**SSY080**

**Transformer, Signaler och System**

**Examiner: Silvia Muceli muceli@chalmers.se**

**Date: 29/10/20, Time: 4 h (8.30-12.30)**

**Grading system**

| | | | |
|---|---|---|---|
| **10 Quest A** | **1 point each** | **10 points in total** | **5/10 necessary to pass** |
| **3 Quest B** | **5 points each** | **15 points in total** | **7/15 necessary to pass** |

Note: only a **complete answer** results in the **full point** (A) / **points** (B).

| **Points** | **[12,16)** | **[16-21)** | **[21-25]** |
|---|---|---|---|
| **Final grade** | **3** | **4** | **5** |

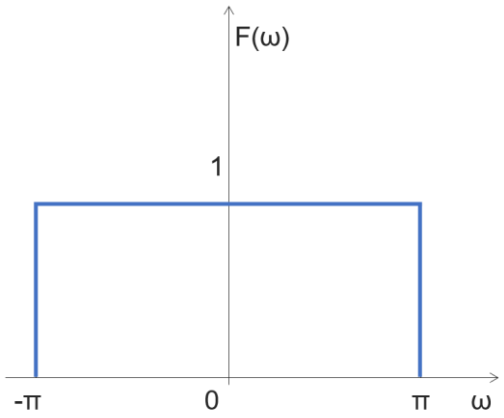
At the top of the first page, report **which questions you have answered** (e.g. A1, A3, A10, B2).

All answers must be written in **English**.

The solutions must be complete and easy to follow.

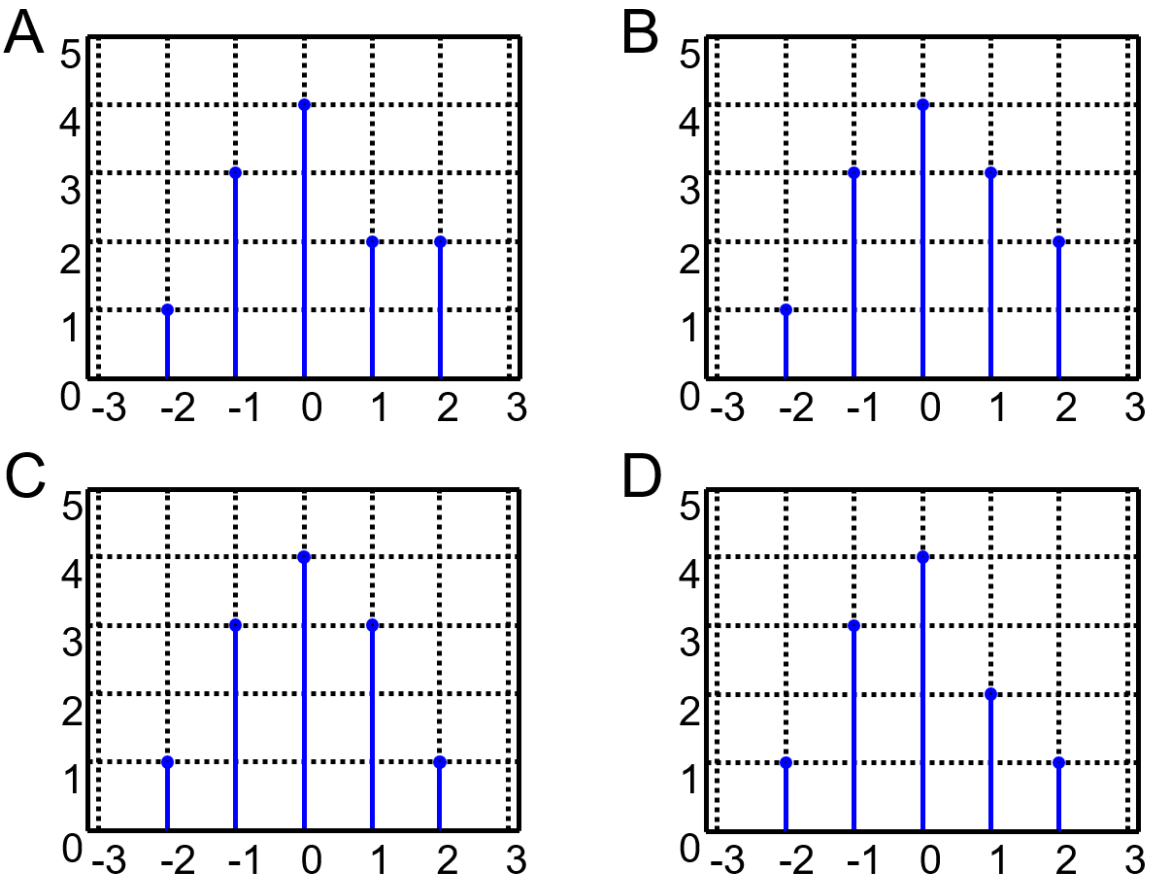
You can either write by hand or on a computer.

**A1.** Given a signal f(t) with Fourier transform F(ω) sketched in the following figure

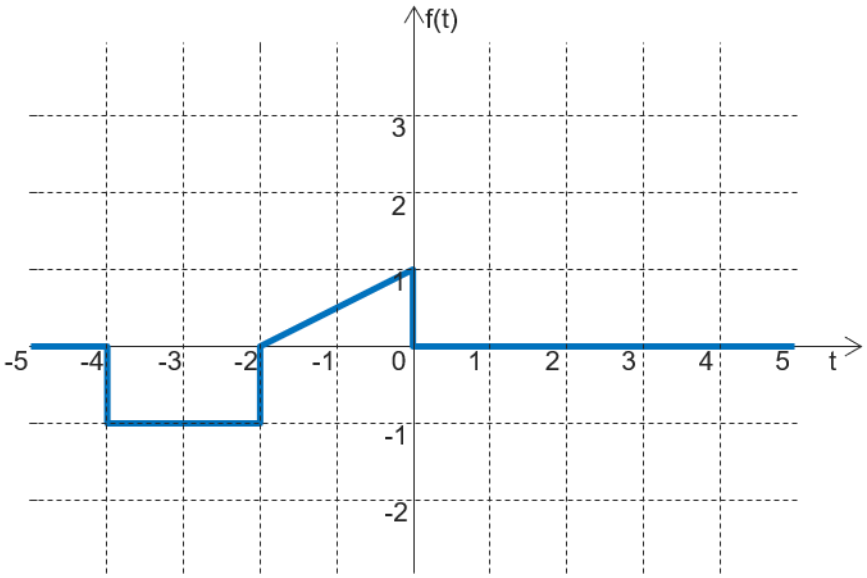



Determine the minimum sampling frequency to recover f(t) from its samples. Motivate your answer.

**A2.** Given the two sequences $x_1[n] = \delta[n+1] + 2\delta[n] + \delta[n-1]$ and $x_2[n] = u[n+1] - u[n-2]$, determine the convolution x[n]= $x_1$[n] * $x_2$[n]. One of the 4 options (A, B, C, D) is correct. Motivate your answer.

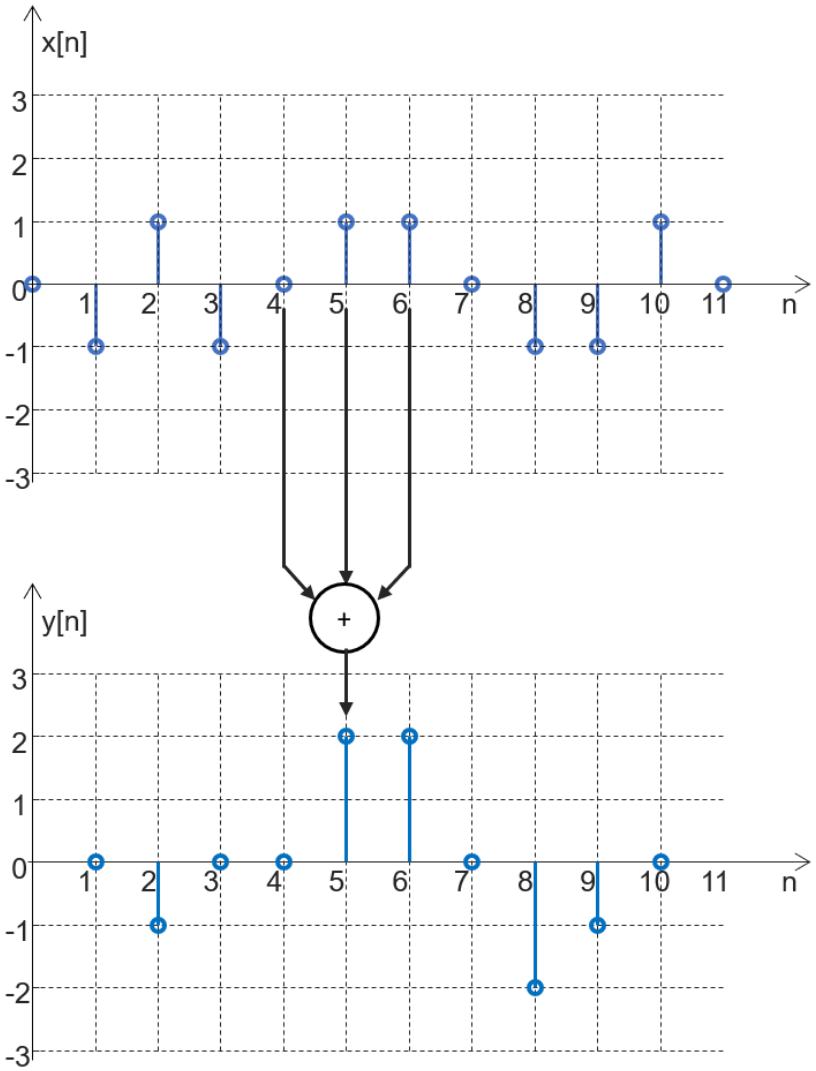



**A3.** Given the signal f(t) in the figure



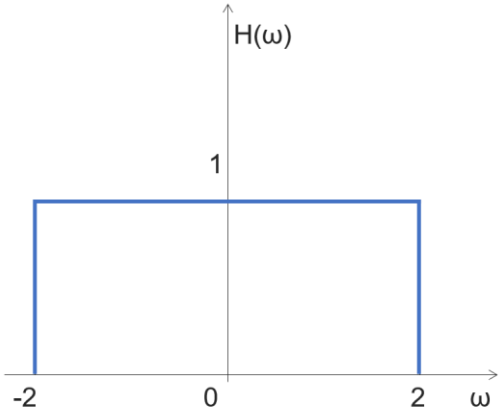

Plot the signal y(t)=f(2t-2). Motivate your answer.

**A4.** The figure represents the input x[n] and the output y[n] of a system. Is the system causal? Motivate your answer.




**A5.** Given a LTI system with

- input x(t)=cos(t)+sin(5t)
- and impulse response h(t) with $H(\omega) = \Im\{h(t)\}$ and $\Im$ indicating the Fourier transform



Which of the following options correspond to the system output y(t)?

A. y(t) = α cos(t) + β sin(5t)
B. y(t) = α cos(t)
C. y(t) = β sin(5t)

with α and β constant and ≠ 0. Motivate your answer.

**A6.** Plot the signal $x[n] = (u[n+1] - u[n-1])\delta[n]$, and determine its simplified analytical expression. Motivate your answer.

**A7.** Given the signal $x(t) = sinc(t)$, with $sinc(t) = \frac{\sin t}{t}$ determine which of the following options corresponds to $y(t) = \underbrace{sinc(t) * sinc(t) * \ \dots \ * sinc(t)}_{n\ times}$, where * is the convolution operator. Motivate your answer.

A. $y(t) = \pi^n sinc(t)$

B. $y(t) = \pi^{n-1} sinc(t)$

C. $y(t) = \big(sinc(t)\big)^{n-1}$

D. $y(t) = \big(sinc(t)\big)^{n}$

**A8.** Consider the following difference equation of a system with x[n] as input and y[n] as output.

$$y[n] - 0.9y[n-1] = 0.1x[n]$$

Find the transfer function H[z] using the z-transform.

**A9.** Given the signal

$$x(t) = \sum_{n=-\infty,\ n\ odd}^{+\infty} \big(u(t-2n) - u(t-2n-1)\big)$$

determine if it is periodic (Y/N). Motivate your answer. In case it is, determine the fundamental period $T_0$.

**A10.** Given a system with transfer function $H(s) = \frac{1}{s+1}$, determine if the system is stable. Motivate your answer.

**B1.** Consider the LTI system described by the following differential equation

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = x(t)$$

x(t) is the system input and y(t) the system output.

Find the unit step response.

Report the calculation to motivate your answer.

**B2.** Given the signal $x(t) = sin(3t) + 4\cos(9t)$,

a. Determine the fundamental frequency $\omega_0$
b. Determine the coefficients $c_k$ of the complex Fourier series

$$x(t) = \sum_{k=-\infty}^{+\infty} c_k e^{jk\omega_0 t}.$$

**B3.** Compute and plot the convolution between $x_1(t)$ and $x_2(t)$.