SSY080

Transformer, Signaler och System

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Date: 25/08/21, Time: 4 h (14.00-18.00)

Grading system

10 Quest A	**1 point each**	**10 points in total**	**5/10 necessary to pass**
3 Quest B	**5 points each**	**15 points in total**	**7/15 necessary to pass**

Note: only a **complete answer** results in the **full point** (A) / **points** (B).

Points	**12-15**	**16-20**	**20-25**
Final grade	**3**	**4**	**5**

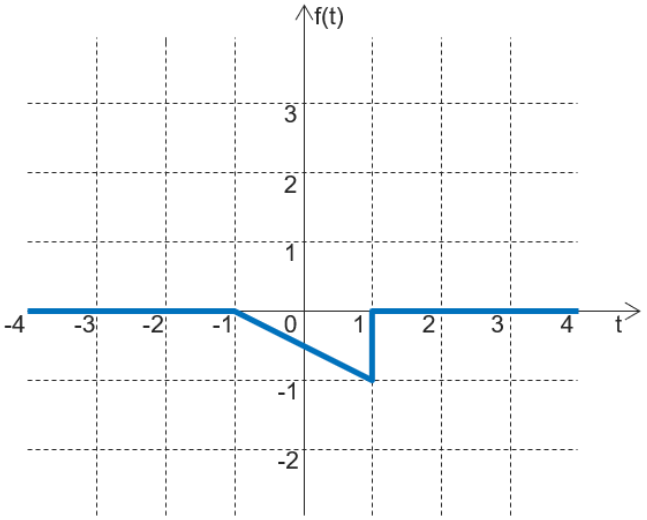
At the top of the first page, report **which questions you have answered** (e.g. A1, A3, A10, B2).

All answers must be written in **English**.

The solutions must be complete and easy to follow.

You can either write by hand or on a computer.

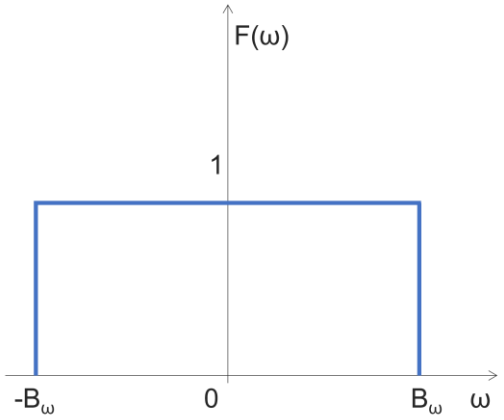
A1. Given the signal f(t) in the figure



Plot the signal y(t) = 1 + 2 f(-t). Motivate your answer.

A2. Given the two sequences $x_1[n] = 2\,\delta[n] + \delta[n-1] + \delta[n+1]$ and $x_2[n] = \delta[n-2] + \delta[n+2]$, plot the convolution x[n]= x_1[n] * x_2[n]. Motivate your answer.

A3. The signal f(t) with Fourier transform F(ω) sketched in the following figure is sampled at the frequency of $\omega_s = 4\pi$ r/s. B_ω is a positive number.



Does spectral aliasing occur? Motivate your answer.

A4. Consider the LTI system described by the following differential equation

$$\frac{d^2y(t)}{dt^2} - 3y(t) = f(t),$$

where f(t) is the system input and y(t) the system output. Determine the system transfer function in the Laplace domain. Motivate your answer.

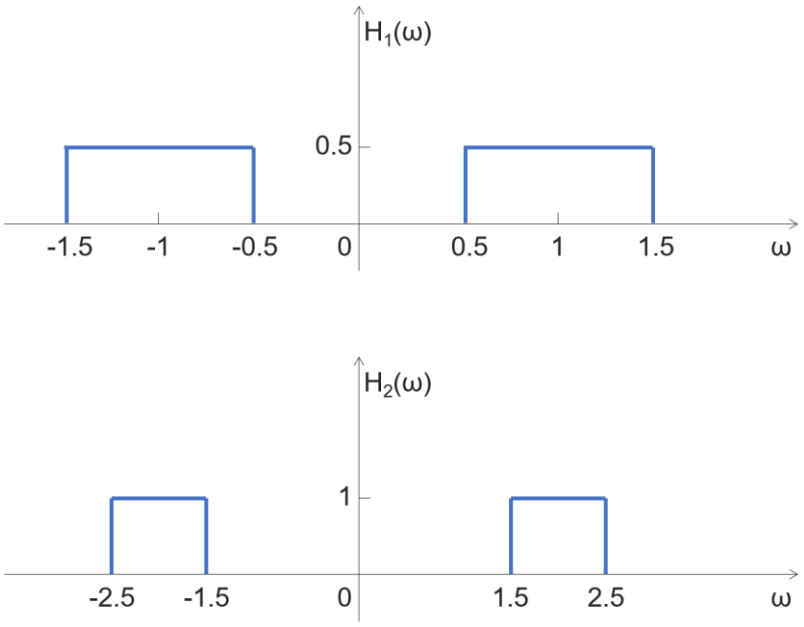
A5. Consider the LTI system described by the following differential equation

$$\frac{d^2y(t)}{dt^2} - 3y(t) = f(t),$$

where f(t) is the system input and y(t) the system output. Is the system stable? Motivate your answer.

A6. Given a LTI system with

- input x(t)=cos(t)
- impulse response h(t)= $h_1(t)$ * $h_2(t)$, with $H_1(\omega)$ Fourier transform of $h_1(t)$, and $H_2(\omega)$ Fourier transform of $h_2(t)$ represented in the figures below (not in scale) (ω expressed in radians / second)



The system output y(t) is equal to

a) y(t) = cos(t)
b) y(t) = 0.5 cos(t)
c) neither a or b

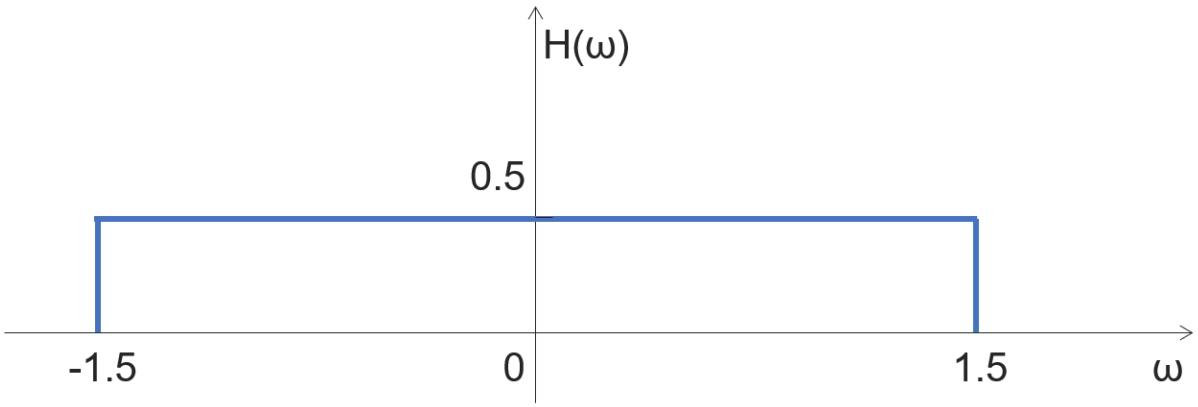
Motivate your answer.

A7. Given the periodic signal $x(t) = 5 + sin(3t) + 2\cos(9t)$, compute the coefficient c_0 of the complex Fourier series

$$x(t) = \sum_{k=-\infty}^{+\infty} c_k e^{jk\omega_0 t}.$$

Motivate your answer.

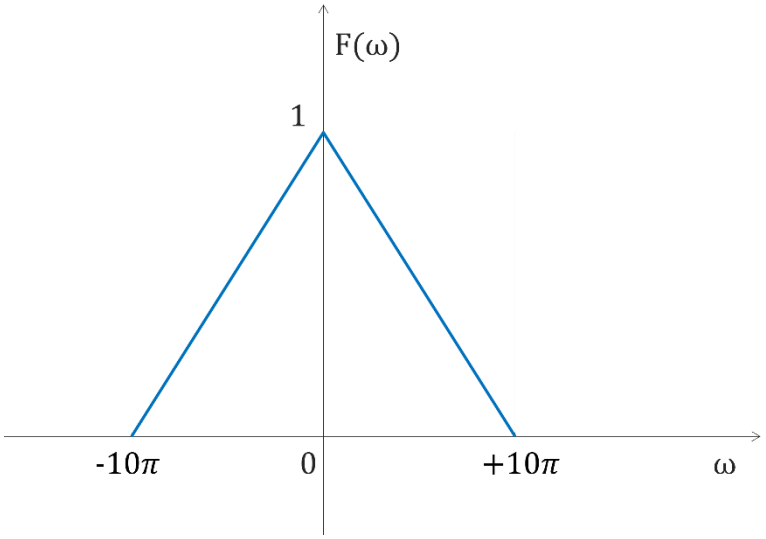
A8. The signal x(t)=cos(t) is filtered by a LTI system with Fourier transform of the impulse response $H(\omega)$ represented in the figures below (ω expressed in radians / second)



Which of the following statements is correct? Motivate your answer.

a. The filter smoothens the signal x(t)
b. The system is a low-pass filter
c. The filter enhances the high-frequency components of the signal x(t)
d. None of the above statements

A9. Given a signal $f(t)$ with Fourier transform $F(\omega)$ sketched in the following figure



with ω expressed in radians / second, plot the Fourier spectrum $\bar{F}(\omega)$ of the signal $\bar{f}(t)$ obtained by sampling the signal $f(t)$ at a rate of $\omega_s = 15\pi$ r/s. Motivate your answer.

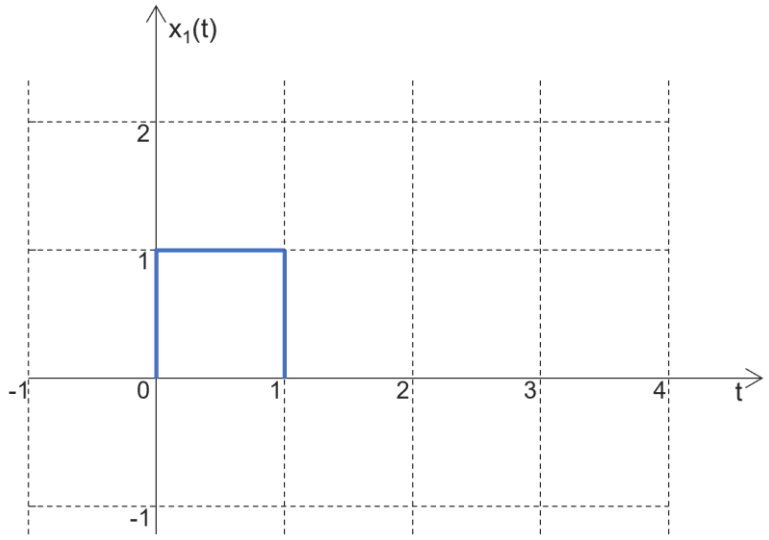
Note: if you want, you can ignore the amplitude of $\bar{F}(\omega)$.

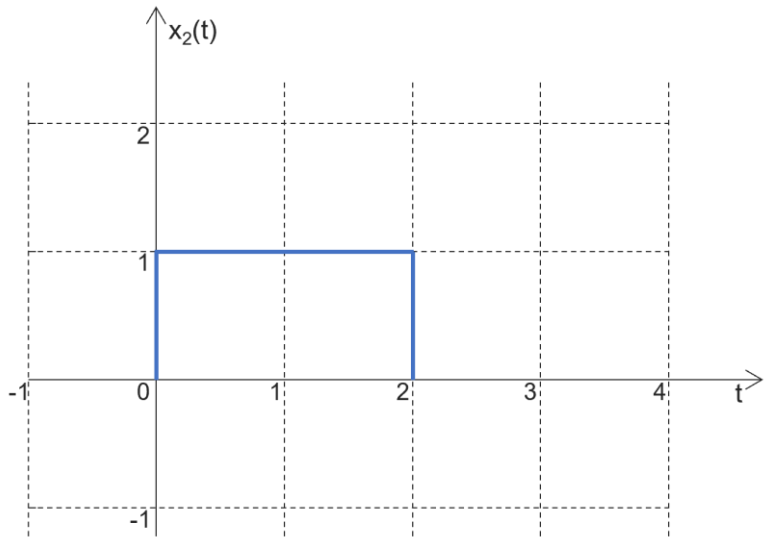
A10. Consider the following difference equation of a system with x[n] as input and y[n] as output.

$$y[n] - 0.8y[n-1] = 0.3x[n]$$

Find the transfer function H[z] using the z-transform.

B1. a. Compute the convolution $x(t) = x_1(t) * x_2(t)$





b. Plot x(t).

c. Using the results determined in a and b, plot $y(t) = x_1(t) * x_2(t-1)$.

NOTE: If you use a software to calculate the integrals, you need to indicate which software you used. Even if you solved the integrals with a software, you need to indicate the mathematical expression of the integrals.

I report a random example to clarify what I mean:

- $\int_a^b x(\tau)d\tau = \frac{b^2-a^2}{2}$ will NOT be considered correct, because the mathematical expression of $x(\tau)$ is not specified
- $\int_a^b x(\tau)d\tau = \int_a^b \tau\, d\tau = \frac{\tau^2}{2}\Big|_a^b = \frac{b^2-a^2}{2}$ will be considered correct
- In case you use a software to calculate the integrals, $\int_a^b x(\tau)d\tau = \int_a^b \tau\, d\tau = \frac{b^2-a^2}{2}$ will also be considered correct because the expression of $x(\tau)$ has been explicitly reported (provided that you indicate which software you used to calculate the integral).

B2. Given the signal $x(t) = 5\,cos\left(2t + \frac{\pi}{3}\right) + sin(4t)$,

a. Determine the fundamental frequency ω_0
b. Determine the coefficients c_k of the complex Fourier series

$$x(t) = \sum_{k=-\infty}^{+\infty} c_k e^{jk\omega_0 t}.$$

B3. Given a LTI system described by the transfer function

$$H(s) = \frac{s+5}{s^2+3s+2},$$

a. Find the impulse response
b. Find the differential equation that describes the system.