SS4080 - D3
2019-10-31

A1 $x_1[n]$ - B

En period sinusformad signal i intervallet $n = [0, N-1]$
$|X[k]|$ max vid $k=1$ och $N-1 = 15$

$x_2[n]$ - A

Också en period i intervallet $n = [0, N-1]$
Fyrkant ger övertoner vid udda k
$|X[k]|$max vid $k=1$ men bidrag också vid $k = 3, 5, 7$ ..

$x_3[n]$ - C

Ser ut som fem upprepningar (perioder) i intervallet $n = [0, N-1]$
$|X[k]|$max vid $k=5$

$x_4[n]$ - D

Två upprepningar (perioder) i intervallet, $|X[k]|$max vid $k=2$

A2. $h[n] = \delta[n] + \delta[n-2] + \delta[n-3]$

Insignal $x[n] = u[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + \ldots$

$y[n] = h[n] + h[n-1] + h[n-2] + h[n-3] + \ldots$

n	0	1	2	3	4	5	6			
h[n]	1		1	1						
h[n-1]		1		1	1					
h[n-2]			1		1	1				
h[n-3]				1		1	1			
h[n-4]					1		1	1		
h[n-5]						1		1	1	
							1		1	1

y[n]	1	1	2	3	3	3	3	......

↑ $y[4] = 3$

SSY080 - D3

2019-10-31

A3

$$H(s) = \frac{K}{(s+4)^2} \quad ; \quad h(t) = \mathcal{L}^{-1}\{H(s)\} = K \cdot t e^{-4t} \cdot u(t)$$

$$h(t)\Big|_{t=0} = 0 \qquad h(t)\Big|_{t\to\infty} = 0 \qquad h(t) \geq 0 \ \ \forall t$$

Svar: d

A4.

Sök nollställe till täljaren

Frekvenssvar: $s = j\omega$

$(j\omega)^2 + 4{,}0 \cdot 10^6 = 0$

$-\omega^2 + 4{,}0 \cdot 10^6 = 0 \quad ; \quad \omega = \sqrt{4{,}0 \cdot 10^6} = \pm 2 \cdot 10^3$

Svar: $\omega = 2 \cdot 10^3$ rad/s

A5.

$$X(z) = \frac{1 + 3z^{-1} + 2z^{-2}}{1 + z^{-1}} = \frac{z^2 + 3z + 2}{z^2 + z} = \{\text{Faktorisera}\}$$

$$= \frac{(z+1)(z+2)}{z(z+1)} = \frac{z+2}{z} = 1 + 2 \cdot z^{-1}$$

Invers z-transform ger $x[n] = \delta[n] + 2\delta[n-1]$

A6

$h(t) = e^{-4(t-1)} u(t) = e^4 \cdot e^{-4t} \cdot u(t)$

$x(t) = u(t) \qquad y(t) = h(t) * u(t) = e^4 \int^{\infty} e^{-4\tau} u(t-\tau)\, d\tau =$

$$= e^4 \int_0^t e^{-4\tau}\, d\tau = e^4 \left[\frac{e^{-4\tau}}{-4} \right]_0^t =$$

$$= \frac{e^4}{4}\left(1 - e^{-4t}\right), \ t \geq 0 \quad \text{Slutvärde } \frac{e^4}{4}$$

Halva slutvärdet ger $e^{-4t} = \frac{1}{2} \Rightarrow e^{4t} = 2$

$4t = \ln 2$ och $\boxed{t = \frac{\ln 2}{4}}$

SSY080 - D3

2019-10-31

A7 $y(t) = A\,x(t-t_0) = A\sin(5000\pi(t-t_0)) =$

$= A\sin(5000\pi t - 5000\pi \cdot t_0)$

$\phi = -5000\pi \cdot t_0 = -5000\pi \cdot 20 \cdot 10^{-6} = -0{,}1\pi = \frac{-\pi}{10}$

A8 $\frac{k}{N} = \frac{f}{f_s}$; $f_s = \frac{1}{T}$

$k = f \cdot T \cdot N = 100 \cdot 6{,}25 \cdot 10^{-3} \cdot 800 = 500$

Reell signal: Bidrag även i $N-k = 300$ (Aliasing!)

Svar: $k = 300$ och 500

A9. $x(t) = \sin(\omega t)$ samplas $t = n \cdot T$

$x[n] = \sin(\omega n T) = \sin(\omega T n) = \sin(\Omega n)$

$\Omega = \omega \cdot T = 2\pi f \cdot T = 2\pi \cdot 100 \cdot 6{,}25 \cdot 10^{-3} = 1{,}25\pi$

Svar: $\Omega = 1{,}25\pi$

A10. Täljarpolynom ordn. $M = 3$

Nämnarpolynom ordn. $N = 6$

Lutning $\{\omega \to \infty\}$ $(M-N) \cdot 20\,dB/dekad =$

$= -60\,dB/dekad$

SSY080 - D3
2019-10-31

B11 $y(t) = \left(1{,}8 + 0{,}2e^{-50t} - 2{,}0e^{-10t}\right)u(t)$

$$Y(s) = \mathcal{L}\{y(t)\} = \frac{1{,}8}{s} + \frac{0{,}2}{s+50} - \frac{2}{s+10} =$$

$$= \frac{1{,}8(s+50)(s+10) + 0{,}2s(s+10) - 2s(s+50)}{s(s+50)(s+10)} =$$

$$= \frac{s^2(1{,}8 + 0{,}2 - 2) + s(60 \cdot 1{,}8 + 0{,}2 \cdot 10 - 2 \cdot 50) + (1{,}8 \cdot 500)}{s(s+50)(s+10)}$$

$$= \frac{10s + 900}{s(s+50)(s+10)} = \frac{1}{s} \cdot H(s) \quad \text{ty } X(s) = \frac{1}{s}$$

$$H(s) = \frac{10s + 900}{(s+50)(s+10)} = \frac{A}{s+50} + \frac{B}{s+10}$$

$$10s + 900 = A(s+10) + B(s+50)$$

$s = -10 \Rightarrow \quad -100 + 900 = B \cdot 40 \quad \Rightarrow B = \frac{800}{40} = 20$

$s = -50 \Rightarrow \quad -500 + 900 = A(-40) \quad \Rightarrow A = -\frac{400}{40} = -10$

$$H(s) = \frac{20}{s+10} - \frac{10}{s+50}$$

$$h(t) = \mathcal{L}^{-1}\{H(s)\} = \left(20e^{-10t} - 10e^{-50t}\right)u(t)$$

Kan även beräknas som $h(t) = \frac{d}{dt}\{y(t)\}$

SSY080 - D3
2019-10-31

B12

$$h[n] = \left(8(0{,}2)^n - 6(-0{,}8)^n \right) u[n]$$

$$H(z) = \mathcal{Z}\{h[n]\} = 8 \cdot \frac{z}{z-0{,}2} - 6\frac{z}{z+0{,}8} =$$

$$= z\left(\frac{8(z+0{,}8) - 6(z-0{,}2)}{(z-0{,}2)(z+0{,}8)} \right) = z \frac{2z+7{,}6}{(z-0{,}2)(z+0{,}8)}$$

$$x[n] = 2(0{,}4)^n u[n] \xrightarrow{\mathcal{Z}} X(z) = 2\frac{z}{z-0{,}4}$$

$$Y(z) = H(z)\cdot X(z) = z\left(\frac{2z(2z+7{,}6)}{(z-0{,}4)(z-0{,}2)(z+0{,}8)} \right)$$

$$\frac{Y(z)}{z} = \frac{2(2z^2+7{,}6z)}{(z-0{,}4)(z-0{,}2)(z+0{,}8)} = \frac{A}{z-0{,}4} + \frac{B}{z-0{,}2} + \frac{C}{z+0{,}8}$$

$$2(2z^2+7{,}6z) = A(z-0{,}2)(z+0{,}8) + B(z-0{,}4)(z+0{,}8) + C(z-0{,}4)(z-0{,}2)$$

$$z=0{,}4 \Rightarrow 2(2\cdot 0{,}4^2 + 7{,}6\cdot 0{,}4) = A(0{,}2)(1{,}2) \Rightarrow A = 6{,}72/0{,}24 = 28$$

$$z=0{,}2 \Rightarrow 2(2\cdot 0{,}2^2 + 7{,}6\cdot 0{,}2) = B(-0{,}2)(1) \Rightarrow B = 3{,}2/-0{,}2 = -16$$

$$z=-0{,}8 \Rightarrow 2(2\cdot 0{,}8^2 - 7{,}6\cdot 0{,}8) = C(-1{,}2)(-1) \Rightarrow C = -9{,}6/1{,}2 = -8$$

$$Y(z) = 28\cdot\frac{z}{z-0{,}4} - 16\frac{z}{z-0{,}2} - 8\frac{z}{z+0{,}8}$$

$$y[n] = \mathcal{Z}^{-1}\{Y(z)\} = \left[28(0{,}4)^n - 16(0{,}2)^n - 8(-0{,}8)^n\right]u[n]$$

SSY080 - D3
2019-10-31

B13 Komplex Fourierserie

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \quad ; \text{ vi har } \omega_0 = \frac{\pi}{2}$$

$$c_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} \, dt \quad \left(\begin{array}{l}\text{integration}\\ \text{över en period}\end{array}\right)$$

T_0 : signalens fundamentala period $T_0 = \frac{2\pi}{\omega_0}$
$T_0 = B - 1$

$k=0$ $$c_0 = \frac{1}{T_0} \int_{T_0} x(t)\,dt = \frac{1}{B-1} \int_1^2 A\,dt =$$

$$= \frac{A}{B-1} = 1 \quad (\text{signalens medelvärde})$$

$$T_0 = \frac{2\pi}{\omega_0} \Rightarrow B - 1 = \frac{2\pi \cdot 2}{\pi} = 4$$

$A = B - 1 = 4$

Svar: $A = 4$
$B = 5$