambda$, where λ is the wavelength at the operation frequency. The aperture efficiency of the antenna is $e_{ap} = -1.5$ dB, and the radiation efficiency is $e_{rad} = -0.25$ dB. What is the realized gain of the antenna? (2p)

A:

$$D_{max} = \frac{4\pi}{\lambda^2}A = \frac{4\pi}{\lambda^2}\pi r^2 = 4\pi^2\cdot 100 = 36\text{dBi}$$

$$G_{ant} = e_{ap}e_{rad}D_{max} = 36 - 1.5 - 0.25 = 34.25\text{dB}$$

1.9. The total antenna efficiency of a paraboloidal reflector antenna can be factorized in several different sub-efficiencies that characterize different losses. Name as many of these sub-efficiencies as possible and explain them (no equations are needed). Try to give typical values of the sub-efficiencies in dB when the antenna is optimized for as high gain as possible. (3.5p)

A:

$$e_{ap} = e_{sp}\, e_{pol}\, e_{ill}\, e_{phase}\, e_{cb}$$

e_{sp} - ratio of power hitting reflector to total power (-0.05dB ~ -0.5dB);

- ratio of co-polar field power to total power within the subtended angle (~ -0.1dB);

e_{ill} - a measure of how efficient the aperture is used (-0.4dB ~ -1.5dB);

e_{phase} - a measure of loss due to phase error over aperture (~ -0.1dB);

e_{cb} - a measure of loss due to center blockage (-0.15dB ~ -0.3dB);

1.10. A large phased array has the directivity of 43 dBi when its main beam is at the direction of $(\theta_0, \varphi_0) = (0°, 0°)$. What is the directivity of the array antenna when the main beam is steered to the direction of $(\theta_0, \varphi_0) = (30°, 30°)$? Assume that there is no grating lobes for the above two cases. (2p)

A:

$$D_{\theta 0=30°} = D_{\theta 0=0°} + 10\log_{10}(cos\theta_0) = 43 - 0.62 = 42.38 \text{ dBi}$$

1.11. When the main beam of the array antenna in 1.9 is steered to the direction of $(\theta_0, \varphi_0) = (45°, 30°)$, a grating lobe appears at the direction of $(\theta_0, \varphi_0) = (75°, 210°)$ with the same level as the main beam. Estimate the directivity of the antenna for this situation. (2p)

A:

$$e_{grt} = \frac{1}{1 + cos45°/cos75°} = -5.7 \text{ dB}$$

$$D_{\theta 0=45°} = e_{grt} cos\theta_0 D_{\theta 0=0°} = 43 + 10\log_{10}(cos45°) - 5.7$$
$$= 43 - 1.5 - 5.7 = 35.8\text{dBi}$$

2.0 Antenna Systems in Multipath Environment (25p)

2.1 Briefly explain the concept of multipath fading experienced by the induced voltage at the terminals of an antenna. (2p)

A: Multipath fading is the fluctuation experienced by the received voltage when moving around in the (multipath) propagation environment. The waves arrive from multiple paths due to scattering, and impinge at the receiver inducing voltages that interfere with each other constructively or destructively.

2.2 Briefly explain what Rich Isotropic MultiPath (RIMP) environment means. (2p)

A: Isotropic means that the Angle-of-Arrivals are uniformly distributed over a sphere of unit radius, i.e., the likelihood of observing a wave impinging at the receiver is the same for all directions. And also that it is power balanced with cross-polar power discrimination equal 1 in linear scale or 0 dB in dB-scale, i.e., the average received power of a vertical polarization is equal to the power in the horizontal polarization.

2.3 What is a MIMO system? What does "MIMO" stand for? What are the benefits to use MIMO systems? (2p)

A: MIMO stand for multi-input and multi-output. It is a system where there are more than one transmitting antennas and more than one receiving antennas. By transmitting more than one independent data stream through MIMO channel, one can achieve higher data rate.

2.4 Mention a simple way to control the coherence bandwidth and the inverse time delay spread of a reverberation chamber. (2p)

A: can be controlled by loading the reverberation, e.g., with absorbers. This is possible since absorbers have a large impact on the mode bandwidth to which the coherence bandwidth is directly proportional.

2.5 What is the fundamental parameter that characterizes the performance of a single-port antenna in rich isotropic multipath (RIMP)? (2p)

A: The total radiation efficiency.

2.6 Write the expression of total embedded efficiency of port 3 of a lossless 4-port antenna. Can this antenna parameter be measured in a reverberation chamber? (2p)

A:

$$e_{totemb,3} = 1 - |S_{31}|^2 - |S_{32}|^2 - |S_{33}|^2 - |S_{34}|^2 .$$

Yes.

2.7 Write an expression for the definition of effective diversity gain (In the exam it was *effective radiation efficiency,* which is a typo. So we decide that everyone got 2 points for this question.) (2p)

A:

$$G_{eff} = e_{rad} G_{app}$$

2.8 State three ways to determine the complex correlation coefficient between port voltages in RIMP. (2p)

A: From embedded element far-field functions, from S-parameters of the antenna (for lossless antennas only), from measured port voltages (channels)

2.9 Briefly explain the main idea behind diversity. (2p)

A: The idea is to exploit received signals with low correlation at different antennas. Fading dips do not occur simultaneously at the different antennas in multipath environments, especially in rich isotropic multipath channels.

2.10 State three antenna diversity methods. (2p)

A: polarization diversity, space diversity, pattern diversity.

2.11 Briefly explain the definition of dBR. (2p)

A: dBR is defined as the diversity gain in dB at a given probability level. The reference is a Rayleigh-shaped CDF corresponding to an ideal antenna with 100% efficiency.

2.12 The throughput of a wireless device equipped with a single antenna is measured in a reverberation chamber. It is noted that the measured throughput can be modelled by the Probability of Detection corresponding to the Rayleigh CDF. What is the throughput at the threshold level if the maximum throughput is $TPUT_{\max} = 1Mbps$? Note: The Rayleigh CDF describes the distribution of voltages. (3p)

A: $TPUT = TPUT_{\max} PoD = TPUT_{\max}(1 - CDF) = 1e(-1) \approx 368kbps$

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PART B (You can use the textbook to solve this problem, but only after PART A has been delivered)

3.0 Boundary Conditions, Reaction Concept, Equivalent Circuit of Antennas (25p)

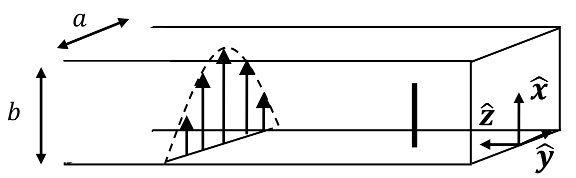

Fig 3.0 A rectangular waveguide fed by a monopole probe.

Consider a hollow semi-infinitely long rectangular waveguide with PEC walls at $z = 0$, $y = \pm a/2$, $x = 0$, and $x = b$. The E-field distribution within the waveguide, when excited from the left by an incident EM field traveling in the negative z-direction (i.e. receiving situation, no probe excitation/current), is given through the TE_{01} mode as $\boldsymbol{E} = E_0 \cos\left(\frac{\pi}{a}y\right)\sin(k_z z)\,\hat{\boldsymbol{x}}$, with $k_z = \sqrt{k^2 - \left(\frac{\pi}{a}\right)^2}$, where E_0 is a constant amplitude, $k = 2\pi/\lambda_0$ is the wavenumber of the medium and λ_0 is the wavelength in vacuum.

3.1 Show that the above E-field satisfies the boundary conditions at the five PEC walls of the waveguide. (3p)

A:
The tangential component of the E-field must vanish at the PEC walls, i.e., $E_x\left(y = \pm\frac{a}{2}\right) = E_z\left(y = \pm\frac{a}{2}\right) = 0$; $E_x(z = 0) = E_y(z = 0) = 0$, and; $E_z(x = b) = E_z(x = 0) = E_y(x = b) = E_y(x = 0) = 0$.

3.2 Next, the probe is excited (i.e. transmitting situation, no incident waveguide field). The probe current is given as $\boldsymbol{J} = I_0 \cos\left(\frac{\pi x}{2L}\right)\delta(y)\delta(z - z_0)[U(x) - U(x - L)]\hat{\boldsymbol{x}}$ A/m^2, where L is the probe length, z_0 is the probe distance from the back wall, $U(x)$ is the Heaviside step function defined as $U(x) = 0$ for $x < 0$ and $U(x) = 1$ for $x \geq 0$, and δ is the Dirac delta function. Derive a closed form solution for the probe open-circuit receive voltage for the E-field defined above through a reaction integral formula. (5p)

A:
The reaction integral formula is given as $V = -\frac{1}{I_0}\langle \boldsymbol{E}, \boldsymbol{J}\rangle = -\frac{1}{I_0}\iiint_{-\infty}^{+\infty} \boldsymbol{E}\cdot\boldsymbol{J}\,\mathrm{d}V$.
Substituting the known quantities in this formula yields:

$V = -E_0 \iiint_{-\infty}^{+\infty} \cos\left(\frac{\pi}{a}y\right) \sin(k_z z) \cos\left(\frac{\pi x}{2L}\right) \delta(y)\delta(z - z_0)[U(x) - U(x - L)]\, \mathrm{d}V$, which reduces to $V = -E_0 \sin(k_z z_0) \int_0^L \cos\left(\frac{\pi x}{2L}\right) \mathrm{d}x = -\frac{2E_0 L}{\pi} \sin(k_z z_0)$.

3.3 Assume that $a = \frac{2}{3}\lambda_0$, then compute the optimal probe position z_0 in terms of the free-space wavelength λ_0 that maximizes the open-circuit receiving voltage. Assume that the open-circuit receive voltage $V \propto \sin(k_z z_0)$. (3p)

A:

The first maximum of $V(z_0)$ appears if $k_z z_0 = \frac{\pi}{2}$, where $k_z = \sqrt{k^2 - \left(\frac{\pi}{a}\right)^2}$ with $k = 2\pi/\lambda_0$ and $a = \frac{2}{3}\lambda_0$. Hence, $k_z = \sqrt{\left(\frac{2\pi}{\lambda_0}\right)^2 - \left(\frac{3\pi}{2\lambda_0}\right)^2} = \sqrt{\frac{7}{4}}\frac{\pi}{\lambda_0}$, so that $z_0 = \frac{4}{\sqrt{7}}\left(\frac{\lambda_0}{4}\right)$. In other words, it is about 51% larger than a quarter wavelength in free space.

3.4 What is the reason that the probe has to be at a greater distance than $\frac{\lambda_0}{4}$ from the back wall to maximize the receiving voltage? (2p)

A:

The *guided wavelength* $\lambda_\mathrm{g} = 2\pi/k_z = \frac{4}{\sqrt{7}}\lambda_0$ for the fundamental TE mode, which is larger than the free space wavelength λ_0. Since the probe-wall distance must be equal to $\lambda_\mathrm{g}/4$, it is therefore larger than a $\lambda_0/4$.

3.5 The next step is to open the waveguide at $z = z_A$ and let it radiate as an open-ended rectangular waveguide antenna. We wish to determine the far field function value $G_x(\hat{\boldsymbol{z}})$ in Volts at $f_0 = 10$ GHz for a unit current excitation. To this end, we let an x-polarized plane wave of 1 V/m be incident on the open-ended waveguide antenna. We measure that the short-circuited port current in the receiving situation is 1µA and also know that the antenna input impedance $Z_{11} = 50\ \Omega$. Calculate $G_x(\hat{\boldsymbol{z}})$ (4p)

A:

The Thevenin open-circuit voltage V is given as $V = -\frac{2j\lambda_0}{\eta I} G_x E_x^\mathrm{i} = -\frac{2j\lambda_0}{\eta} G_x$. From the equivalent circuit for receiving antennas we know that this voltage is also equal to the short-circuited port current multiplied by the antenna input impedance, i.e., $V = 50 * 1\mathrm{E}-6 = 50$ µV. Accordingly, $G_x = -\frac{\eta V}{2j\lambda_0} = j\frac{120\pi * 50\mathrm{E}-6 * 10\mathrm{E}9}{2 * 3\mathrm{E}8} = 0.3j$ Volt.

3.6 At the input of the monopole probe we measure the reflection coefficient over frequency and observe an oscillatory behavior with period $\Delta f = 1$ GHz. This is due to an undesired reflection that occurs directly at the probe input and a second one at the waveguide end. Compute the value $z = z_A$ at which the waveguide has been cut using the theory of small reflections. The reflection at the back wall (z=0) is irrelevant since it has been included in the design to achieve a good impedance match. The central operating frequency $f_0 = 10$ GHz. (3p).

A:

From formula below (2.138) as

$$\beta_2 - \beta_1 = \frac{\pi}{\Delta z}$$

We have

$$\beta_1 = \sqrt{k^2 - \left(\frac{\pi}{a}\right)^2} = \sqrt{\left(\frac{2\pi}{\lambda_1}\right)^2 - \left(\frac{\pi}{\frac{2}{3}\lambda_0}\right)^2}$$

$$\beta_2 = \sqrt{k^2 - \left(\frac{\pi}{a}\right)^2} = \sqrt{\left(\frac{2\pi}{\lambda_2}\right)^2 - \left(\frac{\pi}{\frac{2}{3}\lambda_0}\right)^2}$$

Now

$$\lambda_0 = \frac{300}{10} = 30mm, \lambda_1 = \frac{300}{9.5} = 31.58mm, \lambda_2 = \frac{300}{10.5} = 28.57mm$$

$$\beta_1 = \sqrt{\left(\frac{2\pi}{31.58}\right)^2 - \left(\frac{3\pi}{2\cdot 30}\right)^2} = 0.1221$$

$$\beta_2 = \sqrt{\left(\frac{2\pi}{28.57}\right)^2 - \left(\frac{3\pi}{2\cdot 30}\right)^2} = 0.1536$$

$$Z_A = \Delta z = \frac{\pi}{\Delta\beta} = 98.8mm$$

3.7 Consider now an identical pair of open waveguide antennas. When antenna 1 is excited by 1 A, the *open-circuit* voltage of antenna 2 turns out to be $20e^{j\pi}$ Volt and the source voltage at port 1 is 50 Volt. Next, antenna 2 is excited by a voltage source of $20e^{j\pi}$ Volt. Compute the *short-circuited* current at antenna port 1 and the source current at port 2 using network (matrix) theory. Hint: the inverse of a 2×2 matrix is: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. (5p)

A:

Use that $\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$, where $I_1 = 1$, $V_1 = 50$, $I_2 = 0$ (open circuit), and $V_2 = 20e^{j\pi}$. Hence, $Z_{11} = \frac{V_1}{I_1} = Z_{22} = 50\ \Omega$, $Z_{12} = Z_{21} = \frac{V_2}{I_1} = 20e^{j\pi}\ \Omega$. Next, we use the admittance matrix equation $\frac{1}{Z_{11}Z_{22}-Z_{21}Z_{12}}\begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$ to compute I_1 and I_2 for $V_2 = 20e^{j\pi}$and $V_1 = 0$ (short circuit), this yields $I_1 = -0.19$ A, and $I_2 = -0.48$ A.

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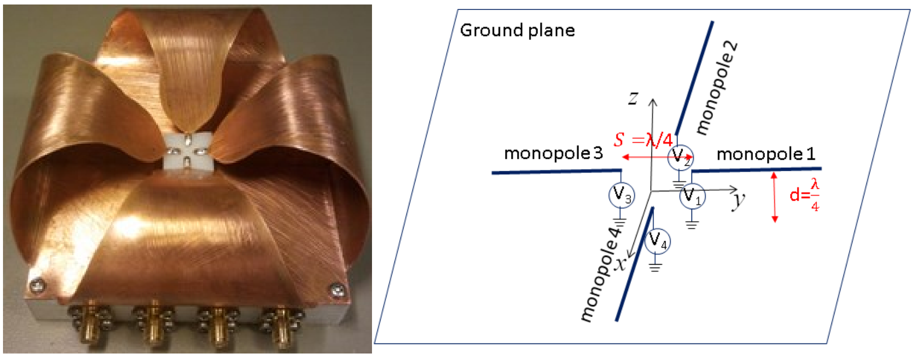
4.0 A bow-tie antenna array (25p)

Fig. 4.0a shows a bow-tie antenna array (developed at Antenna Division, Chalmers), which can be modelled approximately by a four-quarter-wave-monopole array over a ground plane with a distance $d = \lambda/4$ and a spacing $S = \lambda/4$ between monopole input ports, shown in Fig. 4.0b. The coordinate system is defined as in the figure, with the origin at the centre of the ground plane. This array antenna can be used as a classic array antenna (or array element) or a MIMO antenna. In this problem, we use the model of the four quarter-wave monopoles to find the solution, and assume that the self-impedance of the quart-wave monopole in free space is the half of the self-impedance of a half-wave dipole in free space, and the mutual impedance between two quart-wave monopoles in free space is the quarter of the mutual impedance of two half-wave dipoles in free space under the same configuration.



(a) (b)

Fig. 4.0 A bow-tie antenna array: (a) photo, and (b) the simple model of four quart-wave monopoles over a ground plane for the antenna.

4.1 Define the excitations of V_1, V_2, V_3 and V_4 for the following three polarization cases in $+z$ direction: (a) linear y polarization; (b) linear $\varphi = 45°$ polarization; (c) Right hand circular polarization (RHC). (3p)

A:

(a) Linear y-polarized: $V_1 = 1, V_2 = 0, V_3 = -1, V_4 = 0$ or $V_1 = 1, V_2 = 0, V_3 = 0, V_4 = 0$;

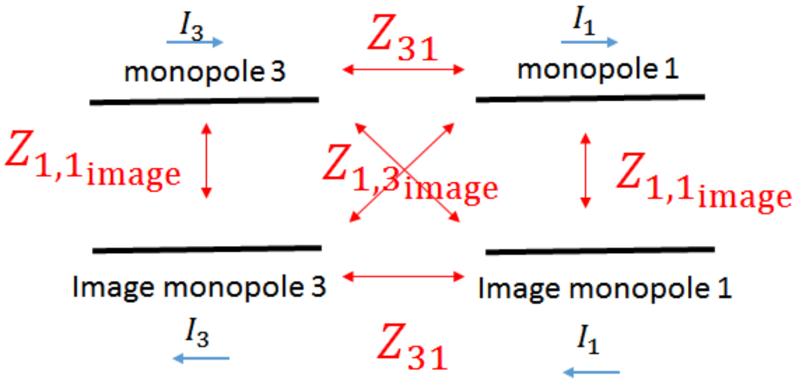
(b) Linear $\varphi = 45°$ polarization: $V_1 = 1, V_2 = -1, V_3 = -1, V_4 = 1$ or $V_1 = 1, V_2 = 0, V_3 = 0, V_4 = 1$

(c) RHC: $V_1 = -j, V_2 = -1, V_3 = j, V_4 = 1$ or $V_1 = e^{j0°}, V_2 = e^{j90°}, V_3 = e^{j180°}, V_4 = e^{j270°}$.

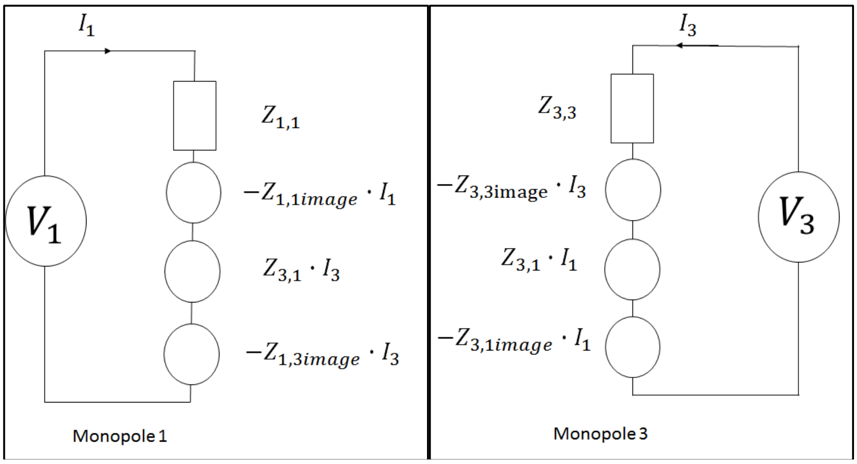
4.2 For the configuration of the four monopoles in Fig. 4.1b, we can ignore the mutual couplings between the orthogonal monopoles for an approximate calculation. Sketch the equivalent circuit for calculation of mutual impedance between the parallel monopoles (such as between monopoles 1 and 3). Explain all parameters used in your equivalent circuit. (2p)

A:

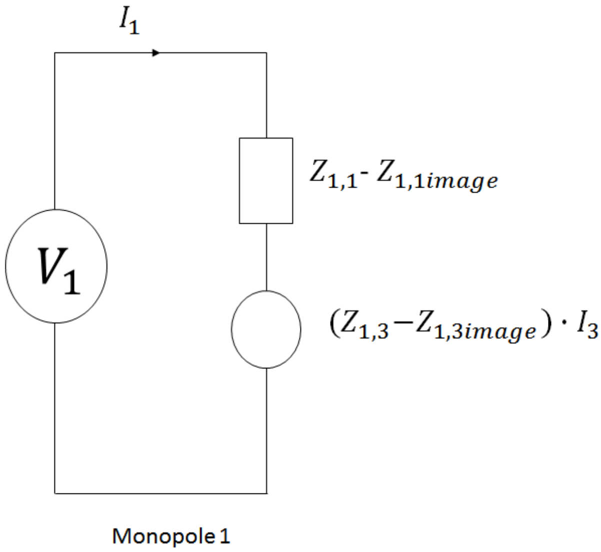
Now the mutual coupling between dipoles 1 and 3 is mainly via the ground plane. We can use imaging method to remove the ground plane and add two image monopoles as



where $Z_{1,1\text{image}}, Z_{3,1}, Z_{1,3\text{image}}$ are the mutual impedances in free space for the above configuration. Therefore, the equivalent circuits are



Note that the current on the image monopole has the same amplitude but opposite direction compared to the current on the monopole, such as $-I_1$ on image monopole 1, and $-I_3$ on image monopole 3. Then, we can simplify the above equivalent circuit as



So the self-impedance of the monopole with the ground plane is $Z_{11} - Z_{1,1image}$, the mutual impedance between monopoles 1 and 3 is $Z_{3,1} - Z_{1,3image}$.

The questions below are under the assumption that all mutual impedances between monopoles are ignored for simplifying the solutions.

4.3 Write out the embedded far field function of monopole 1. (3p)

A:
Since we ignore the mutual coupling between monopoles, the embedded far field function of monopole 1 over the ground plan is the far field function of monopole 1 plus the far field function of its image.

If we put the center of the monopole 1 at the origin of the coordinate system, the far field function is (similar to (5.5) in the text book)

$$\boldsymbol{G}_1(\theta,\varphi) = \eta I_0 \boldsymbol{G}_{idy}(\theta,\varphi)\tilde{j_y}(k\hat{\boldsymbol{l}}\cdot\hat{\boldsymbol{r}})$$

where

$$\boldsymbol{G}_{idy}(\theta,\varphi) = C_k(cos\theta sin\varphi\widehat{\boldsymbol{\theta}} + cos\varphi\widehat{\boldsymbol{\varphi}}) \text{ (eq. (4.79) in the book)}$$

$$\tilde{j_y}(k\hat{\boldsymbol{l}}\cdot\hat{\boldsymbol{r}}) = \int_{-\lambda/8}^{\lambda/8} j(y')e^{jky' sin\theta sin\varphi}dy'$$

Now the monopole is located at $\boldsymbol{r}_1 = \frac{\lambda}{4}\widehat{\boldsymbol{y}} + \frac{\lambda}{4}\hat{\boldsymbol{z}}$, and the image monopole is located at $\boldsymbol{r}_{1\text{image}} = \frac{\lambda}{4}\widehat{\boldsymbol{y}} - \frac{\lambda}{4}\hat{\boldsymbol{z}}$, so the embedded far field function is

$$\boldsymbol{G}_{1embedded}(\theta,\varphi) = \boldsymbol{G}_1(\theta,\varphi)(e^{jk\boldsymbol{r}_1\cdot\hat{\boldsymbol{r}}} - e^{jk\boldsymbol{r}_{1\text{image}}\cdot\hat{\boldsymbol{r}}})$$

The "-" sign is due to the imaging current is opposite to the current of the monopole.

4.4 Write out the far field function of the whole array when the excitations are for the RHC polarization along +z direction. (4p)

A:
Now the excitations are $V_1 = -jV, V_2 = -V, V_3 = jV, V_4 = V$ and the location is

$$\boldsymbol{r}_1 = \frac{\lambda}{4}\hat{\boldsymbol{y}} + \frac{\lambda}{4}\hat{\boldsymbol{z}},\ \boldsymbol{r}_{1\text{image}} = \frac{\lambda}{4}\hat{\boldsymbol{y}} - \frac{\lambda}{4}\hat{\boldsymbol{z}}$$
$$\boldsymbol{r}_2 = -\frac{\lambda}{4}\hat{\boldsymbol{x}} + \frac{\lambda}{4}\hat{\boldsymbol{z}},\ \boldsymbol{r}_{2\text{image}} = -\frac{\lambda}{4}\hat{\boldsymbol{x}} - \frac{\lambda}{4}\hat{\boldsymbol{z}}$$
$$\boldsymbol{r}_3 = -\frac{\lambda}{4}\hat{\boldsymbol{y}} + \frac{\lambda}{4}\hat{\boldsymbol{z}},\ \boldsymbol{r}_{3\text{image}} = -\frac{\lambda}{4}\hat{\boldsymbol{y}} - \frac{\lambda}{4}\hat{\boldsymbol{z}}$$
$$\boldsymbol{r}_4 = \frac{\lambda}{4}\hat{\boldsymbol{x}} + \frac{\lambda}{4}\hat{\boldsymbol{z}},\ \boldsymbol{r}_{4\text{image}} = \frac{\lambda}{4}\hat{\boldsymbol{x}} - \frac{\lambda}{4}\hat{\boldsymbol{z}}$$

So

$$\begin{aligned}\boldsymbol{G}_{\text{tot}}(\theta,\varphi) = & -jV\boldsymbol{G}_1(\theta,\varphi)\left(e^{jk\boldsymbol{r}_1\cdot\hat{\boldsymbol{r}}} - e^{jk\boldsymbol{r}_{1\text{image}}\cdot\hat{\boldsymbol{r}}}\right) \\ & - V\boldsymbol{G}_2(\theta,\varphi)\left(e^{jk\boldsymbol{r}_2\cdot\hat{\boldsymbol{r}}} - e^{jk\boldsymbol{r}_{2\text{image}}\cdot\hat{\boldsymbol{r}}}\right) \\ & + jV\boldsymbol{G}_3(\theta,\varphi)\left(e^{jk\boldsymbol{r}_3\cdot\hat{\boldsymbol{r}}} - e^{jk\boldsymbol{r}_{3\text{image}}\cdot\hat{\boldsymbol{r}}}\right) \\ & + V\boldsymbol{G}_4(\theta,\varphi)\left(e^{jk\boldsymbol{r}_4\cdot\hat{\boldsymbol{r}}} - e^{jk\boldsymbol{r}_{4\text{image}}\cdot\hat{\boldsymbol{r}}}\right)\end{aligned}$$

Where

$$\boldsymbol{G}_1(\theta,\varphi) = \boldsymbol{G}_3(\theta,\varphi) = \eta I_0 \boldsymbol{G}_{idy}(\theta,\varphi)\tilde{j_y}(k\hat{\boldsymbol{l}}\cdot\hat{\boldsymbol{r}})$$

with

$$\boldsymbol{G}_{idy}(\theta,\varphi) = C_k(cos\theta sin\varphi\hat{\boldsymbol{\theta}} + cos\varphi\hat{\boldsymbol{\varphi}})$$
$$\tilde{j_y}(k\hat{\boldsymbol{l}}\cdot\hat{\boldsymbol{r}}) = \int_{-\lambda/8}^{\lambda/8} j(y')e^{jky'\sin\theta\sin\varphi}dy'$$

and

$$\boldsymbol{G}_2(\theta,\varphi) = \boldsymbol{G}_4(\theta,\varphi) = \eta I_0 \boldsymbol{G}_{idx}(\theta,\varphi)\tilde{j_x}(k\hat{\boldsymbol{l}}\cdot\hat{\boldsymbol{r}})$$

with

$$\boldsymbol{G}_{idx}(\theta,\varphi) = C_k(\cos\theta\cos\varphi\hat{\boldsymbol{\theta}} - \sin\varphi\hat{\boldsymbol{\varphi}})$$
$$\tilde{j_x}(k\hat{\boldsymbol{l}}\cdot\hat{\boldsymbol{r}}) = \int_{-\lambda/8}^{\lambda/8} j(x')e^{jkx'\sin\theta\cos\varphi}dx'$$

4.5 Calculate the embedded impedance of monopole 1. You should sketch the equivalent circuit first, and then use data and the figures in the text book for the calculations. (4p)
A: from 4.2, when we ignore the mutual impedance, the embedded impedance of monopole 1 is its self-impedance, equal to $Z_{11} - Z_{1,1image}$

$$Z_{11} = 0.5Z_{11,dipole} = 0.5 \cdot (73 + 0j) = 36.5\Omega$$
$$Z_{1,1\text{image}} = 0.25Z_{1,1\text{image,dipole}}$$

From Figure 10.10 up, the mutual impedance for the side-by-side dipoles with $\lambda/2$ spacing is

$$Z_{1,1\text{image,dipole}} = -15 - 30j\ \Omega$$
$$Z_{11\text{embedded}} = Z_{11} + Z_{1,1image} = 36.5 - 0.25 \cdot (-15 - 30j) = 40.25 + 7.5j\ \Omega$$

4.6 Calculate the effective and apparent diversity gains in rich isotropic multipath of this antenna if selection combining and a CDF-level of 1% are assumed for the case that only monopoles 1 and 2 are used, while dipoles 3 and 4 are terminated with loads. (4p)

A:
The embedded impedance of monopole 1 is $Z_{11\text{embedded}} = 32.75 + 7.5j\ \Omega$, so

$$|S_{11}| = \left|\frac{Z_0 - Z_{11\text{embedded}}}{Z_0 + Z_{11\text{embedded}}}\right| = \left|\frac{50 - (40.25 + 7.5j)}{50 - (40.25 + 7.5j)}\right| = 0.14$$

Since we ignore all mutual coupling between monopoles, $S_{21} = 0$. Therefore, $\rho = 0$.

$$G_{\text{app}} = 10\sqrt{1-\rho^2} = 10 = 10\text{dB}$$

$$G_{\text{eff}} = e_{rad}G_{\text{app}} = (1 - |S_{11}|^2)e_{abs}G_{\text{app}}$$

Since we ignore mutual coupling, $e_{abs} = 0dB$, and $1 - |S_{11}|^2 = 0.9804 = -0.1dB$, so

$$G_{\text{eff}} = 10 - 0.1 = 9.9dB$$

4.7 We now use this antenna array as a feed for a reflector antenna. We excite this array as a *y*-polarized feed. We need to locate the feed with its phase centre at the focus of the reflector. Please find the phase centre of the feed. (2p)

A: The phase center is at the center of the ground plane. Look at the equivalent circuit by using imaging with four monopoles (monopoles 1 and 3 and their images), and using the same way as in problem 1.4, we can get that the phase center is at the center of the ground plane.

4.8 The reflector has a subtended angle of $2 \times 60°$. Estimate the spillover efficiency, illumination efficiency and aperture efficiency (also called feed efficiency) of the reflector antenna. We assume here that the phase efficiency and polarization efficiency are 100%, and the blockage is ignored. (Tips: the feed can be modelled by $\cos^n(\theta_f/2)$ feed, and you need only use figures in the text book to estimate the efficiencies.) (3p)

A:
When the array is excited as y-polarized, the excitation is

$V_1 = 1, V_2 = 0, V_3 = -1, V_4 = 0$. In this case, monopoles 1 and 3 become a dipole. Even the center gap is $\lambda/4$, and with an image dipole, we still can assume the far field function at 60° is similar to that of half-wave dipole. From figure 5.4, you can get the taper at $60°$ is about -8 dB. (it is also OK that you estimate the taper is a bit larger, such as -10 dB.)

From Figure 9.10, when the semi-subtended angle is 60°, the feed efficiency is about -1.05 dB (it is OK if you choose -1 or -1.1 dB). And the spillover efficiency is -0.35 dB. From $e_{ap} = e_{sp}e_{pol}e_{ill}e_{phase}e_{cb}$, and now $e_{pol} =$ 0dB, $e_{phase} =$ 0dB, e_{cb}=0 dB, we have $e_{ill} = e_{ap} - e_{sp} \approx -0.7$ dB.