SSY080

Transformer, Signaler och System

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Date: 29/10/20, Time: 4 h (8.30-12.30)

Grading system

10 Quest A	**1 point each**	**10 points in total**	**5/10 necessary to pass**
3 Quest B	**5 points each**	**15 points in total**	**7/15 necessary to pass**

Note: only a **complete answer** results in the **full point** (A) / **points** (B).

Points	**[12,16)**	**[16-21)**	**[21-25]**
Final grade	**3**	**4**	**5**

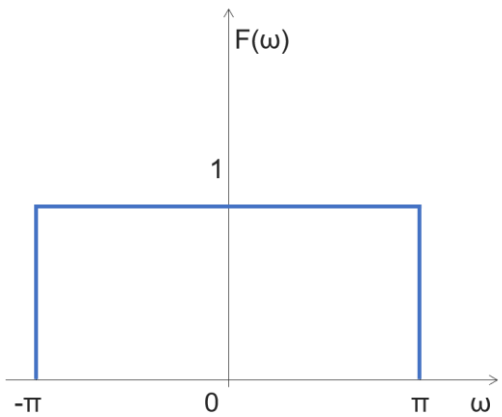
At the top of the first page, report **which questions you have answered** (e.g. A1, A3, A10, B2).

All answers must be written in **English**.

The solutions must be complete and easy to follow.

You can either write by hand or on a computer.

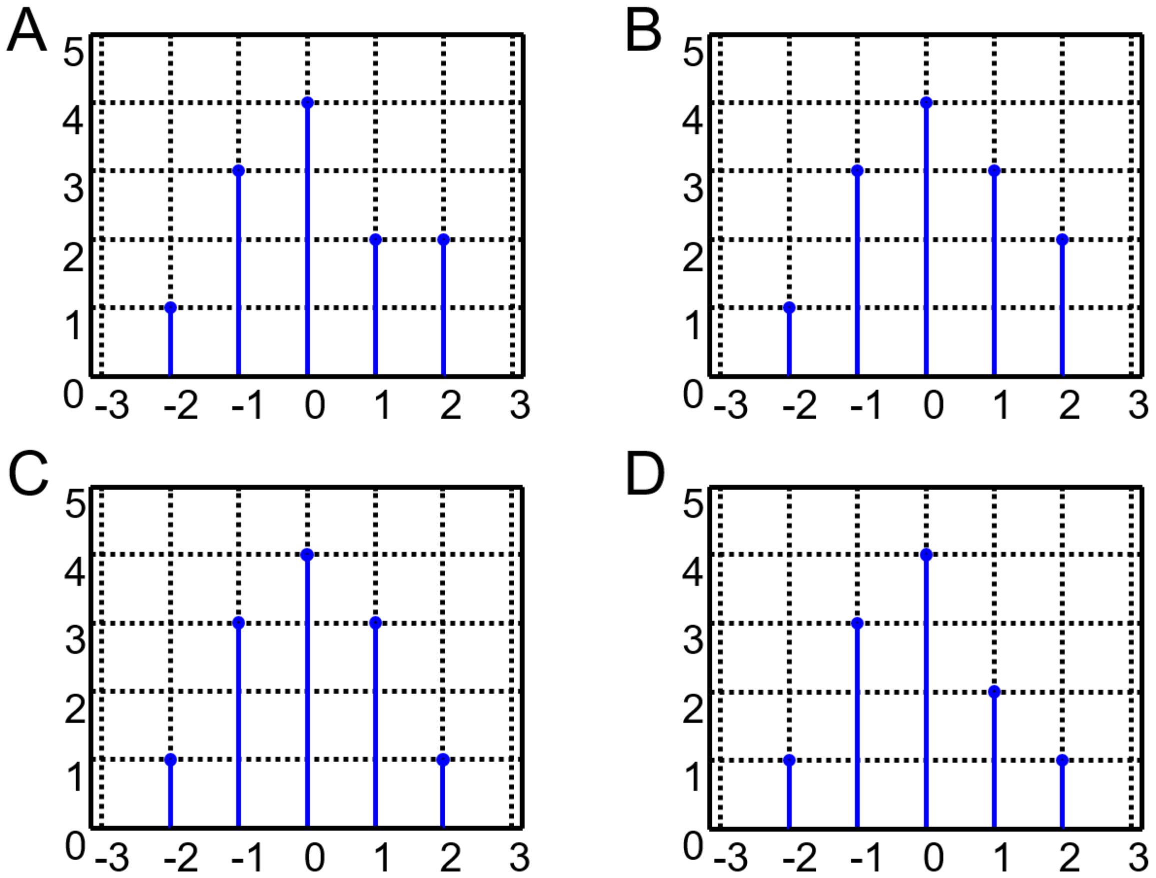
A1. Given a signal f(t) with Fourier transform F(ω) sketched in the following figure



Determine the minimum sampling frequency to recover f(t) from its samples. Motivate your answer.

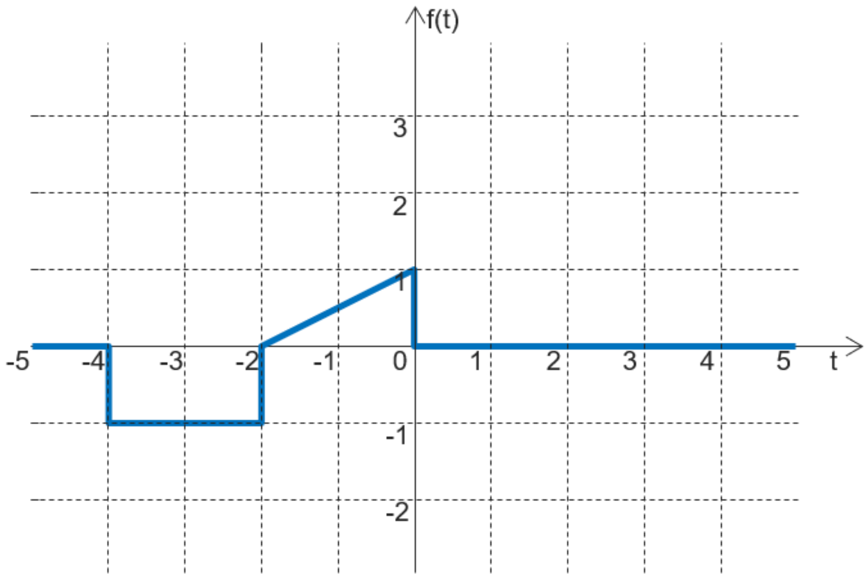
The bandwidth of the signal is B = π r/s. Therefore, the minimum sampling frequency is $\omega_s = 2\pi$ r/s.

A2. Given the two sequences $x_1[n] = \delta[n+1] + 2\delta[n] + \delta[n-1]$ and $x_2[n] = u[n+1] - u[n-2]$, determine the convolution x[n]= $x_1[n]$ * $x_2[n]$. One of the 4 options (A, B, C, D) is correct. Motivate your answer.

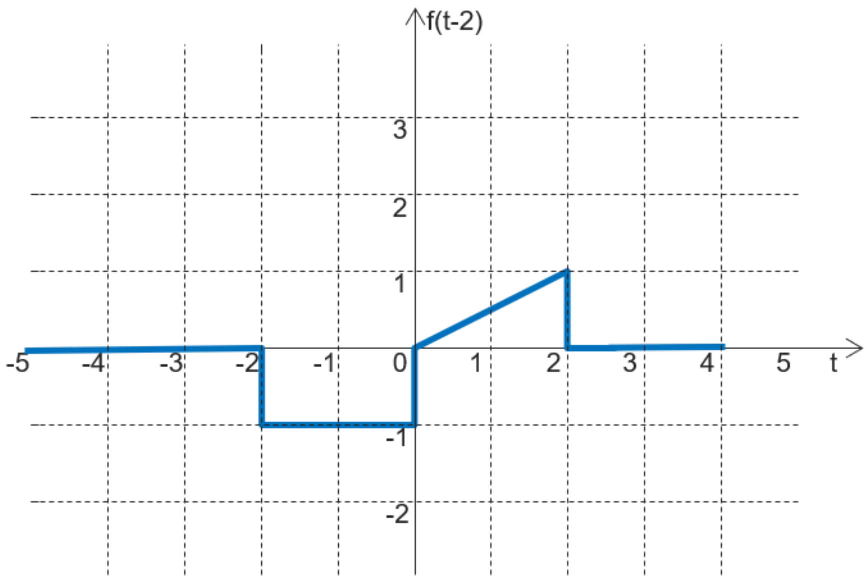


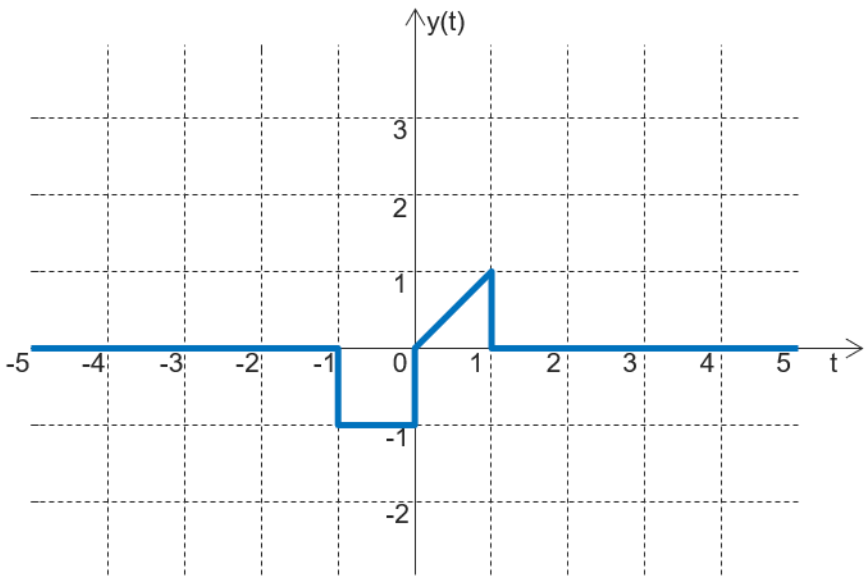
C is the correct answer. It is the only symmetric function (the convolution of two symmetric functions is symmetric).

A3. Given the signal f(t) in the figure

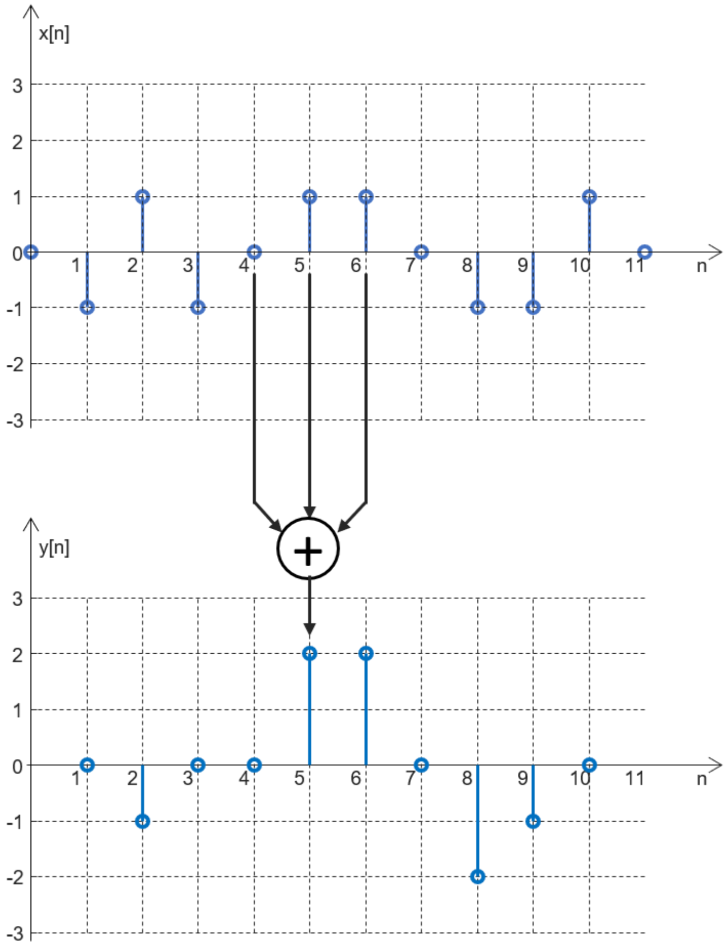


Plot the signal y(t)=f(2t-2). Motivate your answer.





A4. The figure represents the input x[n] and the output y[n] of a system. Is the system causal? Motivate your answer.

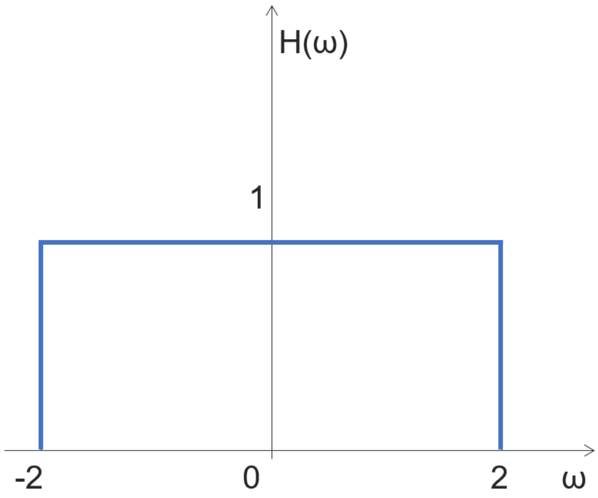


A causal system depends only on earlier times and the present time, not on future times.

In our case, the output of the system also depends on **future time** (y[5]=x[4]+x[5]+**x[6]**). Therefore, the system is not causal.

A5. Given a LTI system with

- **input x(t)=cos(t)+sin(5t)**
- **and impulse response h(t) with $H(\omega) = \mathfrak{I}\{h(t)\}$ and $\mathfrak{I}$ indicating the Fourier transform**



Which of the following options correspond to the system output y(t)?

A. y(t) = α cos(t) + β sin(5t)

B. y(t) = α cos(t)

C. y(t) = β sin(5t)

with α and β constant and ≠ 0. Motivate your answer.

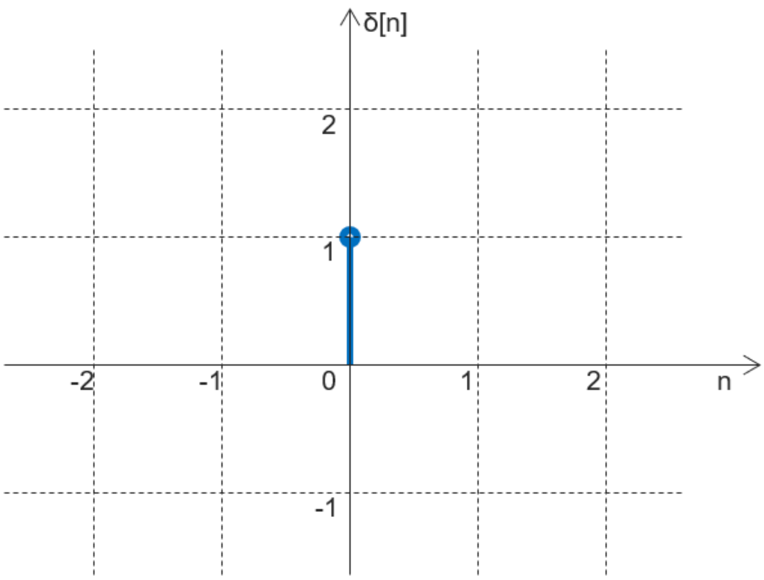
B, because the component sin(5t) is filtered out.

A6. Plot the signal $x[n] = (u[n+1] - u[n-1])\delta[n]$, and determine its simplified analytical expression. Motivate your answer.

$$g[n] = (u[n+1] - u[n-1])$$

Applying the sifting property of the unit impulse

$$x[n] = g[0]\delta[n] = \delta[n]$$



A7. Given the signal $x(t) = sinc(t)$, with $sinc(t) = \frac{\sin t}{t}$ determine which of the following options corresponds to $y(t) = \underbrace{sinc(t) * sinc(t) * \ \dots\ * sinc(t)}_{n\ times}$, where * is the convolution operator. Motivate your answer.

A. $y(t) = \pi^n sinc(t)$

B. $y(t) = \pi^{n-1} sinc(t)$

C. $y(t) = \left(sinc(t)\right)^{n-1}$

D. $y(t) = \left(sinc(t)\right)^{n}$

B is the correct answer.

$$Y(\omega) = \Im\{y(t)\} = \underbrace{\pi\, rect\left(\frac{\omega}{2}\right) \cdot \pi\, rect\left(\frac{\omega}{2}\right) \cdot \ \dots\ \cdot \pi\, rect\left(\frac{\omega}{2}\right)}_{n\ times} = \pi^{n-1} \left(\pi\, rect\left(\frac{\omega}{2}\right)\right)$$

$$y(t) = \Im^{-1}\{Y(\omega)\} = \pi^{n-1}\, \Im^{-1}\left\{\pi\, rect\left(\frac{\omega}{2}\right)\right\} = \pi^{n-1} sinc(t)$$

A8. Consider the following difference equation of a system with x[n] as input and y[n] as output.

$$y[n] - 0.9y[n-1] = 0.1x[n]$$

Find the transfer function H[z] using the z-transform.

$$(1 - 0.9z^{-1})Y[z] = 0.1X[z]$$

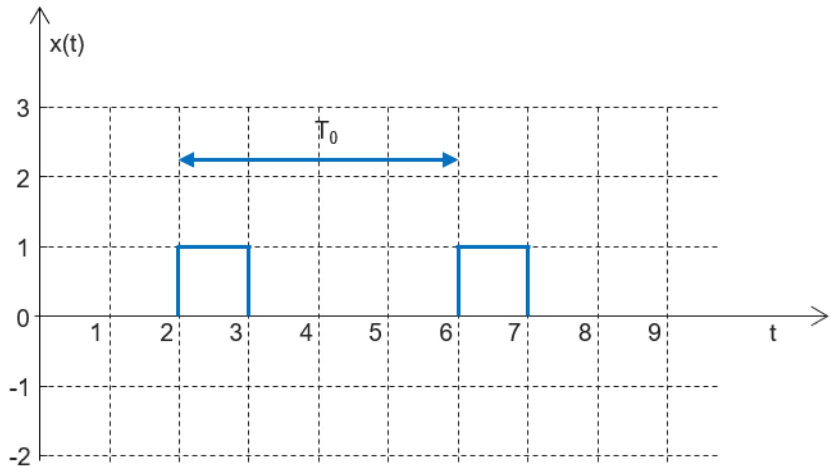
$$H[z] = \frac{Y[z]}{X[z]} = \frac{0.1z}{z - 0.9}$$

A9. Given the signal

$$x(t) = \sum_{n=-\infty,\ n\ odd}^{+\infty} \left(u(t-2n) - u(t-2n-1)\right)$$

determine if it is periodic (Yes/No). Motivate your answer. In case it is, determine the fundamental period T_0.

The signal is periodic (see figure).



$$T_0 = 4$$

A10. Given a system with transfer function $H(s) = \frac{1}{s+1}$, determine if the system is stable. Motivate your answer.

This system has one pole s=-1. The pole is in the LHP. The system is stable.

B1. Consider the LTI system described by the following differential equation

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = x(t)$$

x(t) is the system input and y(t) the system output.

Find the unit step response.

Report the calculation to motivate your answer.

The unit step response is the system output when the input x(t)=u(t).

Applying the Laplace transform

$$s^2\, Y(s) + 3\, s\, Y(s) + 2\, Y(s) = \frac{1}{s}$$

Therefore

$$Y(s) = \frac{1}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$$

$$A(s^2 + 3s + 2) + B(s^2 + 2s) + C(s^2 + s) = 1$$

$$\begin{cases} A + B + C = 0 \\ 3A + 2B + C = 0 \\ 2A = 1 \end{cases}$$

$$\begin{cases} A = \frac{1}{2} \\ B + C = -\frac{1}{2} \\ 2B + C = -\frac{3}{2} \end{cases}$$

$$\begin{cases} A = \frac{1}{2} \\ B = -1 \\ C = \frac{1}{2} \end{cases}$$

$$Y(s) = \frac{1}{2}\frac{1}{s} - \frac{1}{s+1} + \frac{1}{2}\frac{1}{s+2}$$

$$y(t) = \left(\frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t}\right)u(t)$$

B2. Given the signal $x(t) = sin(3t) + 4\cos(9t)$,

a. **Determine the fundamental frequency ω_0**
b. **Determine the coefficients c_k of the complex Fourier series**

$$x(t) = \sum_{k=-\infty}^{+\infty} c_k e^{jk\omega_0 t}.$$

a. Fundamental frequency

$$\omega_1 = 3 \quad \left[\frac{\text{r}}{\text{s}}\right], T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{3} \quad [\text{s}]$$

$$\omega_2 = 9 \quad \left[\frac{\text{r}}{\text{s}}\right], T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{9} \quad [\text{s}]$$

Common period $T = m_1 T_1 = m_2 T_2$, with $m_1, m_2 \in \mathbb{Z}$

$$T = \frac{2\pi}{3} \quad [s]\,, m_1 = 1, m_2 = 3$$

$$\omega_0 = \frac{2\pi}{T} = 3 \quad \left[\frac{\text{r}}{\text{s}}\right]$$

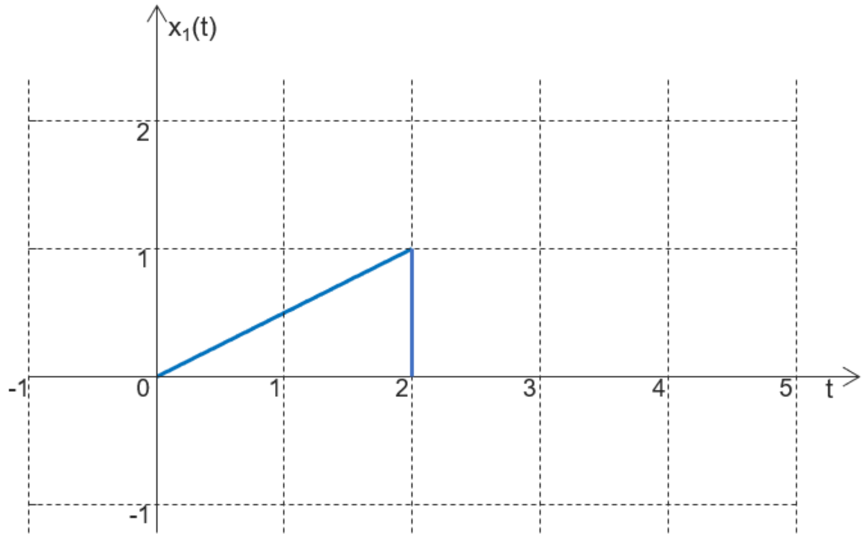
b. Coefficients

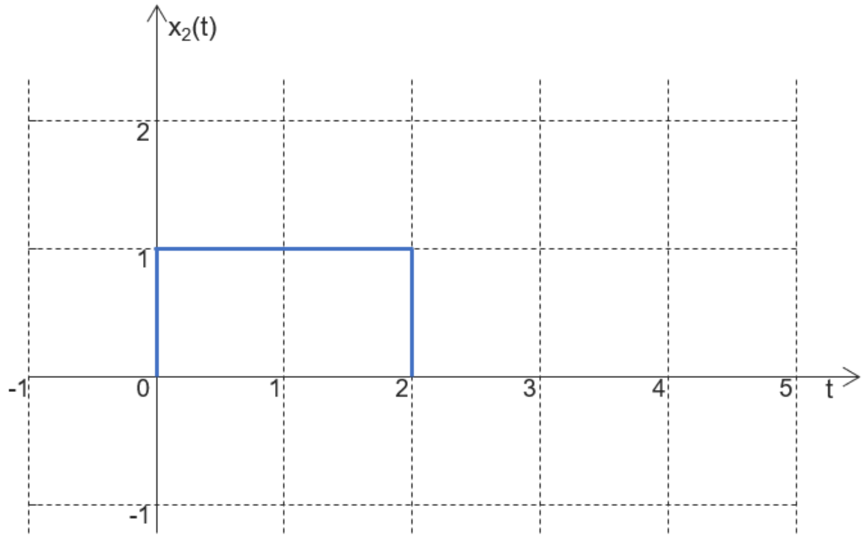
$$x(t) = \frac{1}{2j}\left(e^{j3t} - e^{-j3t}\right) + \frac{4}{2}\left(e^{j9t} + e^{-j9t}\right) =$$

$$\frac{1}{2j}e^{jk\omega_0 t}\Big|_{k=1} - \frac{1}{2j}e^{jk\omega_0 t}\Big|_{k=-1} + 2e^{jk\omega_0 t}\Big|_{k=3} + 2e^{jk\omega_0 t}\Big|_{k=-3}$$

$$c_k = \begin{cases} \frac{1}{2j} & k = 1 \\ -\frac{1}{2j} & k = -1 \\ 2 & k = \pm 3 \\ 0 & otherwise \end{cases}$$

B3. Compute and plot the convolution between $x_1(t)$ and $x_2(t)$.





$y(t) = x_1(t) * x_2(t)$

$t \leq 0$

$y(t) = 0$

$0 \leq t \leq 2$

$$y(t) = \frac{1}{2}\int_0^t \tau \, d\tau = \frac{1}{4}\tau^2\Big|_0^t = \frac{1}{4}t^2$$

$2 \leq t \leq 4$

$$y(t) = \frac{1}{2}\int_{-2+t}^{2} \tau \, d\tau = \frac{1}{4}\tau^2\Big|_{t-2}^{2} = \frac{1}{4}(4-(t-2)^2) = \frac{1}{4}(4t-t^2)$$

$t \geq 4$

$y(t) = 0$