Name:

Exam in the course Antenna Engineering

2013-05-30

ANTENNA ENGINEERING (SSY100)

(E4) 2012/13 (Period IV)

Thursday 30 May 0830–1230 hours.

Teachers: Prof. Per-Simon Kildal, Associate Prof. Marianna Ivashina, Associate Prof. Jian Yang, Assistant Prof. Rob Maaskant, and Post Doc. Esperanza Alfonso.

Questions: Jian Yang, tel. 1736 (mobil: 0733 678841)

The exam consists of 2 parts. Part A is printed on colored paper and must be solved without using the textbook. When you have delivered the colored text and the solutions to Part A (latest 11:30), the textbook can be used for Part B of the exam.

You are allowed to use the following:

For Part A:	**Pocket calculator of your own choice**
For Part B only:	**Mathematical tables including Beta**
	Pocket calculator of your own choice
	Kildal's compendium "Foundations of Antennas: A Unified Approach for LOS and Multipath"

(The textbook can contain own notes and marks on its original printed pages. No other notes are allowed.)

Tentamen består av 2 delar. Del A har tryckts på färgade papper och skall lösas utan att använda läroboken. När du har inlämnat dom färgade arken med uppgifterna för del A och dina svar på dessa uppgifter (senast 11:30), kan du ta fram läroboken för att lösa del B.

Tillåtna hjälpmedel:

För del A:	Valfri räknedosa
För del B:	Matematiska Tabeller inkluderad Beta
	Valfri räknedosa
	Kildals lärobok "Foundations of Antennas: A Unified Approach for LOS and Multipath"

(Boken kan innehålla egna noteringar skrivna på de inbundna sidorna. Extra ark med noteringar tillåts inte.)

Name:

PART A (must be delivered before textbook can be used)

1.0 Aperture and Array antennas (25p)

A circular aperture with a diameter of 20λ (λ is the wavelength at the operating frequency) has a uniform aperture field distribution as

$$\boldsymbol{E}_{\mathrm{a}}(\rho', \varphi') = \hat{\boldsymbol{y}}, \quad 0 \leq \rho' \leq 10\lambda, \quad 0 \leq \varphi' \leq 2\pi. \tag{1}$$

The origin of the coordinate system is located at the center of the aperture and the main beam direction is along the z-axis.

1.1. What is the directivity of this aperture antenna? What is the level of the first sidelobe compared to the maximum co-polar level? Explain how you get the results. (3p)

A:

Using (7.63), you have

$$D_0 = \frac{4\pi}{\lambda^2} A = (\frac{\pi d}{\lambda})^2 = 400\pi^2 = 36 dBi.$$

The level of the first sidelobe compared to the maximum co-polar level is -17.6 dB (pp. 206–207).

1.2. Explain why the antenna is a BOR1 antenna. Can BOR1 antennas have different radiation patterns in E- and H-plane? Will this aperture have different radiation patterns in E- and H-plane? Explain. (3p)

A:

The antenna is a BOR1 antenna because 1) the geometry of the antenna is rotationally symmetric; 2) The far-field function of an aperture antenna is the product of the far-field function of its incremental source and the Fourier transform of the aperture field distribution. The incremental source of an aperture is either a magnetic current (PEC aperture) or Huygens' source (free space aperture), and both these sources have BOR1-type far-field function. In addition, the Fourier transform of a uniform distribution is only a function of θ. Therefore, the whole far-field function is a BOR1 far-field function.

BOR1 antennas can have different radiation patterns in E- and H-plane.

This aperture has the same radiation patterns in E- and H-plane when it is a free space aperture (Huygens' source). For PEC aperture, the magnetic current source gives the different far-field function in E- and H-planes so the whole aperture antenna has different far-field function E- and H-planes.

1.3. What are the 3dB semi beam widths in E- and H-planes? Explain and show how you get the results. (2p)

A:

$$\theta_{3dB} = \texttt{asin}(0.5\lambda/d) = 1.4deg.$$

for both E- and H-plane.

1.4. What is the maximum level of the cross polar beam compared to the maximum co-polar beam level. (1p)

A: There is no cross polar component in this aperture for a PEC aperture. There is a cross pol of the incremental source, but this will be very small near broadside. Therefore, the maximum level of the cross polar beam compared to the maximum co-polar beam level is close to $-\infty dB$.

Now we will realize this aperture distribution by using a planar array of halfwave slots in an infinite ground plane. It is pointed out that we consider the broadside scan direction.

1.5. Sketch the layout of the slots in a square grid in such a way that the orientation and spacing between them can be seen. Note that you must make the layout in such a way that grating lobes do not appear and there is as few slots as possible. (2p)

A: See Fig. 1 on next page. For no grating lobes, we need element spacing $d_{max} < \frac{1}{1+\lambda/D} = \lambda/1.05$. Thus minimum number of elements is $N_x = N_y = 21$.

1.6. What is the directivity of the array antenna, the approximate beamwidth, and the level of the first sidelobe? (3p)

A:

The same as that in the above aperture if your array is a correct one: $D_0 = 36$ dBi, $\theta_{3dB} = 1.4$ deg, and the level of the first sidelobe compared to the maximum co-polar level is -17.6 dB.

1.7. Explain the difference between the directivity and the gain of an antenna in terms of efficiency factors? (3p)

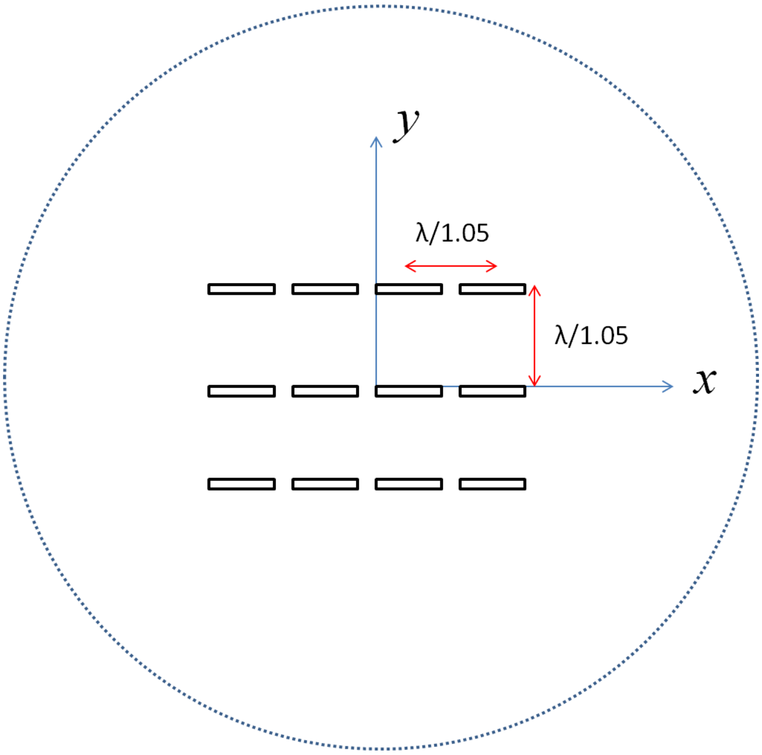

Figure 1: layout of slot array.

A:

$$G_0 = e_{rad} \cdot e_{pol} \cdot D_0.$$

where $e_{rad} = e_r e_{ohmic}$, and e_{pol} is the polarization efficiency on axis due to deviation from desired polarization on axis.

1.8. If the element spacing is λ along both x- and y-axis, grating lobes will appear at 90°. Please estimate the reduction of the directivity of this array compared to the directivity you obtained in 1.6. Explain how you do it. (4p)

A:

Replacing the slots with magnetic currents, then you see that the magnetic current has non radiation at 90° in H-plane but has a uniform radiation in E-plane. Therefore, there are two grating lobes in E-plane at $(\theta = 90°, \varphi = 90°)$ and $(\theta = 90°, \varphi = 270°)$. Note that these two grating lobes have the same level as the main beam and much wider baemwidth than the main beam, even tough they are cut by the infinite ground plane. Therefore, the grating lobes take much more power than the main beam. The reduction of the directivity should be larger than 6 dB.

1.9. There is mutual coupling between the elements of an array. The closer the elements are, the larger mutual coupling. Explain the differences between the directivities and gains of the three realizations of the array having different element spacings, when we assume that the elements are impedance matched to a return loss better than 15 dB in all three cases. The element spacings in the three considered case are $\lambda/2$; $3\lambda/4$; and $3\lambda/2$. (5p)

A: The directivities of the three cases satisfy

$$D_{\lambda/2} = D_{3\lambda/4} > D_{3\lambda/2}.$$

because the array with element spacing of $3\lambda/2$ has grating lobes and the others not.

$$G_{3\lambda/2} < G_{\lambda/2} \approx G_{3\lambda/4}.$$

because the array with element spacing of $\lambda/2$ has more ohmic loss due to the larger mutual couplings.

2.0 Antenna Measurements and Labs (25p)

There are mainly two types of chambers for antenna measurements: anechoic chambers and reverberation chambers.

2.1. Describe briefly the anechoic chamber and reverberation chamber, and which types of environments these two chambers emulate. (3p)

A:

An anechoic chamber is a chamber which has absorbing materials on all walls (including ceiling and floor) with a certain shape such that there is no reflection. An anechoic chamber emulates the free-space environment (no refection...). In contrast, a reverberation chamber is a chamber with inwards reflecting walls, so exists strong wave reflection. A reverberation chamber emulates the rich isotropic multipath environment (due to multiple reflections...).

2.2. Describe the antenna characteristics that can be measured in an anechoic chamber. (3p)

A:

We can measure co- and cross-polar radiation patterns, realized gain,directivity ... in an anechoic chamber.

2.3. Describe briefly how to measure realized gain in an anechoic chamber. You must sketch the setup and should point out which data you need to know for the measurement. (3p)

A: You can use one of three methods in Figs. 2, 3 and 4.

Fig. 2: Replacement method: You need to know the gain of the replaced antenna (often the standard gain horn).

Fig. 3: Two antenna method: You need to know the distance R between the two antennas (and the two antennas should be identical).

Fig. 4: Three antenna method: You need to know the distance R between the antennas.

2.4. The turntable in the anechoic chamber is horizontal and can be rotated around a vertical axis. Describe how you will locate and orient your antenna in order to measure E- and H-plane radiation patterns, and patterns in 45°- planes. (3p)

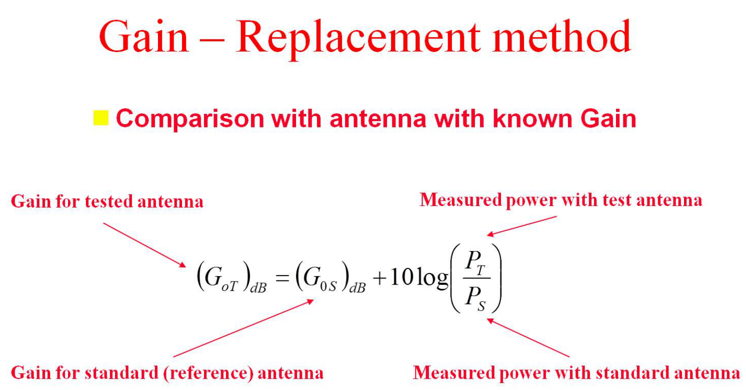

Figure 2: Gain measurement– Replacement methods (we used this in Lab 2).

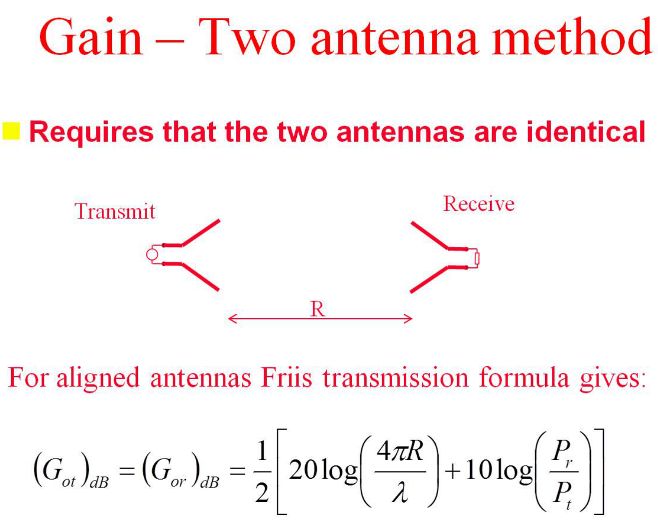

Figure 3: Gain measurement – Two antenna method.

A: The setups for E- and H-plane, and 45°- plane radiation patterns are shown in Figs. 5, 6 and 7

2.5. Describe the characteristics of multi-port antennas that can be measured in a reverberation chamber. Describe in particular different efficiency factors and other things that will affect performance in a MIMO system. (5p)

A:

We can measure the diversity gain, embedded element efficiency, and MIMO capacity in a

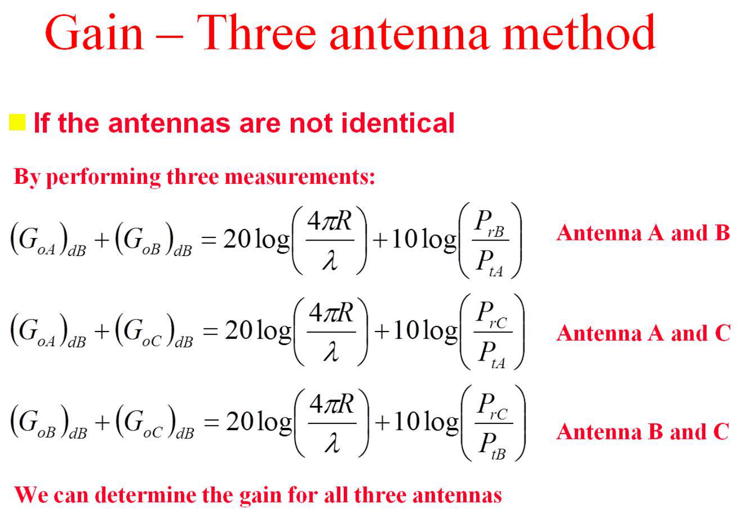

Figure 4: Gain measurement – Three antenna method.

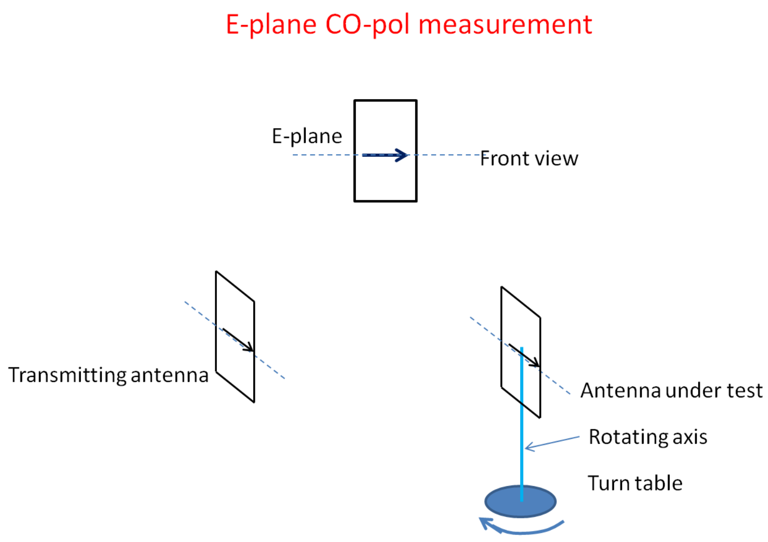

Figure 5: setup for co-polar E-plane radiation patterns.

reverberation chamber. (Mentioning three quantities is enough.)

MIMO system is affected by embedded radiation efficiency and correlation between the antenna ports. The diversity gain and capacity is reduced when the correlation is large.

2.6. Describe how to calibrate the reverberation chamber and why we need to do this. (2p)

A: This can be found in section 3.8.1 in the book. The key points are

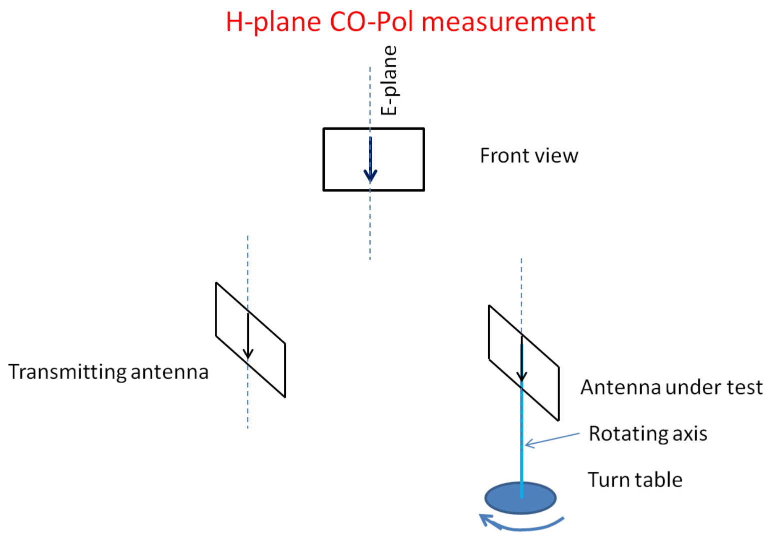

Figure 6: setup for co-polar H-plane radiation patterns.

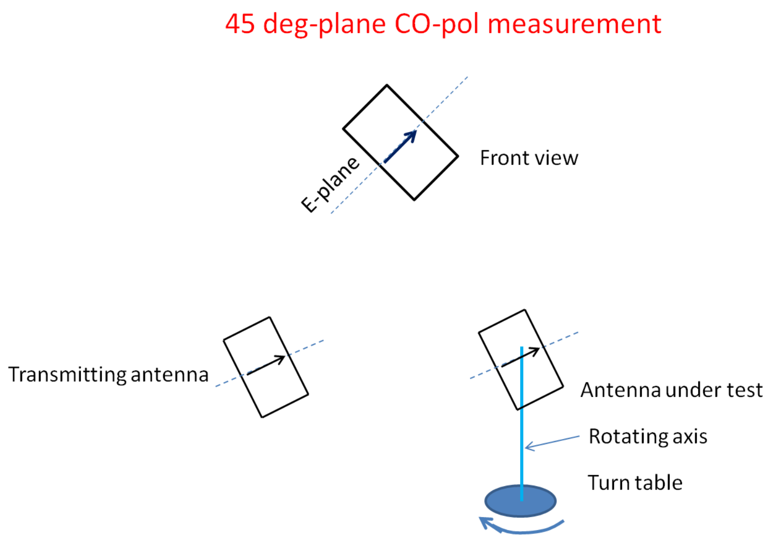

Figure 7: setup for co-polar 45°-plane radiation patterns.

1) Both the AUT (antenna under test) and a reference antenna are located inside the RC (half wavelength away from from any walls and stirrers).

2) Connect one of the AUT ports, and terminate all the other ports and the reference antenna in 50 ohm.

3) We measure S-parameters between the port and the three chamber antennas (used for polarization stirring) for all positions of the platform and the mechanical stirrers and for all frequency points.

4) We connect the reference antenna to the network analyzer and perform the same measurements as for the array. This is for calibration.

5) During the reference measurements, the AUT with all its ports terminated in 50 ohm must be present in the chamber. This is necessary because the loading of the chamber (and thereby the Q-factor) needs to be the same during all measurements.

2.7. We have a 2-port MIMO antenna in which both ports are impedance matched to -7 dB, and we know that the mutual coupling between all ports are -10 dB. Which apparent and effective diversity gain will we measure on this antenna if we assume that it is lossless? (5p)

You may need to use these two equations, and assume that the S-parameters are real-valued, i.e. make use of the worst case upper-bound:

$$\rho = \frac{S_{11}^* S_{12} + S_{21}^* S_{22}}{\left[1 - \left(|S_{11}|^2 + |S_{21}|^2\right)\right]\left[1 - \left(|S_{21}|^2 + |S_{22}|^2\right)\right]} \tag{2}$$

$$G_{\text{app}} = 10.5 e_{\text{p}}, \quad \text{with} \quad e_{\text{p}} = \sqrt{1 - |\rho|^2} \tag{3}$$

A:

$$\begin{aligned} |S_{11}| = 0.4467, \quad |S_{21}| = 0.3162 \\ \rho = \frac{2 \cdot 0.4467 \cdot 0.3162}{\left[1 - \left(0.4467^2 + 0.3162^2\right)\right]\left[1 - \left(0.3162^2 + 0.4467^2\right)\right]} = 0.58 \end{aligned}$$

$$\begin{aligned} G_{\text{app}} = 10.5\sqrt{1 - |\rho|^2} = 10.5 \cdot 0.8 = 8.4 = 9.3dB \\ G_{\text{app}} = (1 - |S_{11}^2 - |S_{12}|^2|)G_{\text{app}} = -1.55 + 9.3(dB) = 7.8dB \end{aligned} \tag{4}$$

PART B (You can use the textbook to solve this problem, but only after PART A has been handed in)

3.0 Equivalent Circuits for Antennas (25p)

Consider the circularly-polarized field $\boldsymbol{E}^{\mathrm{i}} = (\hat{\boldsymbol{x}} + j\hat{\boldsymbol{y}})e^{jkz}$ that is incident on the $\lambda/2$-dipole which is aligned along the y-axis as shown in Fig. 8.

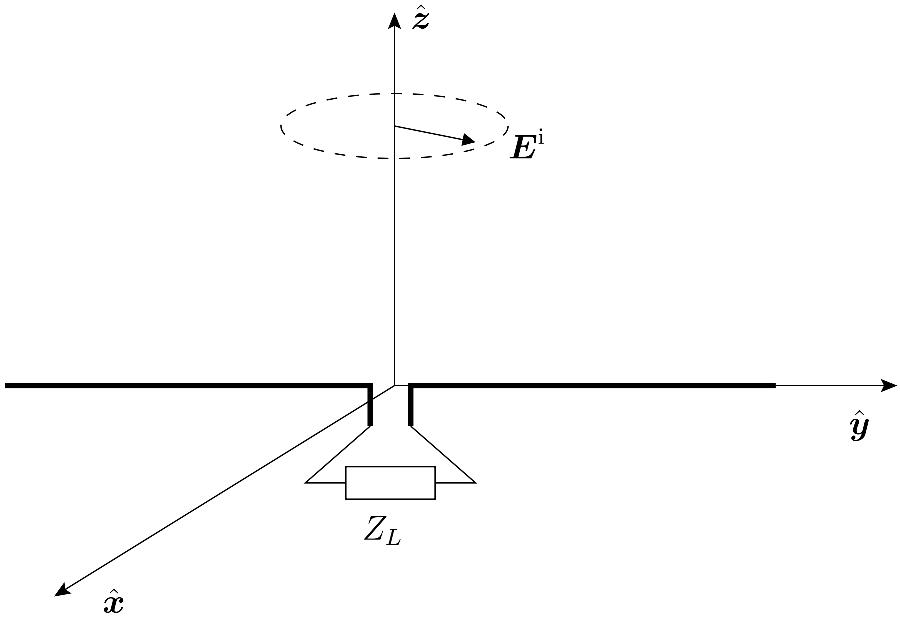

Figure 8: Dipole in receiving mode.

3.1 The expression of the time-harmonic incident field $\boldsymbol{E}^{\mathrm{i}}$, with time factor $e^{j\omega t}$, represents (choose one answer) (3p):
(a) LHC polarization traveling in the +z direction
(b) LHC polarization traveling in the -z direction
(c) RHC polarization traveling in the +z direction
(d) RHC polarization traveling in the -z direction.

Answer: (b)

3.2 The input impedance of the dipole antenna in free-space is Z_{11}. The antenna is power matched at its port, so that $Z_{11} = Z_L^*$. Compute the delivered power to the power-matched load if the broadside directivity of the dipole antenna at 1 GHz is $D_0 = 2.16$ dBi and has unit radiation efficiency ($\eta_{\mathrm{rad}} = 100\%$). Hint: first compute the total power density W_{av} of the incident field in $\mathrm{W/m^2}$, then compute the delivered power to the load using the effective area A_e of the dipole. Note the polarization mismatch, since the antenna is y-polarized. (7p)

Answer: Eq. (2.10): $W_{av} = \frac{1}{2\eta}|\boldsymbol{E}^i|^2 = \frac{1}{2\cdot 120\pi}2 = \frac{1}{120\pi} = 2.65$ mW/m^2. Furthermore, $G_0 = D_0 = 2.16$ dBi, which is 1.644. Hence, $A_e = \frac{\lambda^2}{4\pi}G_0 = \frac{0.09}{4\pi}1.644 = 0.0118$ m^2. Thus, $P_r = 0.0118 * 2.65/2 = 15.6$ μW, where the last division by 2 comes from the polarization mismatch.

3.3 When supplying the antenna with an input current $I_{at} = 1$ Ampère in the transmitting situation, the y-polarized radiated E-field at $\boldsymbol{r} = 100\hat{\boldsymbol{z}}$ is $\boldsymbol{E}^t = E_y^t\hat{\boldsymbol{y}}$, where $E_y^t = 0.497e^{-jk100}$ V/m. The antenna far field function (in Volts) is therefore (choose one answer) (3p):
(a) $\boldsymbol{G}(\hat{\boldsymbol{z}}) = 49.7\hat{\boldsymbol{y}}$
(b) $\boldsymbol{G}(\hat{\boldsymbol{z}}) = 49.7e^{-jk200}\hat{\boldsymbol{y}}$
(c) $\boldsymbol{G}(\hat{\boldsymbol{z}}) = 0.0497\hat{\boldsymbol{y}}$
(d) $\boldsymbol{G}(\hat{\boldsymbol{z}}) = 0.0497e^{-jk200}\hat{\boldsymbol{y}}$.

Answer: (a)

3.4 Another way to compute the power delivered to the load is through the equivalent circuit for a receiving antenna as shown in Fig. 9. The antenna input impedance is given as $Z_{11} = 50 + 10j\Omega$, and $Z_L = Z_{11}^*$.

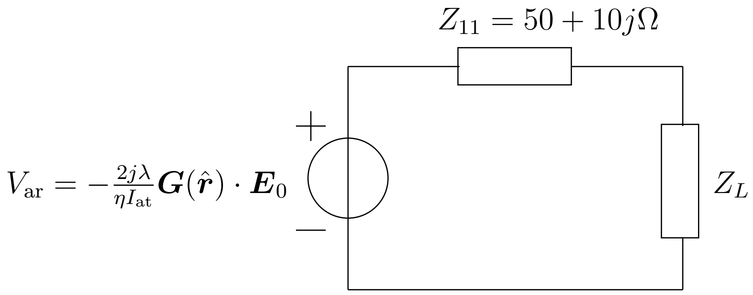

Figure 9: Equivalent circuit of a receiving antenna.

Compute the complex-valued open-circuited antenna port voltage in the receiving situation using the far-field function found above. Hint: $\boldsymbol{E}_0$ is the incident E-field at the phase reference point of $\boldsymbol{G}(\hat{\boldsymbol{z}})$ of the antenna. (4p)

Answer: $V_{ar} = -\frac{2j\lambda}{\eta I_{at}}\boldsymbol{G}(\hat{\boldsymbol{r}})\cdot\boldsymbol{E}_0 = -\frac{j0.6}{120\pi}49.7j = 79.0$ mV.

3.4 Calculate once again the power dissipated in the load Z_L. Show how it is calculated. (you should obtain the same result as obtained in 3.2). (5p)

Answer: $P_L = \frac{1}{2}|I|^2\Re\{Z_L\}$, where $I = V_{\rm ar}/2\Re\{Z_L\} = 79/100$ mA, so that $P_L = 15.6$ μW, which is the same answer as above.

3.5 Describe the difference between a power-matched and a characteristically terminated situation at an antenna output transmission line. (3p)

Answer: in a power-matched situation maximum power is extracted from the antenna system, while if the system is terminated with the characteristic impedance, the received power may be less since we match only to the (typically real-valued) characteristic impedance of the output transmission line to avoid a wave reflection at the load.

4.0 A dipole array (25p)

Figure 10 shows a dipole array consisting of four half wavelength dipoles, which can be used as a classic array antenna or a MIMO antenna. In this problem, the mutual couplings between the orthogonal dipoles are ignored, i.e. $Z_{12} = Z_{23} = Z_{34} = Z_{41} = 0$. But the mutual coupling between the parallel dipoles should be taken into account in the solution to the problem. $Z_0 = 50$ Ohms is a termination load.

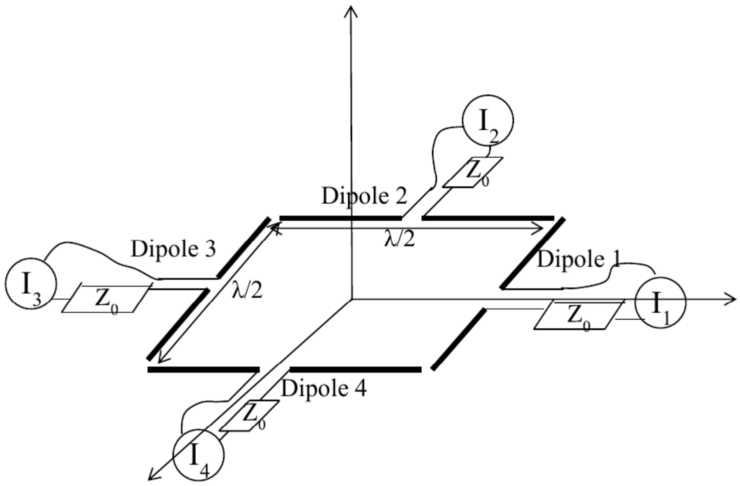

Figure 10: Four-element dipole array.

4.1 When the excitations are set as $I_1 = I_2 = I_3 = I_4 = I_0$, what is the polarization of the far-field function of the antenna along the $+z$ axis? State the direction of the polarization vector if linear polarization, or RHC (right hand circular) or LHC (left hand circular) if circular polarization. (2p)

A:

Since the excitation $I_1 = I_2 = I_3 = I_4 = I_0$, the same amplitude and the same phase, it is linear polarization in 45 deg plane (both $\pm 45°$ plane is correct).

4.2 Write out the embedded far-field function of dipole 1. (4p)

A:

The equivalent circuit of embedded dipole 3 is shown in Fig. 11, where dipoles 2 and 4 are ignored since the mutual couplings between the orthogonal dipoles are ignored. From the equivalent circuit, we can obtain

$$I_3 = \frac{Z_{31}}{Z_0 + Z_{33}} \cdot I_1$$

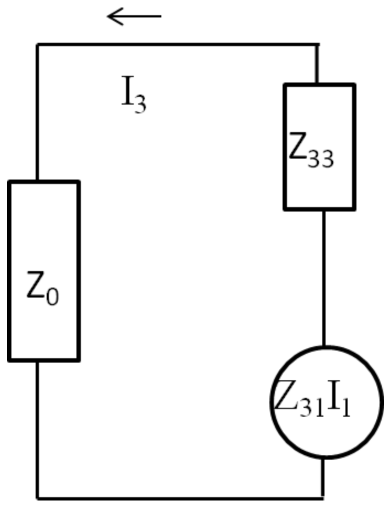

Figure 11: Equivalent circuit for dipole 3.

Here notice that we use current source at dipole 1 so the excitation for dipole 1 is always I_1. If you use here voltage source, it is also correct.

Now $Z_{11} = Z_{33} \approx 73(\Omega)$ (resonate halfwave dipole) and $Z_{31} \approx -10 - j30(\Omega)$ (Fig. 10.9). Therefore, the embedded far-field function of dipole 1 is

$$\mathbf{G}_{emb\_d1}(\theta, \varphi) = \mathbf{G}_{dy\_d1}(\theta, \varphi) \cdot e^{jk\mathbf{r}_{d1} \cdot \hat{r}} + \mathbf{G}_{dy\_d3}(\theta, \varphi) \cdot e^{jk\mathbf{r}_{d3} \cdot \hat{r}}$$

where from (5.11) and (5.12),

$$\mathbf{G}_{dy\_d1}(\theta, \varphi) = C_k \eta I_1 (cos\theta sin\varphi \hat{\theta} + cos\varphi \hat{\varphi}) \tilde{j}(\theta, \varphi)$$
$$\mathbf{G}_{dy\_d3}(\theta, \varphi) = C_k \eta I_3 (cos\theta sin\varphi \hat{\theta} + cos\varphi \hat{\varphi}) \tilde{j}(\theta, \varphi)$$

and from (2.52) we have the term of $e^{jk\mathbf{r}_{d1} \cdot \hat{r}}$.

4.3 Write out the far-field function of the whole array when the excitations are $I_1 = I_2 = I_3 = I_4 = I_0$. (4p)

A:

Although there are mutual couplings, since we use current source, the excitation currents on all dipoles are the same. Therefore, we have

$$\mathbf{G}_{whole}(\theta, \varphi) = \mathbf{G}_{d1}(\theta, \varphi) \cdot e^{jk\mathbf{r}_{d1} \cdot \hat{r}} + \mathbf{G}_{d2}(\theta, \varphi) \cdot e^{jk\mathbf{r}_{d2} \cdot \hat{r}}$$
$$+\mathbf{G}_{d3}(\theta, \varphi) \cdot e^{jk\mathbf{r}_{d3} \cdot \hat{r}} + \mathbf{G}_{d4}(\theta, \varphi) \cdot e^{jk\mathbf{r}_{d4} \cdot \hat{r}}$$

where (using (4.79) and (5.5-5.7))

$$\mathbf{G}_{d1}(\theta,\varphi) = C_k\eta I_0(cos\theta sin\varphi\hat{\theta} + cos\varphi\hat{\varphi})\tilde{j}_y(\theta,\varphi)$$
$$\mathbf{G}_{d2}(\theta,\varphi) = C_k\eta I_0(cos\theta cos\varphi\hat{\theta} - sin\varphi\hat{\varphi})\tilde{j}_x(\theta,\varphi)$$
$$\mathbf{G}_{d3}(\theta,\varphi) = C_k\eta I_0(cos\theta sin\varphi\hat{\theta} + cos\varphi\hat{\varphi})\tilde{j}_y(\theta,\varphi)$$
$$\mathbf{G}_{d2}(\theta,\varphi) = C_k\eta I_0(cos\theta cos\varphi\hat{\theta} - sin\varphi\hat{\varphi})\tilde{j}_x(\theta,\varphi)$$

and

$$\mathbf{r}_{d1} = \frac{\lambda}{4}\hat{x}; \mathbf{r}_{d2} = \frac{\lambda}{4}\hat{y}; \mathbf{r}_{d3} = -\frac{\lambda}{4}\hat{x}; \mathbf{r}_{d4} = -\frac{\lambda}{4}\hat{y};$$

4.4 Calculate the embedded impedance of dipole 1. You should sketch the equivalent circuit first, and then use data and the figures in Per-Simon's book for the calculations. (4p)

A:

The equivalent circuit is shown in Fig. 12.

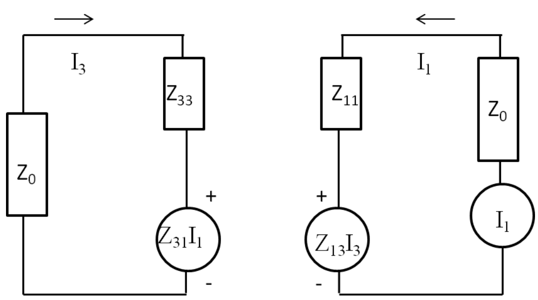

Figure 12: Equivalent circuit for embedded dipole 1.

$$Z_{in} = Z_{11} + Z_{13}\frac{I_3}{I_1} = Z_{11} - Z_{13}\frac{Z_{31}}{Z_0 + Z_{33}}$$
$$= 73 - (-10 - j30)\frac{-10 - j30}{50 + 73} = 79.5 - j5 \approx 79.5(\Omega)$$

Now we use two dipoles in the array as a diversity antenna.

4.5 Please sketch the equivalent circuit for the diversity antenna. (4p)

A:

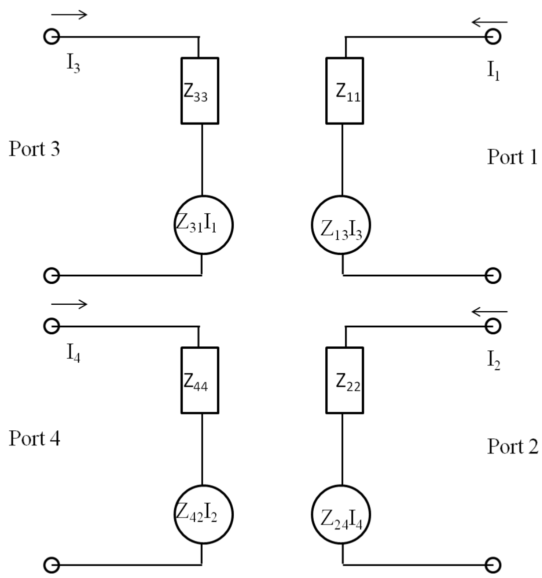

Figure 13: Equivalent circuit for the diversity antenna.

The equivalent circuit for the 4-dipole diversity antenna is (note there is no coupling between the orthogonal dipoles) Then using any two of it can make a two-dipole diversity antenna.

4.6 Please calculate the effective and apparent diversity gains of this antenna if selection combining and a CDF-level of 1% are assumed for the following two cases: (a) use only dipoles 1 and 2, while dipoles 3 and 4 are terminated with loads (i.e. $I_3 = I_4 = 0$); (b) use only dipoles 1 and 3, while dipoles 2 and 4 are terminated with loads (i.e. $I_2 = I_4 = 0$). (7p)

A:

(a)

Now the equivalent circuit is shown in Fig. 14.

We know that $S_{12} = 0$ (no mutual couplings between the orthogonal dipoles), and (from 4.4)

$$S_{11} = \frac{Z_0 - Z_{in}}{Z_0 + Z_{in}} \approx \frac{50 - (79.5)}{50 + 79.5} = -0.23$$

Therefore, we have

$$|\rho| = |\frac{S_{11}^* S_{12} + S_{21}^* S_{22}}{\left[1 - \left(|S_{11}|^2 + |S_{21}|^2\right)\right]\left[1 - \left(|S_{21}|^2 + |S_{22}|^2\right)\right]}| = 0 \quad (5)$$

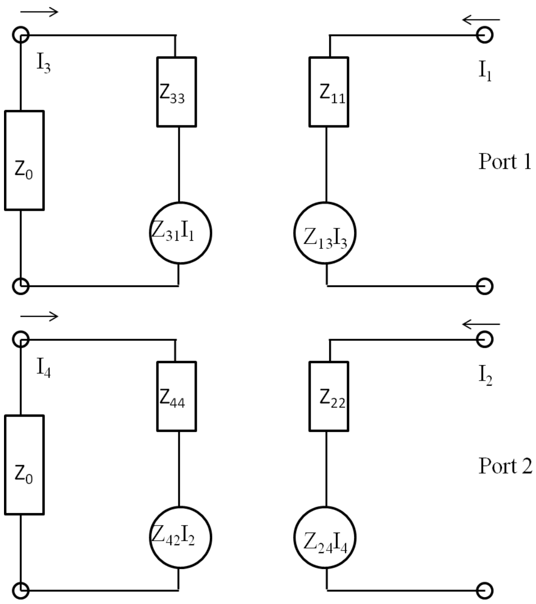

Figure 14: Equivalent circuit od the diversity antenna using dipoles 1 and 2.

The apparent diversity gain is given by equation 3.12 in the book:

$$G_{app} = 10\sqrt{1 - |\rho|^2} = 10 = 10dB$$

The effective diversity gain is defined as the apparent diversity gain multiplied with the radiation efficiency, equation 3.11. We have by using equation 3.7 (note that dipole 3 is terminated with the load so this load will absorb some power)

$$G_{eff} = e_{rad}G_{app} = (1 - |S_{11}|^2)e_{abs}G_{app}$$

where (3.6 in the book)

$$e_{abs} = 1 - \frac{Z_0|I_3|^2}{Re(Z_{in})|I_1|^2} = 1 - \frac{Z_0}{Re(Z_{in})} \cdot |\frac{Z_{31}}{Z_0 + Z_{33}}|^2$$
$$= 1 - \frac{50}{66.5)} \cdot |\frac{-10 - j30}{50 + 73}|^2 = 0.9503 = -0.2214dB$$

$$(1 - |S_{11}|^2) = 0.9481 = -0.23dB$$

So

$$G_{eff} = e_{rad}G_{app} = 10 - 0.2214 - 0.23 = 9.55dB$$

(b)

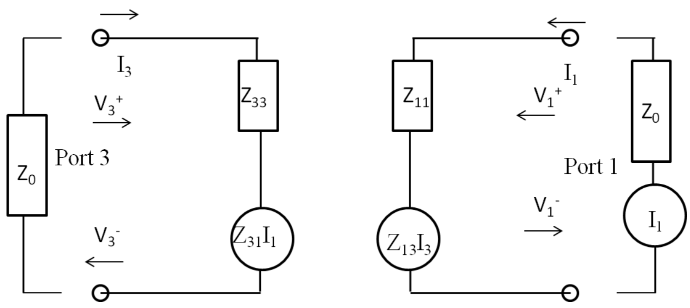

Figure 15: Equivalent circuit od the diversity antenna using dipoles 1 and 2.

The equivalent circuit is shown in Fig.15

This is the same as last year's exam question 3. Please refer the solution to exam 20120525. We have

$$S_{31} = S_{13} = \frac{100Z_{31}}{(Z_{11}+50)(Z_{33}+50) - Z_{31}Z_{13}} = -0.0556 - 0.1904i$$

Then

$$e_{rad} = 1 - |S_{11}|^2 - |S_{13}|^2 = 1 - (-0.23)^2 - |-0.0556 - 0.1904i|^2 = 0.9078 = -0.42dB$$

$$|\rho| = |\frac{S_{11}^* S_{13} + S_{31}^* S_{33}}{\left[1 - \left(|S_{11}|^2 + |S_{31}|^2\right)\right]\left[1 - \left(|S_{31}|^2 + |S_{33}|^2\right)\right]}| = 0.0310$$
$$\sqrt{1-|\rho|^2} = -0.0023dB \approx 0dB$$
$$G_{app} = 10dB$$
$$G_{eff} = e_{rad}G_{app} = -0.42 + 10 = 9.58dB$$