SSY080

Transformer, Signaler och System

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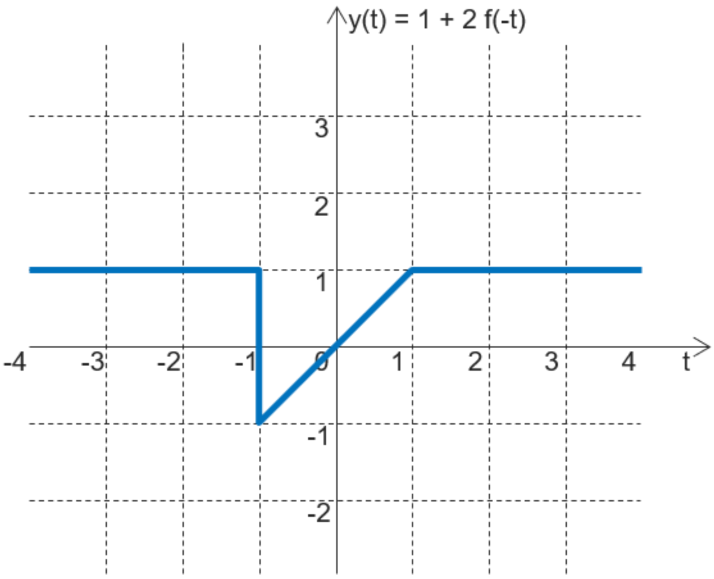
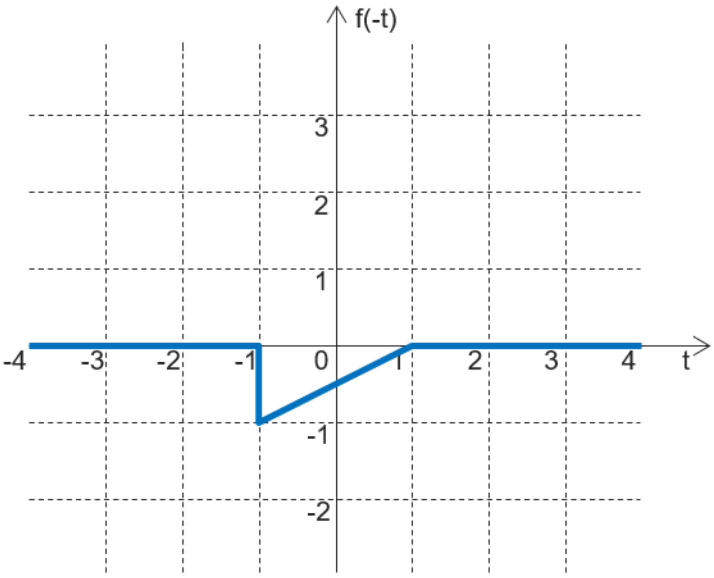
Date: 25/08/21, Time: 4 h (14.00-18.00)

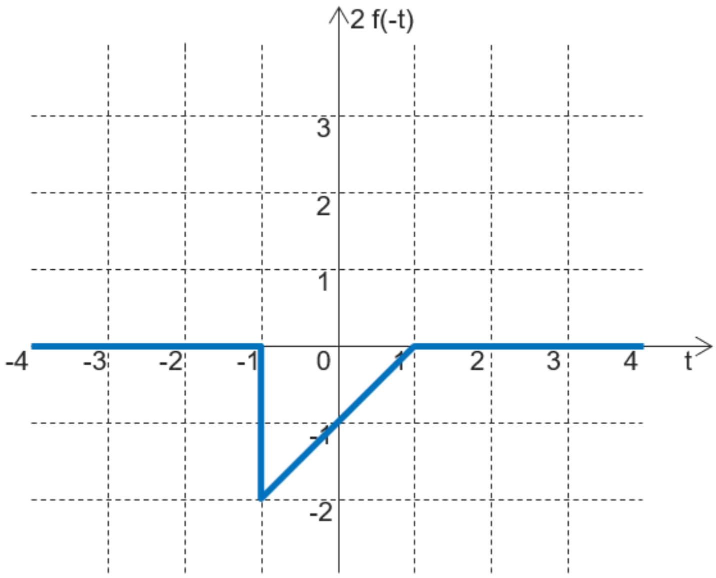
Solution

A1.

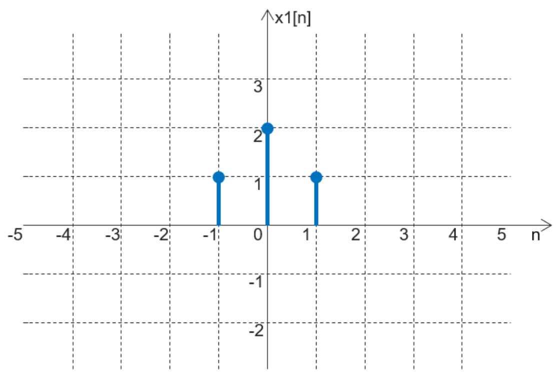


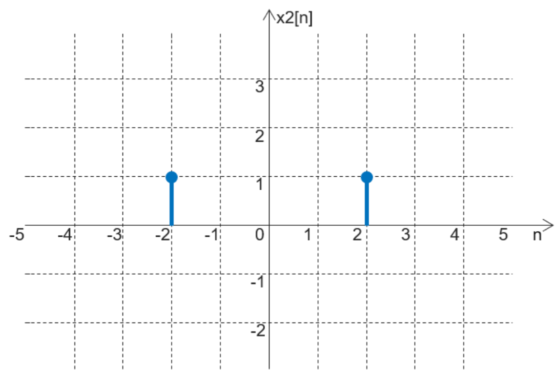
Motivation





A2.

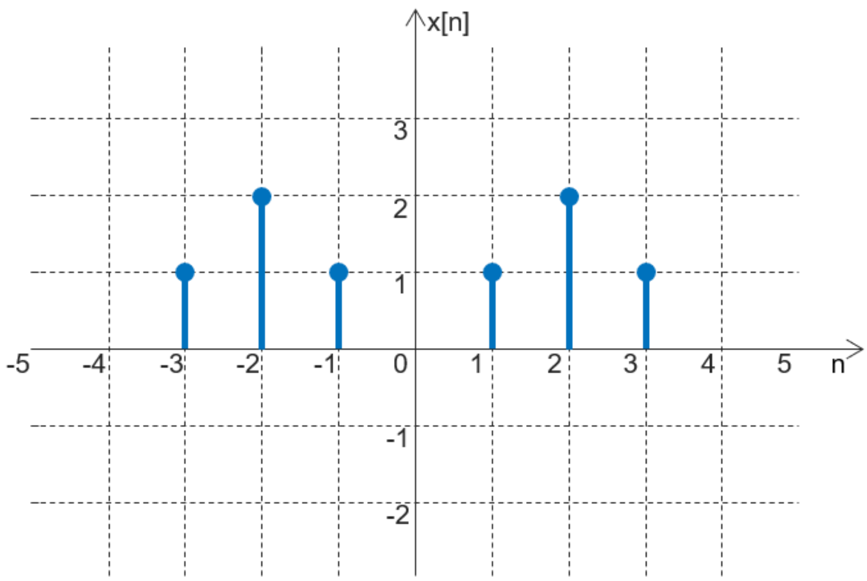




$$x[n] = (2\,\delta[n] + \delta[n-1] + \delta[n+1]) * (\delta[n-2] + \delta[n+2]) =$$

$$= (2\,\delta[n] + \delta[n-1] + \delta[n+1]) * \delta[n-2] + (2\,\delta[n] + \delta[n-1] + \delta[n+1]) * \delta[n+2] =$$

$$= 2\,\delta[n-2] + \delta[n-3] + \delta[n-1] + 2\,\delta[n+2] + \delta[n+1] + \delta[n+3]$$



A3. It depends on the value of B_ω.

According to the Nyquist theorem, $\omega_s \geq 2\,B_\omega$ for aliasing not to occur.

$4\pi \geq 2\,B_\omega$

$B_\omega \;\leq 2\pi$

A4.

$$s^2\,Y(s) - 3\,Y(s) = F(s)$$

$$H(s) = \frac{Y(s)}{F(s)} = \frac{1}{s^2 - 3}$$

A5. It stems from A4 that the system has poles $-\sqrt{3}$ and $\sqrt{3}$. The latter is in RHP. The system is therefore unstable.

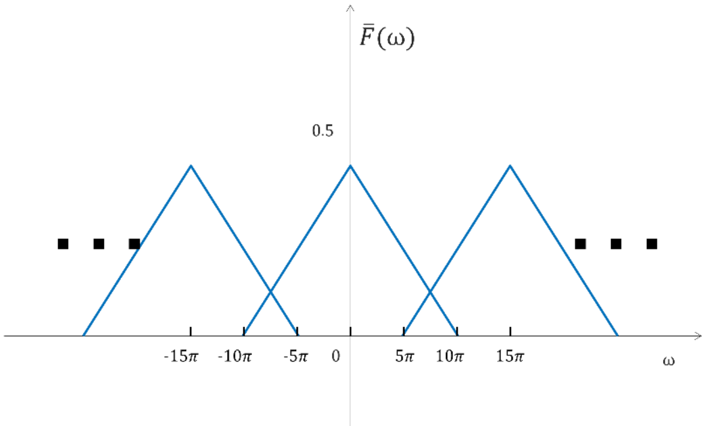
A6. C is the correct answer, $y(t) = 0$. In fact, $Y(\omega) = X(\omega)H_1(\omega)$, $X(\omega)$ is constituted by two pulses at $\omega = \pm 1$ radians/s, which are filtered out by $H_1(\omega)$.

A7. $c_0 = 5$ (DC component).

A8. b is the correct answer. It can be observed from $H(\omega)$ that the system retains only the low-frequency components. Usually, a low-pass filter smooths the signal, but not in this case because the whole spectrum of the signal is within the ideal filter bandwidth.

A9.

$$\bar{F}(\omega) = \frac{1}{T}\sum_{n=-\infty}^{+\infty} F(\omega - n\,\omega_s)$$



A10.

$(1 - 0.8z^{-1})Y[z] = 0.3X[z]$

$H[z] = \frac{Y[z]}{X[z]} = \frac{0.3z}{z-0.8}$

B1. a. $x(t) = x_1(t) * x_2(t) = x_2(t) * x_1(t) = \int_{-\infty}^{+\infty} x_2(\tau)\, x_1(t-\tau)\ d\tau$

$t \le 0$

$x(t) = 0$

$0 \le t \le 1$

$x(t) = \int_0^t d\tau = \tau \,|_0^t = t$

$1 \le t \le 2$

$x(t) = \int_{-1+t}^{t} d\tau = \tau \,|_{t-1}^{t} = t - (t-1) = 1$

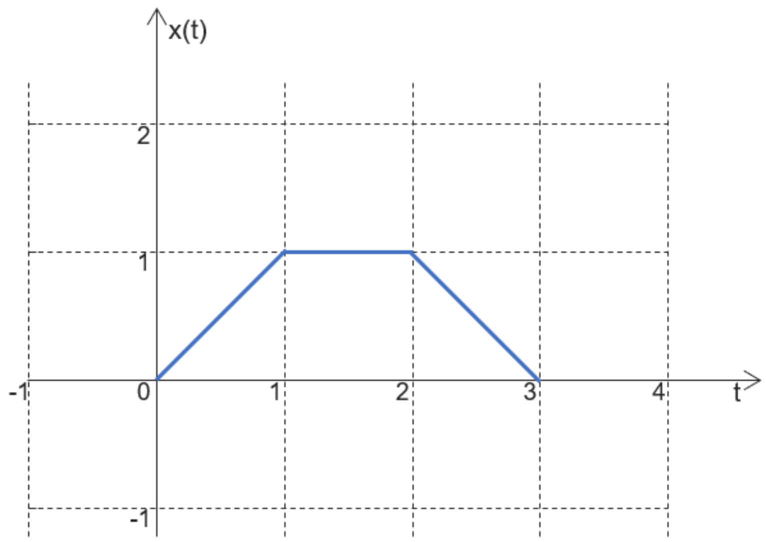
$2 \le t \le 3$

$x(t) = \int_{-1+t}^{2} d\tau = \tau \,|_{t-1}^{2} = 2 - (t-1) = 3 - t$

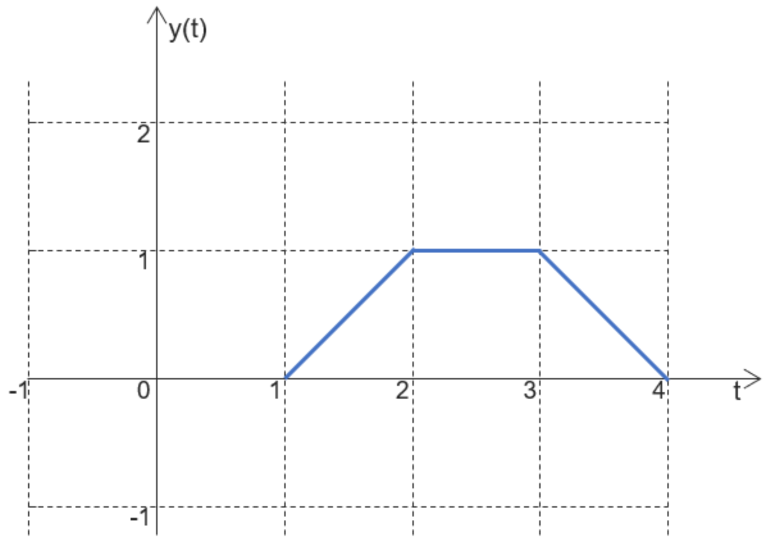
$t \ge 3$

$x(t) = 0$

b.



c. $y(t) = x_1(t) * x_2(t-1) = x(t-1)$



B1. a. Fundamental frequency

$$\omega_1 = 2 \quad \left[\frac{\text{r}}{\text{s}}\right], T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{2} = \pi \quad [\text{s}]$$
$$\omega_2 = 4 \quad \left[\frac{\text{r}}{\text{s}}\right], T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{4} = \frac{\pi}{2} \quad [\text{s}]$$

Common period $T = m_1 T_1 = m_2 T_2$, with $m_1, m_2 \in \mathbb{Z}$

$$T = \pi \ [s], m_1 = 1, m_2 = 2$$
$$\omega_0 = \frac{2\pi}{\pi} = 2 \quad \left[\frac{\text{r}}{\text{s}}\right]$$

b. Coefficients

$$x(t) = 5\,cos\left(2t + \frac{\pi}{3}\right) + sin(4t) = 5\,\frac{e^{j2t}e^{\frac{j\pi}{3}} + e^{-j2t}e^{-\frac{j\pi}{3}}}{2} + \frac{e^{j4t} - e^{-j4t}}{2j} =$$
$$= \frac{5}{2}e^{\frac{j\pi}{3}}\,e^{jk\omega_0 t}\Big|_{k=1} + \frac{5}{2}\,e^{-\frac{j\pi}{3}}\,e^{jk\omega_0 t}\Big|_{k=-1} + \frac{1}{2j}e^{jk\omega_0 t}\Big|_{k=2} - \frac{1}{2j}e^{jk\omega_0 t}\Big|_{k=-2}$$

$$c_k = \begin{cases} \frac{5}{2}e^{\frac{j\pi}{3}} & k = 1 \\ \frac{5}{2}e^{-\frac{j\pi}{3}} & k = -1 \\ \frac{1}{2j} = -\frac{j}{2} & k = 2 \\ -\frac{1}{2j} = \frac{j}{2} & k = -2 \\ 0 & otherwise \end{cases}$$

B3. a.

$$H(s) = \frac{s+5}{s^2+3s+2} = \frac{s+5}{(s+1)(s+2)} =$$

$$= \frac{A}{s+1} + \frac{B}{s+2}$$

$$A(s+2) + B(s+1) = s+5$$

$$\begin{cases} A+B=1 \\ 2A+B=5 \end{cases}$$

$$\begin{cases} B=1-A \\ 2A+1-A=5 \end{cases}$$

$$\begin{cases} A=4 \\ B=-3 \end{cases}$$

$$H(s) = \frac{4}{s+1} - \frac{3}{s+2}$$

$$h(t) = (4e^{-t} - 3e^{-t})u(t)$$

b.

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt} + 5x(t)$$