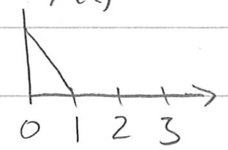
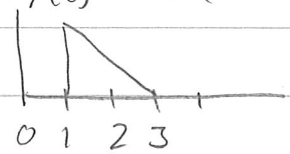
Transformer, Signaler & System

D3, SS4080 2019-08-28

A1.

$x(t)$ $\qquad\qquad$ $y(t) = x(at+b)$





$x(0) \longleftrightarrow y(1) \qquad 0 = a\cdot 1 + b \qquad a = -b$

$x(1) \longleftrightarrow y(3) \qquad 1 = a\cdot 3 + b \qquad a = \frac{1}{2}$

$b = -\frac{1}{2}$

$$y(t) = x\left(\tfrac{1}{2}t - \tfrac{1}{2}\right) = x\left(\tfrac{1}{2}(t-1)\right)$$

$$a = \frac{1}{2}, \quad b = -\frac{1}{2}$$

A2.

$$x[n+N_0] = x[n] = e^{jn\Omega_0} \qquad j\Omega_0 = \frac{5\pi}{13}$$

$$x[n+N_0] = e^{j(n+N_0)\Omega_0} = \underbrace{e^{jn\Omega_0}}_{x[n]} \cdot \underbrace{e^{jN_0\Omega_0}}_{=1}$$

$$N_0 \cdot \Omega_0 = k \cdot 2\pi$$

$$N_0 = \frac{k\,2\pi}{\Omega_0} = \frac{k\,2\pi \cdot 13}{5\pi} = \frac{k\,26}{5}$$

N_0 minsta heltal för $k=5$ $\Rightarrow$ $N_0 = 26$

A3.

Uppgiften på tesen var ofullständigt formulerad.

Uppgiften utgår. Alla ges 1p.

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A4 $x(t) = \cos(t)\cdot\delta(t-\frac{\pi}{4}) = \cos(\frac{\pi}{4})\cdot\delta(t-\frac{\pi}{4}) =$

$= \frac{1}{\sqrt{2}}\,\delta(t-\frac{\pi}{4})$

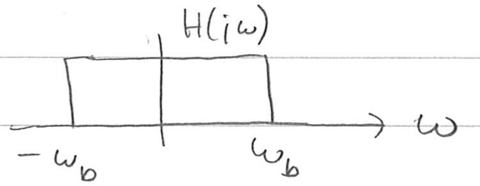
$$\delta(t) \xleftrightarrow{FT} 1$$

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

$$x(t-t_0) \xleftrightarrow{FT} e^{-j\omega t_0} X(j\omega)$$

Alltså: vårt $x(t) \xleftrightarrow{FT} \frac{1}{\sqrt{2}} e^{-j\omega\frac{\pi}{4}}$

A5 $h(t) = \frac{\sin(\omega_b t)}{\pi t} \xleftrightarrow{FT} H(j\omega) = \begin{cases} 1, & |\omega| < \omega_b \\ 0, & |\omega| > \omega_b \end{cases}$



$\omega_b = \frac{16\pi}{3T}$

Signal frekvenser: $\omega_1 = \frac{2\pi}{T}$, $\omega_2 = \frac{220}{4\pi T}$, $\omega_3 = \frac{3\pi^2}{2T}$

Jämför

x_1: $\frac{\omega_1}{\omega_b} = \frac{2\pi\ 3T}{T\cdot 16\pi} = \frac{6}{16} < 1$ Passerar

x_2: $\frac{\omega_2}{\omega_b} = \frac{220\ \ 3T}{4\pi T\cdot 16\pi} = \frac{220\cdot 3}{4\cdot 16\cdot \pi^2} \approx 1{,}04 > 1$ Passerar ej

x_3: $\frac{\omega_3}{\omega_b} = \frac{3\pi^2\cdot 3T}{2T\cdot 16\pi} = \frac{9\pi}{32} \approx 0{,}88 < 1$ Passerar

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A6.

$$H(z) = \frac{z}{(z-2)(z-1)} = \frac{z}{z^2-3z+2}$$

$$x[n] = \delta[n] - 3\delta[n-1] + 2\delta[n-2]$$

$$X(z) = \mathcal{Z}\{x[n]\} = 1 - 3z^{-1} + 2z^{-2} = \frac{z^2-3z+2}{z^2}$$

$$Y(z) = H(z) \cdot X(z) = \frac{z}{z^2-3z+2} \cdot \frac{z^2-3z+2}{z^2} = \frac{1}{z} = z^{-1}$$

$$y[n] = \mathcal{Z}^{-1}\{Y(z)\} = \delta[n-1]$$

A7 Begränsa signalens (iii) Bandbredd.

A8

$$H(s) = \frac{bs}{s+a} \quad ; \quad x(t) = u(t) \xleftrightarrow{\mathcal{L}} X(s) = \frac{1}{s}$$

$$Y(s) = X(s) \cdot H(s) = \frac{1}{s} \cdot \frac{bs}{s+a} = \frac{b}{s+a}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = be^{-at} \cdot u(t) = \frac{b}{e}$$

$$\frac{b}{e} = be^{-at} \Rightarrow e^{-1} = e^{-at} \Rightarrow at = 1$$

Svar: $t = \frac{1}{a}$

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A9 $y[n] + 0{,}5\, y[n-1] = 1{,}5\, x[n]$

$$Y(z)\left(1 + 0{,}5 z^{-1}\right) = 1{,}5\, X(z)$$

$$H(z) = \frac{1{,}5}{1 + 0{,}5 z^{-1}}$$

Frekvenssvar: $z = e^{j\Omega}$

$$H(e^{j\Omega}) = \frac{1{,}5}{1 + 0{,}5 e^{-j\Omega}}$$

Stämmer med

$$H(e^{j\Omega})\Big|_{\Omega=0} = \frac{1{,}5}{1+0{,}5} = 1$$ 2, 3

$$H(e^{j\Omega})\Big|_{\Omega=\pi} = \frac{1{,}5}{1+0{,}5 e^{-j\pi}} = \frac{1{,}5}{1-0{,}5} = 3$$ 2

Svar: Kurva nr 2

A10 $H(s) = \dfrac{K s^2}{(s+100)^n}$; $H(j\omega) = \dfrac{-K\omega^2}{(j\omega + 100)^n}$

Låga frekvenser $\omega \ll 100$ $|H(j\omega)|$ stiger med ω^2
+40 dB/dekad

Höga frekvenser $\omega \gg 100$ $|H(j\omega)|$ sjunker med -60 dB/dekad
som svarar mot $\frac{1}{\omega^3}$

Då måste $n = 2 + 3 = 5$

Fas ändras från 180 till $-270°$ vilket är

$180 + 270 = n \cdot 90° \Rightarrow n = 5$

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B11

$$h(t) = 10e^{-10t}u(t) \xleftrightarrow{\mathcal{L}} H(s) = \frac{10}{s+10}$$

$$x(t) = \cos(\omega_0 t)u(t) \xleftrightarrow{\mathcal{L}} X(s) = \frac{s}{s^2+\omega_0^2} \quad ; \omega_0 = 4\pi\ r/s$$

$$Y(s) = H(s)X(s) = \frac{10}{s+10} \cdot \frac{s}{s^2+\omega_0^2} = \frac{A}{s+10} + \frac{Bs+C}{s^2+\omega_0^2}$$

$$10s = A(s^2+\omega_0^2) + \overbrace{(Bs+C)(s+10)}^{Bs^2+(10B+C)s+10C}$$

$s^2: \quad 0 = A+B$	$A = -B$	$0 = A(\omega_0^2+10^2)+10^2$
$s^1: \quad 10 = 10B+C$	$C = 10(1-B)$	$A = -\frac{100}{\omega_0^2+100} = -B$
$s^0: \quad 0 = A\omega_0^2+10C$	$0 = A\omega_0^2+10^2(1+A)$	

$$C = 10\left(1-\frac{100}{\omega_0^2+100}\right) \approx 6{,}12 \qquad B \approx 0{,}388$$

$$y(t) = Ae^{-10t} + B\frac{s}{s^2+\omega_0^2} + \frac{C}{\omega_0}\cdot\frac{\omega_0}{s^2+\omega_0^2} =$$

$$= \left[-0{,}388\left(e^{-10t} - \cos\omega_0 t\right) + 0{,}487\sin\omega_0 t\right]u(t)$$

Kan skrivas om som

$$B\cos(\omega_0 t) + \frac{C}{\omega_0}(\sin\omega_0 t) = \sqrt{B^2+\left(\frac{C}{\omega_0}\right)^2}\cdot\sin(\omega_0 t+\varphi)$$

med $\varphi = \arcsin\frac{B}{\sqrt{B^2+\left(\frac{C}{\omega_0}\right)^2}} = 38{,}5^\circ$

och $y(t) = \left[0{,}623\cdot\sin\left(\omega_0 t+38{,}5^\circ\right) - 0{,}388\cdot e^{-10t}\right]u(t)$

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B12. $y[n] + 0{,}6\,y[n-1] - 0{,}16\,y[n-2] = x[n-1] + 0{,}5\,x[n-2]$

z-transf.

$$Y(z)\left(1 + 0{,}6z^{-1} - 0{,}16z^{-2}\right) = X(z)\left(z^{-1} + 0{,}5z^{-2}\right)$$

$$H(z) = \frac{z^{-1} + 0{,}5z^{-2}}{1 + 0{,}6z^{-1} - 0{,}16z^{-2}} = \frac{z + 0{,}5}{z^2 + 0{,}6z - 0{,}16} = \frac{Y(z)}{X(z)}$$

Poler: $z_{1,2} = -0{,}3 \pm \sqrt{0{,}3^2 + 0{,}16} = -0{,}3 \pm 0{,}5 = \begin{cases} 0{,}2 \\ -0{,}8 \end{cases}$

$$H(z) = \frac{z + 0{,}5}{(z - 0{,}2)(z + 0{,}8)}$$

$y[n] = \left[0{,}2^n - (-0{,}8)^n\right] u[n]$ z-transf.

$$Y(z) = \frac{z}{z - 0{,}2} - \frac{z}{z + 0{,}8} = z \cdot \frac{z + 0{,}8 - z + 0{,}2}{(z - 0{,}2)(z + 0{,}8)}$$

$$X(z) = \frac{Y(z)}{H(z)} = \frac{z}{(z - 0{,}2)(z + 0{,}8)} \cdot \frac{(z - 2)(z + 0{,}8)}{(z + 0{,}5)} =$$

$$= \frac{z}{z + 0{,}5}$$

$$x[n] = \mathcal{Z}^{-1}\left\{X(z)\right\} = (-0{,}5)^n\, u[n]$$

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B13

$T = 1{,}25$ ms $\quad \omega_s = \frac{2\pi}{T} = \frac{2\pi}{1{,}25 \cdot 10^{-3}} = 800 \cdot 2\pi$ rad/s

$f_s = 800$ Hz

$\frac{f}{f_s} = \frac{k}{N} \Rightarrow k = \frac{f}{f_s} \cdot N \qquad N = 64$

X_1: $f = 210$ Hz $\quad k = \frac{210}{800} \cdot 64 = 16{,}8 \approx \underline{17} \quad N-k \approx \underline{47}$

X_2: $f = 270$ Hz $\quad k = \frac{270}{800} \cdot 64 = 21{,}6 \approx \underline{22} \quad N-k \approx \underline{42}$

X_3: $f = 555$ Hz $\quad k = \frac{555}{800} \cdot 64 \approx 44.4 \approx \underline{44} \quad N-k \approx \underline{20}$.

OBS! Aliasing.

Jämför k och N-k värden med "toppar" i $|X[k]|$ figur.

			$\underline{k}$
Svar:	X_1 —	B	[22, 42]
	X_2 —	A	[17, 47]
	X_3 —	C	[20, 44]