SSY080 2020-01-8

D3

A1. Periodtid $T = 8$

$$\omega_0 = \frac{2\pi}{T} = \frac{\pi}{4}$$

a_0 är signalens medelvärde: $a_0 = \frac{14}{2} = 7$

$x(t)$ jämn signal, $b_n \sin(n\omega_0 t)$ är alla udda $\Rightarrow b_n = b_1 = 0$

A2. $y(t) = \sin(3t) + \cos\left(\frac{15}{4}t\right)$, $\forall t$

$x_1 = \sin 3t$, $\omega_1 = 3$, $T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{3}$

$x_2 = \cos\left(\frac{15}{4}t\right)$, $\omega_2 = \frac{15}{4}$, $T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi \cdot 4}{15} = \frac{8\pi}{15}$

Gemensam periodtid (fundamental)

$T = k_1 \cdot T_1 = k_2 \cdot T_2$; $\frac{k_1}{k_2} = \frac{T_2}{T_1} = \frac{8\pi}{15} \cdot \frac{3}{2\pi} = \frac{4}{5}$

$T = k_1 T_1 = 4 \cdot \frac{2\pi}{3} = \frac{8\pi}{3}$ eller $T = k_2 T_2 = 5 \cdot \frac{8\pi}{15} = \frac{8\pi}{3}$

Svar: Ja! med $T = \frac{8\pi}{3}$

A3. $x(t) = e^{-at}u(t) \xleftrightarrow{FT} X(j\omega) = \frac{1}{a+j\omega}$

a) $|X(j\omega)| = \frac{1}{\sqrt{a^2+\omega^2}} = |X(-j\omega)| \Rightarrow$ Jämn

b) $\arg\{X(j\omega)\} = -\arctan\left(\frac{\omega}{a}\right) = -\arg\{X(-j\omega)\} \Rightarrow$ Udda

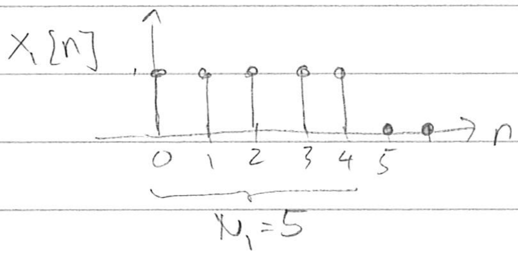
A4. $H_1(z) = \frac{1}{z-0{,}8} = z^{-1} \cdot \frac{z}{z-0{,}8} \Rightarrow h_1[n] = (0{,}8)^{n-1} u[n-1] \Rightarrow C$

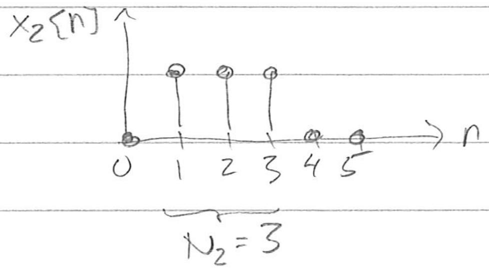
$H_2(z) = \frac{z}{z-0{,}8} \Rightarrow h_2[n] = 0{,}8^n \cdot u[n] \Rightarrow B$

$H_3(z) = \frac{0{,}8z}{z^2-1{,}6z+0{,}64} = \frac{0{,}8z}{(z-0{,}8)^2} \Rightarrow h_3 = n \cdot 0{,}8^n \cdot u[n] \Rightarrow D$

$H_4(z) = \frac{0{,}1z}{z^2-2z+1} = 0{,}1 \frac{z}{(z-1)^2} \Rightarrow h_4 = 0{,}1 \cdot n \cdot u[n] \Rightarrow A$

A5. $y[n] = x_1[n] * x_2[n]$





$y[n]$ har $N_1 + N_2 - 1 = 5+3-1 = \underline{\underline{7}}$ nollskilda värden

A6. Krav: poler innanför enhetscirkeln (EC)
Beräkna poler

(i) $z_{1,2} = +0{,}2 \pm \sqrt{0{,}2^2 + 0{,}45} = +0{,}2 \pm 0{,}7 = \begin{cases} +0{,}9 \\ -0{,}5 \end{cases}$

Alla poler innanför EC $\Rightarrow$ Stabilt

(ii) $z_{1,2} = +0{,}2 \pm \sqrt{0{,}2^2 + 0{,}96} = 0{,}2 \pm 1 = \begin{cases} 1{,}2 \\ -0{,}8 \end{cases}$

En pol utanför EC $\Rightarrow$ Instabilt

A7. $H(s) = \dfrac{s(s+b_1)(s^2+sb_2+b_3)}{(s+a_1)^2(s+a_2)^N}$ $\leftarrow M=4$ gradtal täljare
$\leftarrow N_1 = N+2$ gradtal nämnare

Lutning:
Krav: -40 dB / dekad för $|H(j\omega)|_{dB}$ då $\omega \to \infty$
Gradtal i nämnare 2 ggr större än gradtal på polynom i täljare
$N_1 - M = 2$; $N+2-4=2 \Rightarrow N=4$

A8. $x(t) = \sin(\omega t)$ Insignal
$y(t) = A\sin(\omega t - \phi) = A\sin(\omega(t-t_0))$ Utsignal

$\phi = \frac{\pi}{8}$; $t_0 = 250\,\mu s$
$\omega t_0 = \phi$ $\qquad \omega = \dfrac{\phi}{t_0} = \dfrac{\pi}{8\cdot 250\cdot 10^{-6}} = 500\pi$ rad/s

A9. $\dfrac{f_0}{f_s} = \dfrac{k}{N}$ $\qquad k=20$, $N=2^7=128$
$f_s = \dfrac{1}{T} = \dfrac{1}{50\cdot 10^{-6}}$

$f_0 = \dfrac{k}{N\cdot T} = \dfrac{20}{128\cdot 50\cdot 10^{-6}} = 3125$ Hz

(Dock med en osäkerhet på $\pm\Delta f = \dfrac{f_s}{N} = 156$ Hz)

A10. $H(s) = \dfrac{10}{s^2+sa+8}$

Krav: Poler med negativ realdel (i VHP)

$s_{1,2} = -\dfrac{a}{2} \pm \underbrace{\sqrt{\dfrac{a^2}{4}-8}}_{<\frac{a}{2} \text{ om } a>\sqrt{32} \text{ (reella poler)}}$ $\qquad$ Svar: $\underline{\underline{a>0}}$

Imag. om $a<\sqrt{32}$

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B11. $h(t) = \left(e^{-t} - e^{-2t}\cos(3t)\right) u(t)$

Laplacetransf.

a) $H(s) = \dfrac{1}{s+1} - \dfrac{s+2}{(s+2)^2+9} = \dfrac{(s^2+4s+4+9)-(s+1)(s+2)}{(s+1)(s^2+4s+4+9)} =$

$= \dfrac{s^2+4s+13-s^2-3s-2}{s^3+4s^2+13s+s^2+4s+13} = \dfrac{s+11}{s^3+5s^2+17s+13}$

b) $H(s) = \dfrac{Y(s)}{X(s)}$

$Y(s)(s^3+5s^2+17s+13) = X(s)(s+11)$

Differentialekv.

$\dfrac{d^3y(t)}{dt^3} + 5\dfrac{d^2y(t)}{dt^2} + 17\dfrac{dy(t)}{dt} + 13y(t) = \dfrac{dx(t)}{dt} + 11x(t)$

c) Frekvenssvar $|H(j\omega)|$

$H(j\omega) = \dfrac{j\omega+11}{(j\omega)^3+5(j\omega)^2+17j\omega+13} = \dfrac{j\omega+11}{-j\omega^3-5\omega^2+j17\omega+13}$

$H(j\omega) = \dfrac{j\omega+11}{(13-5\omega^2)+j(17\omega-\omega^3)}$

$|H(j\omega)| = \dfrac{\sqrt{\omega^2+121}}{\sqrt{(13-5\omega^2)^2+(17\omega-\omega^3)^2}}$

Alternativt

$H(j\omega) = \dfrac{j\omega+11}{(j\omega+1)(-\omega^2+j4\omega+13)}$

$|H(j\omega)| = \dfrac{\sqrt{\omega^2+121}}{\sqrt{\omega^2+1}\cdot\sqrt{(13-\omega^2)^2+(4\omega)^2}}$

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B12. $H_1(z) = \dfrac{z+0,5}{z-0,5}$

$h_2[n] = 2(0,5)^n u[n] - \delta[n]$

$H_2(z) = \mathcal{Z}\{h_2[n]\} = \dfrac{2z}{z-0,5} - 1 =$

$= \dfrac{2z-(z-0,5)}{z-0,5} = \dfrac{z+0,5}{z-0,5}$

$H_1(z) = H_2(z)$

$H(z) = H_1(z) - H_2(z) = 0$

Alltså $y[n] = 0$, $\forall n$

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B13 $y[n] - 0{,}8\,y[n-1] = x[n]$

z-transformera

$$Y(z) - 0{,}8\,z^{-1}Y(z) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-0{,}8z^{-1}}$$

Frekvenssvar, $\boxed{z = e^{j\Omega}}$,

$$H(e^{j\Omega}) = \frac{1}{1-0{,}8e^{-j\Omega}} = \frac{1}{1-0{,}8(\cos\Omega - j\sin\Omega)} =$$

$$= \frac{1}{1-0{,}8\cos\Omega + j0{,}8\sin\Omega}$$

$$|H(e^{j\Omega})| = \frac{1}{\left[(1-0{,}8\cos\Omega)^2 + (0{,}8\sin\Omega)^2\right]^{1/2}} =$$

$$= \frac{1}{\left[1-1{,}6\cos\Omega + 0{,}8^2\cos^2\Omega + 0{,}8^2\sin^2\Omega\right]^{1/2}} =$$

$$= \frac{1}{\sqrt{1{,}64-1{,}6\cos\Omega}}$$

$$\arg\{H(e^{j\Omega})\} = -\arctan\left(\frac{0{,}8\sin\Omega}{1-0{,}8\cos\Omega}\right)$$

Insignal: $x[n] = \cos(\Omega n)$ med $\Omega = \frac{\pi}{6}$

Amplitudpåverkan $|H(e^{j\Omega})|_{\Omega=\frac{\pi}{6}} = \frac{1}{\sqrt{1{,}64-1{,}6\cos(\frac{\pi}{6})}} \approx 1{,}98$

Fas: $-\arctan\left(\frac{0{,}8\sin(\pi/6)}{1-0{,}8\cos(\pi/6)}\right) = -0{,}92$

Svar: $y[n] = 1{,}98\cos\left(\frac{\pi}{6}n - 0{,}92\right)$